

## Some Graph Parameters on the Composite Order Cayley Graph

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**ABSTRACT.** In this paper, the composite order Cayley graph is introduced. It is denoted by  $Cay(G, S)$ , where  $G$  is a group and  $S$  is the set of all composite order elements of  $G$ . Some graph parameters such as diameter, girth, clique number, independence number, vertex chromatic number and domination number are calculated for the composite order Cayley graph of some certain groups. Moreover, the planarity of composite order Cayley graph is discussed.

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### 1. INTRODUCTION

Arthur Cayley defined the Cayley graph for the first time which encodes the abstract structure of a group. Suppose that  $G$  is a group and  $S$  is a generating set. The Cayley graph  $\Gamma = \Gamma(G, S)$  is a colored directed graph with the vertex set  $V(\Gamma)$  which is identified with the elements of  $G$ , for each generator  $s$  of  $S$  is assigned a color  $c_s$ . Moreover, the vertices corresponding to the elements  $g$  and  $gs$  are joined by a directed edge of color  $c_s$ , for any  $g \in G$ ,  $s \in S$ . Thus the edge set  $E(\Gamma)$  consists of pairs of the form  $(g, gs)$ , with  $s \in S$  providing the color. The set  $S$  is usually assumed to be finite, symmetric  $S = S^{-1}$  and the identity element of the group is excluded  $S$ . In this paper, we consider simple Cayley graph.

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The uncolored Cayley graph is a simple undirected graph. In general, we can assume  $S$  as a subset of non-identity elements  $G$  instead of being a generating set.

The research about the Cayley graph have been done by many authors for instance see [2, 5]. Recently, the author discussed about the prime order Cayley graph in [6]. The prime order Cayley graph is associated to the group  $G$  such that  $S$  is the set of prime order elements of  $G$ . The prime order Cayley graph is the inspiration of defining of the composite order Cayley graph. In this new graph consider  $S$  as the set of elements of the group of composite order. This graph is complement of the prime order Cayley graph. Let us denote the composite order Cayley graph by  $Cay(G, S)$ . Under this assumption we may treat  $Cay(G, S)$  as an undirected graph because  $S = S^{-1}$ . Moreover as  $S$  does not contain the identity, so that  $Cay(G, S)$  does not have any loop.

Apparently the definition of the prime order Cayley graph is similar to the composite order Cayley graph, but note that these two graphs are completely different as the set  $S$  changed. There are some parameters for a graph, that can not achieve by discussing about them in the complement of the graph. We discuss about the general properties of the composite order Cayley graph of cyclic groups of different orders. We try to find the degree of vertices in this graph. Since the composite order Cayley graph is  $|S|$ -regular, degree of a vertex in this graph shows the number of elements of composite order. The structure of composite order Cayley graph of a cyclic group of order  $p^n$  is completely clarify, where  $p$  is a prime number. Furthermore, we prove that if the composite order Cayley graph  $Cay(G, S)$  is planar, then  $|S| = 0, 2$  or  $4$ . The composite order Cayley graph associated to the group  $G$  is planar if and only if  $G \cong \mathbb{Z}_4, \mathbb{Z}_6, S_3, D_8, \mathbb{Z}_2 \times D_8, D_{2p}, A_4$ , a group whose cyclic subgroups are of order  $4, 6$  or  $p$  and a group whose elements are of prime order, where  $p$  is a prime number.

Throughout the paper, graphs are simple and all the notations and terminologies about the graphs are found in [1, 4].

## 2. PRELIMINARY RESULTS

Let us start with the definition of composite order Cayley graph.

**Definition 2.1.** The composite order Cayley graph which is assigned to the group  $G$  is a graph with vertex set whole elements of the group  $G$  and two distinct vertices  $x$  and  $y$  are adjacent whenever  $xy^{-1} \in S$ , where  $S$  is a subset of  $G$  which contains all elements of composite order. We denote the composite order Cayley graph by  $Cay(G, S)$ .

It is clear that the composite order Cayley graph of cyclic group of order  $t$  is connected and Hamiltonian, where  $t$  is not a prime number.

**Theorem 2.2.** *Let  $\mathbb{Z}_t$  be a cyclic group of order  $t$ ,  $x$  a vertex and  $\varphi(t)$  is the Eulerian function.*

- (i) *If  $t$  is a prime number, then  $\text{Cay}(\mathbb{Z}_t, S)$  is an empty graph.*
- (ii) *If  $t = p^\alpha$ , then  $\deg(x) = \varphi(t) + \sum_{\alpha-\beta \geq 2} \varphi(p^{\alpha-\beta}) = p^\alpha - p$ , where  $\beta < \alpha$  is a positive integer.*
- (iii) *If  $t = p_1 p_2$ , then  $\deg(x) = \varphi(t)$ , where  $x$  is a vertex and  $p_i$ ,  $i = 1, 2$  are distinct prime numbers.*
- (iv) *If  $t = \prod_{i=1}^l p_i$ , where  $p_i, p_j$  are distinct prime numbers for  $i \neq j$  and  $l > 2$ , then*

$$\deg(x) = \varphi(t) + \sum_{j=2}^{l-1} \binom{M}{j},$$

where  $M$  is the set of all prime numbers which divides  $t$  and the notation  $\binom{M}{j}$  stands for the sum of the Eulerian function of multiplication of  $j$  prime numbers belongs to  $M$  which are chosen randomly.

- (v) *If  $t = \prod_{i=1}^l p_i^{\alpha_i}$ , where  $p_i, p_j$  are distinct prime numbers for  $i \neq j$ ,  $1 < \alpha_i$  for some  $i$  and  $l > 2$ , then*

$$\deg(x) = \sum_{j=2}^l \binom{M}{j},$$

where  $M$  is the set of all power of prime numbers which divides  $t$  and the notation  $\binom{M}{j}$  stands for the sum of the Eulerian function of multiplication of  $j$  power of prime numbers belongs to  $M$  which are chosen randomly.

*Proof.* (i) Suppose  $m$  and  $n$  are distinct adjacent vertices in the graph. The definition of the composite order Cayley graph implies that the order of  $n - m$  is a composite but it is not possible. Thus  $m$  and  $n$  are not adjacent.

(ii) If  $(p^\alpha, n - m) = 1$  or  $p^\beta$ , then  $n$  and  $m$  are adjacent, where  $\alpha - \beta \geq 2$ . Therefore, assume  $m$  is a fixed vertex, there are  $\varphi(t)$  vertex which are adjacent to  $m$  for the first case. Moreover, for the second case as we have  $n - m = p^\beta k$ , there are  $\varphi(p^{\alpha-\beta})$  vertices join to  $m$ , where  $k$  is an integer  $(k, p^{\alpha-\beta}) = 1$ .

(iii) Clearly, if the order of  $n - m$  is composite if and only if  $m$  and  $n$  are adjacent. This will occur whenever  $(t, n - m) = 1$ . Hence the number of vertices which are adjacent to a vertex like  $m$  is  $\varphi(t)$ .

(iv) Similar to the second part, two vertices  $m$  and  $n$  are adjacent if and only if  $(t, n - m) = 1$  or  $(t, n - m) = \prod_{i=1}^\gamma p_i$ , where  $1 \leq \gamma \leq l - 2$ .

Hence the result is clear.

(v) It is enough to count the number of elements of composite order.  $\square$

In fact the degree of vertices in this graph shows the number of elements of the group of composite order. Furthermore, as degree of vertices are even number the composite graph  $\text{Cay}(\mathbb{Z}_t, S)$  is Eulerian graph, where  $t$  is a composite number.

**Theorem 2.3.** *Let  $\mathbb{Z}_t$  be a cyclic group of order  $t$ , where  $p, p_i$  are prime numbers.*

- (i) *If  $t = \prod_{i=1}^n p_i$ , then  $\text{diam}(\text{Cay}(\mathbb{Z}_t, S)) = 3$ , where  $p_i$ 's are distinct prime numbers  $1 \leq i \leq n$ .*
- (ii) *If  $t = p^\alpha$ , then  $\text{diam}(\text{Cay}(\mathbb{Z}_t, S)) = 2$ ,  $1 < \alpha$ .*
- (iii) *If  $t = \prod_{i=1}^n p_i^{\alpha_i}$ , then  $\text{diam}(\text{Cay}(\mathbb{Z}_t, S)) = 2$ ,  $1 < \alpha_i$  for some  $i$  and  $p_i$ 's are distinct prime numbers.*

*Proof.* (i) Suppose  $x$  and  $y$  is two vertex such that they are not adjacent. Moreover  $x$  and  $y$  are of prime and composite order respectively. Since  $x$  and  $y$  does not join  $|x - y| = p_j$ ,  $1 \leq j \leq n$ . Therefore  $x = p_1 \cdots p_{j-1} p_{j+1} \cdots p_n k + y$ ,  $1 \leq k < p_j$ . Consequently  $|x - 1|$  is a composite number. Thus  $x, x - 1, 0, y$  form a path which means  $d(x, y) = 3$ . If  $x, y$  are two non-adjacent vertices of prime orders, then we deduce that  $|x| = |y| = p_j$ ,  $1 \leq j \leq n$ . Similar to the previous argument  $x = p_1 \cdots p_{j-1} p_{j+1} \cdots p_n k + y$ ,  $1 \leq k < p_j$  which implies  $|x - 1 - y|$  is a composite number. Hence  $x, x - 1, y$  form a path and  $d(x, y) = 2$ .

(ii) Assume  $x, y$  are two non-adjacent vertices. Clearly there is an element of order  $p$  in this group and so  $x - y = kp^{\alpha-1}$ . Thus  $x - 1, y$  are adjacent and  $d(x, y) = 2$ .

(iii) Suppose  $x, y$  are two non-adjacent vertices. Therefore  $x - y = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_{j-1}^{\alpha_{j-1}} \cdots p_{j+1}^{\alpha_{j+1}} p_n^{\alpha_n} k$ , where  $1 \leq k < p_j$ . By an easy computation we deduce that  $x$  and  $y$  join to  $p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_{j-1}^{\alpha_{j-1}} p_{j+1}^{\alpha_{j+1}} \cdots p_n^{\alpha_n} k + x + 1$ . Hence  $d(x, y) = 2$ .  $\square$

By computation  $\text{girth}(\text{Cay}(\mathbb{Z}_4, S)) = 4$ ,  $\text{girth}(\text{Cay}(\mathbb{Z}_6, S)) = 6$  and  $\text{diam}(\text{Cay}(\mathbb{Z}_6, S)) = 3$ .

**Theorem 2.4.** *Suppose  $\text{Cay}(\mathbb{Z}_t, S)$  is a composite order Cayley graph where  $p$  and  $p_i$ 's are prime numbers such that  $p_i \neq p_j$  for  $i \neq j$ .*

- (i) *If  $t = p^\alpha$ , then  $\text{girth}(\text{Cay}(\mathbb{Z}_t, S)) = 3$ , where  $\alpha > 2$ .*
- (ii) *If  $t = p_1 p_2 \neq 6$ , then  $\text{girth}(\text{Cay}(\mathbb{Z}_t, S)) = 4$ .*
- (iii) *If  $t = \prod_{i=1}^n p_i$ , then  $\text{girth}(\text{Cay}(\mathbb{Z}_t, S)) = 3$ , where  $n \geq 3$ .*
- (iv) *If  $t = \prod_{i=1}^n p_i^{\alpha_i}$ , then  $\text{girth}(\text{Cay}(\mathbb{Z}_t, S)) = 3$ ,  $1 < \alpha_i$  for some  $i$ .*

*Proof.* (i) There are two elements of composite order which are adjacent with zero we can make a triangle.

(ii) Assume  $\varphi(p_1) \geq 2$ . Since the number of elements of order  $p_1$  is  $\varphi(p_1)$ , there are at least 2 elements  $x_1$  and  $x_2$  of order  $p_1$ . Moreover, if  $y$  is an element of order  $p_2$ , then  $x_1 \sim y \sim x_2$ . It is clear that  $x_1 + 1$  is a vertex distinct from  $x_2$  and  $y$  such that it joins to  $x_1$  and  $x_2$ . Hence we have a square.

(iii) Since there are three elements of order  $p_1$ ,  $p_2$  and  $p_3$  we can form a triangle by these three elements.

(iv) Suppose  $i = 2$  and  $x, y$  are adjacent vertices. Therefore  $x - y = kp_1^{\beta_1}p_2^{\beta_2}$ ,  $0 \leq \beta_i < \alpha_i$ ,  $1 \leq k < p_1^{\alpha_1 - \beta_1}p_2^{\alpha_2 - \beta_2}$ . By computation,  $y$  is adjacent to  $k'p_1^{\beta'_1}p_2^{\beta'_2} + y$ ,  $0 \leq \beta'_i < \alpha_i$ ,  $1 \leq k' < p_1^{\alpha_1 - \beta'_1}p_2^{\alpha_2 - \beta'_2}$ . Now if we choose suitable  $k$  and  $k'$ , then  $x$  joins  $k'p_1^{\beta'_1}p_2^{\beta'_2} + y$  and we have a triangle. Moreover, if  $i \geq 3$ , then the assertion is similar to the third part.  $\square$

In a graph a 3-cycle is called a triangle. For the vertices  $a, b, c$  in a graph  $\Gamma$  the triangle formed by the distinct vertices  $a, b, c$  is denoted by  $(a, b, c)$ . The triangle  $(0, a, b)$  is called a fundamental triangle and it is denoted by  $\Delta_{ab}$ . T. Chalapati et al. enumerate the number of fundamental triangles in a divisor Cayley graph (see [2] for more details). By Theorem 2.4 we observe that the composite Cayley graph of class of some cyclic groups has triangle. Similar to the Lemma 3.6 and Theorem 3.7 in [2] we conclude that the number of fundamental triangles and all triangles in the composite Cayley graph  $\text{Cay}(\mathbb{Z}_t, S)$  is

$$\frac{1}{2} \left( \sum_{a \in S} |S \cap (S + a)| \right) \text{ and } \frac{|\mathbb{Z}_t|}{6} \left( \sum_{a \in S} |S \cap (S + a)| \right),$$

respectively.

A subset  $X$  of the vertices of the graph  $\Gamma$  is called a clique if the induced subgraph on  $X$  is a complete graph. The maximum size of a clique in a graph  $\Gamma$  is called the clique number of  $\Gamma$  and denoted by  $\omega(\Gamma)$ . A subset  $X$  of the vertices of  $\Gamma$  is called an independent set if the induced subgraph on  $X$  has no edges. The maximum size of an independent set in a graph  $\Gamma$  is called the independence number of  $\Gamma$  and denoted by  $\alpha(\Gamma)$ . Let  $k > 0$  be an integer. A  $k$ -vertex coloring of a graph  $\Gamma$  is an assignment of  $k$  colors to the vertices of  $\Gamma$  such that no two adjacent vertices have the same color. The vertex chromatic number  $\chi(\Gamma)$  of a graph  $\Gamma$ , is the minimum  $k$  for which  $\Gamma$  has a  $k$ -vertex coloring.

**Theorem 2.5.** *If  $G$  is a cyclic group of order  $p^n$ , then  $\text{Cay}(G, S)$  is complete  $p^{n-1}$ -partite graph. Moreover,  $\omega(\text{Cay}(G, S)) = \chi(\text{Cay}(G, S)) = p^{n-1}$  and  $\alpha(\text{Cay}(G, S)) = p$ . Furthermore,  $\text{Cay}(G, S)$  is not planar whenever  $p \geq 3$  and  $n \geq 2$ .*

*Proof.* Consider the complement of the composite order Cayley graph, then we have  $p^{n-1}$  complete components which contain  $p$  vertices (See [6, Proposition 3.3]). Therefore the assertion follows.  $\square$

For a graph  $\Gamma$  and a subset  $S$  of the vertex set  $V(\Gamma)$ , denote by  $N_\Gamma[S]$  the set of vertices in  $\Gamma$  which are in  $S$  or adjacent to a vertex in  $S$ . If  $N_\Gamma[S] = V(\Gamma)$ , then  $S$  is said to be a dominating set of vertices in  $\Gamma$ . The domination number of a graph  $\Gamma$ , denoted by  $\gamma(\Gamma)$ , is the minimum size of a dominating set of the vertices in  $\Gamma$ .

**Proposition 2.6.** *Consider the composite order Cayley graph of cyclic group of order  $t$ . Then*

- (i) *If  $t = p^\alpha$ , then  $\gamma(\text{Cay}(\mathbb{Z}_t, S)) = p$ .*
- (ii) *If  $t = \prod_{i=1}^n p_i^{\alpha_i}$ , then  $\gamma(\text{Cay}(\mathbb{Z}_t, S)) = p_i$ , where  $p_i$  is the smallest prime number which divides  $t$ .*

*Proof.* (i) In this case the vertices of  $\text{Cay}(\mathbb{Z}_t, S)$  are identity element of the group, elements of order of power of  $p$  and the elements of order  $p$ . All the elements of composite order join the identity element. Moreover, the elements of order  $p$  are non-adjacent. Hence  $\{0, p-1$  elements of order  $p\}$  is a dominating set.

(ii) The vertices are the identity element, elements of composite order and elements of order  $p_i$ ,  $1 \leq i \leq n$ . Since the elements of order  $p_i$  and  $p_j$  are adjacent,  $\{0, p_i-1$  elements of order  $p_i\}$  is a dominating set, where  $p_i$  is the smallest prime number which divides  $t$ .  $\square$

By the above argument we become familiar with the structure of the composite order Cayley graph of a cyclic group. This knowledge will be useful to discuss about the composite order Cayley graph of other groups by their cyclic subgroups.

**Example 2.7.** Let  $D_{2n} = \langle a, b | a^n = b^2 = 1, a^b = a^{-1} \rangle$  be dihedral group of order  $2n$  and  $p, p_i$  are prime numbers  $1 \leq i \leq n$ . By the presentation of  $D_{2n}$  and considering the cyclic subgroup  $\langle a \rangle$  we deduce the following results.

- (i) If  $n = p$  a prime number, then  $\text{Cay}(D_{2n}, S)$  is made of  $2p$  isolated vertices.
- (ii) If  $n = p^\alpha$ , then the powers of  $a, a^i$  and  $a^j$  are adjacent whenever  $j \neq kp^{\alpha-1} + i$ ,  $1 \leq k < p$ . Moreover,  $1$  joins to all powers of  $a$  except  $a^{kp^{\alpha-1}}$  and  $ab^j$  are isolated vertices,  $1 \leq j \leq p^\alpha - 1$ .
- (iii) If  $n = \prod_{i=1}^n p_i^{\alpha_i}$ , then  $a^i$  and  $a^j$  are adjacent whenever  $j \neq kp_1^{\alpha_1} p_2^{\alpha_2} \cdots p_{j-1}^{\alpha_{j-1}} p_{j+1}^{\alpha_{j+1}} \cdots p_n^{\alpha_n} + i$ , where  $1 \leq k < p_j$ . Moreover,  $1$  joins to all powers of  $a$  of composite order and  $ab^j$  are isolated vertices,  $1 \leq j \leq \prod_{i=1}^n p_i^{\alpha_i} - 1$ .

### 3. MAIN RESULTS

In this section we discuss about the planarity of the composite order Cayley graph. K. Kuratowski provided a characterization of planar graphs in terms of forbidden graphs, now known as Kuratowski's Theorem,

A finite graph is planar if and only if it does not contain a subgraph that is a subdivision of  $K_5$  or  $K_{3,3}$ .

In practice, it is difficult to use Kuratowski's criterion to quickly decide whether a given graph is planar.

If  $\Gamma$  is a simple, connected, planar graph with  $v \geq 3$  vertices and  $e$  edges, then  $e \leq 3v - 6$ .

**Lemma 3.1.** *If the connected composite order Cayley graph  $Cay(G, S)$  is planar, then  $|S| = 0, 2$  or  $4$ .*

*Proof.* Since  $Cay(G, S)$  is planar,  $|E(Cay(G, S))| \leq 3|V(Cay(G, S))| - 6$ . Therefore,  $|G||S| \leq 6|G| - 12$  which implies  $|S| \leq 5$ . The number of elements of composite order is clearly even, because each such element can be paired with its inverse. Hence the assertion follows.  $\square$

A finite group having all (non-trivial) elements of prime order if it is a  $p$ -group of exponent  $p$  or a non-nilpotent group of order  $p^n q$  or it is isomorphic to the simple group  $A_5$ , where  $n$  is a non-negative integer and  $p, q$  are prime numbers (see [3]). Therefore the composite order Cayley graphs of such groups are an empty graph which is planar. In the following theorem we present all the groups whose composite Cayley graphs are planar.

**Theorem 3.2.** *The composite order Cayley graph associated to the group  $G$  is planar if and only if  $G \cong \mathbb{Z}_4, \mathbb{Z}_6, S_3, D_8, \mathbb{Z}_2 \times D_8, D_{2p}, A_4$ , a group whose cyclic subgroups are of order 4, 6 or  $p$  and a group of prime order, where  $p$  is a prime number.*

*Proof.* If the order of the group  $G$  is a prime number, then  $Cay(G, S)$  is an empty graph and clearly is planar. Assume  $|G|$  is a composite number. If  $G$  is a cyclic group, then  $|S| \geq \varphi(|G|)$  by Theorem 2.2. Since  $Cay(G, S)$  is planar the Lemma 3.1 implies  $4 \geq \varphi(|G|)$ . Therefore it is enough to consider cyclic groups of order 1, 2, 3, 4, 5, 6, 8, 10, 12. In the sequel, we check the planarity of the composite order Cayley graph of all the groups of order less than 12. If  $G \cong \mathbb{Z}_2 \times \mathbb{Z}_2$  or  $S_3$ , then  $G$  does not have any element of composite order and so its Cayley composite graph is formed by 4 or 6 isolated vertices, respectively. Suppose  $G \cong \mathbb{Z}_6$ . Then  $Cay(\mathbb{Z}_6, S)$  is a cycle of length 6. There are 5 groups of order 8.

The composite order Cayley graph of  $D_8 = \langle a, b | a^4 = b^2 = 1, a^b = a^{-1} \rangle$  is made of a path  $a \sim 1 \sim a^3$  and five isolated vertices  $a^2, b, ab, a^2b, a^3b$  while  $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$  is an empty graph. But the composite order Cayley graph of  $Q_8 = \{\pm i, \pm j, \pm k, \pm 1\}$ ,  $\mathbb{Z}_8$  and  $\mathbb{Z}_4 \times \mathbb{Z}_2$  contain  $K_{3,3}$  as induced subgraph and are not planar. The composite order Cayley graph  $\mathbb{Z}_{10}, \mathbb{Z}_{12}, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3, D_{12}, T = \langle x, y | x^4 = y^3 = 1, yxy = x \rangle$  have the induced subgraph  $K_{3,3}$  so they are not planar. Since the order of elements of  $A_4$  is 1, 2, 3 so its composite order Cayley graph is an empty graph with 12 isolated vertices. For the groups of greater order, it is easy to see that if all of their elements are of prime order, then their composite order Cayley graphs are planar. Thus we consider the groups which contains an element of composite order. If  $D_{2n}$  is a dihedral group of order  $2n$ , then the composite order Cayley graph of this group is not planar, where  $n > 6$  and  $n$  is not a prime number. Because its cyclic subgroup of order  $n$  has at least 3 generators so there exists  $K_{3,3}$  as its induced subgraph.

Let  $G$  be a group. Consequently every cyclic subgroup of  $G$  has at most 4 elements of composite order. Consider the cyclic subgroup  $\langle x \rangle$ . If  $|x| = m$ , then  $m \leq 6$  because otherwise we have  $K_{3,3}$  by vertices  $1, x, x^t, x^s, x^{t+1}, x^{s+1}$  as the induced subgraph of  $\text{Cay}(G, S)$ , where  $x, x^t, x^s$  are the generators of  $\langle x \rangle$ . This means elements of composite order for this group are of order 4 or 6. This group contains cyclic groups of order 4, 6 or  $p$ , where  $p$  is a prime number.  $\square$

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