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# Endpoints of generalized $\phi$-contractive multivalued mappings of integral type

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Abstract. Recently, some researchers have established some results on existence of endpoints for multivalued mappings. In particular, Mohammadi and Rezapour's [Endpoints of Suzuki type quasi-contractive multifunctions, U.P.B. Sci. Bull., Series A, 2015] used the technique of $\alpha$-$\psi$-contractive mappings, due to Samet et al. (2012), to give some results about endpoints of Suzuki type quasi-contractive multifunctions satisfing property (BS). In this paper, we prove existence and uniqueness of endpoint for multivalued mappings satisfing the weaker conditions generalized $\phi$-contractivity of integral type and property (HS). This result generalize and improve Mohammadi and Rezapour's result. Also, we give an example to illustrate the usability of the result.

Keywords: endpoint, $\phi$-contractions, multivalued mappings, property (HS), integral type.

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## 1. Introduction and preliminaries

Let $(X, d)$ be a metric space, $2^X$ the set of all nonempty subsets of $X$ and $CB(X)$ the set of all nonempty closed bounded subsets of $X$.

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Assume that $H$ be the Hausdorff metric on $CB(X)$ defined by

$$H(A,B) = max\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(y,A)\},$$

for all $A, B \in CB(X)$ where $d(x,B) = \inf_{y\in B} d(x,y)$. An element $x \in X$ is said to be an endpoint of $T$ whenever $Tx = \{x\}$. It is said that $T$ has the approximate endpoint property whenever $\inf_{x\in X} \sup_{y\in Tx} d(x,y) = 0$ (see [1]). In 2010, Amini-Harandi [1] proved that some multifunctions have unique endpoint if and only if have approximate endpoint property . Then, Moradi and Khojasteh [7] generalized Amini-Harandi's result for generalized contractive multivalued mappings. Denote by $\Psi$ the family of nondecreasing functions $\psi : [0,\infty) \to [0,\infty)$ such that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for all $t > 0$ ([9]). The technique of $\alpha$-$\psi$-contractive mappings introduced by Samet, Vetro and Vetro in 2012 (see [9]). Then, some authors generalized it in different subjects (see, for example, [3, 5, 8]). It is said that a multifunction $T : X \to CB(X)$ has the property (BS) whenever for each $x \in X$ there exists $y \in Tx$ such that $H(Tx, Ty) = \sup_{b\in Ty} d(y,b)$. Recently, Mohammadi and Rezapour [6] used this technique to give some results about endpoints of Suzuki type quasi-contractive multifunctions satisfing property (BS) without using the approximate endpoint property.

**Theorem 1.1.** *([6]) Let $(X,d)$ be a complete metric space, $\psi \in \Psi$, $\alpha : X \times X \to [0,\infty)$ a mapping and $T : X \to CB(X)$ an $\alpha$-admissible such that $T$ has the property (BS) and $\alpha(x,y)H(Tx,Ty) \le \psi(M(x,y))$ for all $x, y \in X$, where*

$$M(x,y) = max\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty) + d(y,Tx)}{2}\}.$$

*Suppose that there exist $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0,x_1) \ge 1$. Let either $T$ is continuous or $\psi$ is right upper semi-continuous and for each sequence $\{x_n\}$ in $X$ with $\alpha(x_n,x_{n+1}) \ge 1$ for all $n$ and $x_n \to x$, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha(x_{n_k},x) \ge 1$ for all $k$. Then $T$ has an endpoint.*

On the other hand, in 2001, Branciari 's [2] generalized the Banach contraction principle to integral type contractive self-mappings by using an Lebesgue integrable mapping $\varphi : [0,+\infty) \to [0,+\infty)$, which is summable on each compact subset of $[0,+\infty)$, such that $\int_0^{\epsilon} \varphi(t)dt > 0$ for any $\epsilon > 0$. Througout this paper, we denote by $\Phi_1$ the family of all this functions which is bounded on $[0,+\infty)$. Denote by $\Phi_2$ the collection of all continuous nondecreasing functions $\phi : [0,+\infty) \to [0,+\infty)$ such that $\phi(t) < t$ for all $t > 0$. It is well known that $\phi(0) = 0$ and $\lim_{n\to\infty} \phi^n(t) = 0$ for all $t > 0$. Also it is known that for any $\phi \in \Psi$ we have $\phi \in \Phi_2$ but the reverse is not true. We say that a multivalued mapping $T : X \to CB(X)$ has the property (HS) whenever for each $x \in X$ there exists $y \in Tx$ such that $H(Tx, Ty) \geq \sup_{b \in Ty} d(y, b)$. Note that the property (HS) is weaker than the property (BS). Also, we say that $T$ is a generalized $\phi$-contractive multivalued mapping of integral type whenever there exist $\varphi \in \Phi_1$ and $\phi \in \Phi_2$ such that

$$\int_0^{H(Tx,Ty)} \varphi(t)dt \leq \phi(\int_0^{M(x,y)} \varphi(t)dt),$$

for all $x, y \in X$, where

$$M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), \frac{1}{2}[d(x, Ty) + d(y, Tx)]\}.$$

In this paper, we prove existence and uniqueness of endpoint for such mappings having the conditions $\phi$-contractivity and property (HS) which are weaker conditions with respect to Mohammadi and Rezapour's [6].

## 2. Main results

The following theorem is the main result of this study.

**Theorem 2.1.** *Let $(X, d)$ be a complete metric space and $T : X \to CB(X)$ be a generalized $\phi$-contractive multivalued mapping of integral type which has the property (HS). Then $T$ has a unique endpoint.*

*Proof.* Choose a fixe element $x_0 \in X$. Since $T$ has the property (HS), there exists $x_1 \in Tx_0$ such that $H(Tx_0, Tx_1) \geq \sup_{b \in Tx_1} d(x_1, b)$. Continuing this process, we obtain a sequence $\{x_n\}$ such that $x_{n+1} \in Tx_n$ and $H(Tx_n, Tx_{n+1}) \geq \sup_{b \in Tx_{n+1}} d(x_{n+1}, b)$ for all $n \geq 0$. Then we have

$$\begin{aligned} \int_0^{d(x_n,x_{n+1})} \varphi(t)dt &\leq \int_0^{\sup_{b \in Tx_n} d(x_n,b)} \varphi(t)dt \\ &\leq \int_0^{H(Tx_{n-1},Tx_n)} \varphi(t)dt \\ &\leq \phi(\int_0^{M(x_{n-1},x_n)} \varphi(t)dt), \end{aligned} \tag{2.1}$$

for all $n \geq 1$. Note that

$$\begin{aligned} M(x_{n-1}, x_n) &= \max\{d(x_{n-1}, x_n), d(x_{n-1}, Tx_{n-1}) \\ &\quad , d(x_n, Tx_n), \tfrac{1}{2}[d(x_{n-1}, Tx_n) + d(x_n, Tx_{n-1})]\} \\ &\leq \max\{d(x_{n-1}, x_n)), d(x_n, x_{n+1}), \tfrac{1}{2}[d(x_{n-1}, x_{n+1})]\}) \\ &\leq \max\{d(x_{n-1}, x_n)), d(x_n, x_{n+1})\}, \end{aligned} \tag{2.2}$$

for all $n \geq 1$. If $d(x_n, x_{n+1}) > d(x_{n-1}, x_n)$, then, from (2.1) and (2.2), we have

$$\int_0^{d(x_n,x_{n+1})} \varphi(t)dt \leq \phi(\int_0^{d(x_n,x_{n+1})} \varphi(t)dt),$$

which is a contradiction. Hence, $d(x_n,x_{n+1}) \leq d(x_{n-1},x_n)$ and so from (2.1) and (2.2),

$$\int_0^{d(x_n,x_{n+1})} \varphi(t)dt \leq \phi(\int_0^{d(x_{n-1},x_n)} \varphi(t)dt), \tag{2.3}$$

for all $n \geq 1$. From (2.3), we get $\int_0^{d(x_n,x_{n+1})} \varphi(t)dt \leq \phi^n(\int_0^{d(x_0,x_1)} \varphi(t)dt) \to 0$ as $n \to \infty$. From this, we conclude that $d(x_n,x_{n+1}) \to 0$ as $n \to \infty$. We claim that $\{x_n\}$ is a Cauchy sequence. Suppose to the contrary, $\{x_n\}$ is not a Cauchy sequence. Then, there exists $\varepsilon > 0$ such that for any $i \in \mathbb{N}$ there exist natural numbers $m_i, n_i$ with $m_i > n_i$ and $d(x_{n_i}, x_{m_i}) \geq \varepsilon$. Put

$$k_i = \min\{m_i | m_i > n_i, d(x_{n_i}, x_{m_i}) \geq \varepsilon\}.$$

Hence, for any $i \in \mathbb{N}$, we have $d(x_{n_i}, x_{k_i}) \geq \varepsilon$ and $d(x_{n_i}, x_{k_i-1}) < \varepsilon$. Now

$$\begin{aligned} \varepsilon &\leq d(x_{n_i}, x_{k_i}) \\ &\leq d(x_{n_i}, Tx_{n_i}) + H(Tx_{n_i}, \{x_{k_i}\}) \\ &\leq d(x_{n_i}, Tx_{n_i}) + H(Tx_{n_i}, Tx_{k_i-1}) + H(Tx_{k_i-1}, \{x_{k_i-1}\}) + d(x_{k_i-1}, x_{k_i}). \end{aligned} \tag{2.4}$$

But we have $d(x_{n_i}, Tx_{n_i}) \leq d(x_{n_i}, x_{n_i+1}) \to 0$ and

$$\begin{aligned} \int_0^{H(Tx_{k_i-1},\{x_{k_i-1}\})} \varphi(t)dt &= \int_0^{\sup_{b\in Tx_{k_i-1}} d(x_{k_i-1},b)} \varphi(t)dt \\ &\leq \int_0^{H(Tx_{k_i-2},Tx_{k_i-1})} \varphi(t)dt \\ &\leq \phi(\int_0^{M(x_{k_i-2},x_{k_i-1})} \varphi(t)dt. \end{aligned} \tag{2.5}$$

From inequality (2.2), we conclude that $M(x_{k_i-2}, x_{k_i-1}) \to 0$ as $i \to \infty$. Hence, from (2.5), $H(Tx_{k_i-1}, \{x_{k_i-1}\}) \to 0$. Now from (2.4), we have

$$\begin{aligned} \int_0^{\varepsilon} \varphi(t)dt &\leq \int_0^{d(x_{n_i},Tx_{n_i})+H(Tx_{n_i},Tx_{k_i-1})+H(Tx_{k_i-1},\{x_{k_i-1}\})+d(x_{k_i-1},x_{k_i})} \varphi(t)dt \\ &= \int_0^{H(Tx_{n_i},Tx_{k_i-1})} \varphi(t)dt \\ &+ \int_{H(Tx_{n_i},Tx_{k_i-1})}^{d(x_{n_i},Tx_{n_i})+H(Tx_{n_i},Tx_{k_i-1})+H(Tx_{k_i-1},\{x_{k_i-1}\})+d(x_{k_i-1},x_{k_i})} \varphi(t)dt \\ &\leq \phi(\int_0^{M(x_{n_i},x_{k_i-1})} \varphi(t)dt) \\ &+ M[d(x_{n_i}, Tx_{n_i}) + H(Tx_{k_i-1}, \{x_{k_i-1}\}) + d(x_{k_i-1}, x_{k_i})] \end{aligned} \tag{2.6}$$

where $M$ is a positive number such that $\varphi(t) \leq M$ for all $t \geq 0$. Taking $\limsup_{i\to\infty}$ we obtain

$$\int_0^{\varepsilon} \varphi(t)dt \leq \limsup_{i\to\infty} \phi(\int_0^{M(x_{n_i},x_{k_i-1})} \varphi(t)dt), \tag{2.7}$$

where

$$\begin{array}{rcl} M(x_{n_i}, x_{k_i-1}) & = & \max\{d(x_{n_i}, x_{k_i-1}), d(x_{n_i}, Tx_{n_i}) \\ & & , d(x_{k_i-1}, Tx_{k_i-1}), \frac{1}{2}[d(x_{n_i}, Tx_{k_i-1}) + d(x_{k_i-1}, Tx_{n_i})]\} \\ & \leq & \max\{d(x_{n_i}, x_{k_i-1}), d(x_{n_i}, x_{n_i+1}), d(x_{k_i-1}, x_{k_i}) \\ & & , \frac{1}{2}[d(x_{n_i}, x_{k_i-1}) + d(x_{k_i-1}, x_{k_i}) + d(x_{k_i-1}, x_{n_i}) + d(x_{n_i}, x_{n_i+1})]\}. \end{array} \tag{2.8}$$

Taking $\limsup_{i\to\infty}$ in (2.8), we get $\limsup_{i\to\infty} M(x_{n_i}, x_{k_i-1}) \leq \max\{\varepsilon, 0, 0, \varepsilon\} = \varepsilon$. Therefore, from (2.7), we obtain $\int_0^\varepsilon \varphi(t)dt \leq \phi(\int_0^\varepsilon \varphi(t)dt)$, which is a contradiction. Hence $\{x_n\}$ is a Cauchy sequence. Since $X$ is complete, there exists $x \in X$ such that $\lim_{n\to\infty} d(x_n, x) = 0$. Next, we shall show that $x$ is an endpoint of $T$. To see this, we have

$$\begin{array}{rcl} \int_0^{H(\{x\},Tx)} \varphi(t)dt & \leq & \int_0^{d(x,x_n)+H(\{x_n\},Tx_n)+H(Tx_n,Tx)} \varphi(t)dt \\ & = & \int_0^{H(Tx_n,Tx)} \varphi(t)dt + \int_{H(Tx_n,Tx)}^{d(x,x_n)+H(\{x_n\},Tx_n)+H(Tx_n,Tx)} \varphi(t)dt \\ & \leq & \phi(\int_0^{M(x_n,x)} \varphi(t)dt) + M(d(x,x_n) + H(\{x_n\}, Tx_n)). \end{array} \tag{2.9}$$

Similar to way in (2.5), it is easy to check that $H(\{x_n\}, Tx_n) \to 0$ as $n \to \infty$. Taking $\limsup_{i\to\infty}$ in (2.9), we have

$$\int_0^{H(\{x\},Tx)} \varphi(t)dt \leq \limsup_{n\to\infty} \phi(\int_0^{M(x_n,x)} \varphi(t)dt), \tag{2.10}$$

where

$$M(x_n, x) = \max\{d(x_n, x), d(x_n, Tx_n), d(x, Tx), \frac{1}{2}[d(x_n, Tx)+d(x, Tx_n)]\}$$

$$\leq \max\{d(x_n, x), d(x_n, x_{n+1}), d(x, Tx), \frac{1}{2}[d(x_n, x)+d(x, Tx)+d(x, x_{n+1})]\}.$$

Tending $n$ to $\infty$, we obtain $\lim_{n\to\infty} M(x_n, x) = d(x, Tx)$. Consequently, we obtain from (2.10),

$$\int_0^{H(\{x\},Tx)} \varphi(t)dt \leq \phi(\int_0^{d(x,Tx)} \varphi(t)dt) \leq \phi(\int_0^{H(\{x\},Tx)} \varphi(t)dt).$$

From the above inequality, we conclude that $H(\{x\}, Tx) = 0$ which implies $Tx = \{x\}$. To show the uniqueness of endpoint of $T$ assume that $x, y$ are two endpoints of $T$ such that $x \neq y$. Then

$$\int_0^{d(x,y)} \varphi(t)dt = \int_0^{H(Tx,Ty)} \varphi(t)dt \leq \phi(\int_0^{M(x,y)} \varphi(t)dt) = \phi(\int_0^{d(x,y)} \varphi(t)dt),$$

which is a contradiction. □

**Example 2.1.** *Let* $X = \{\frac{1}{n} | n \in \mathbb{N}\} \cup \{0\}$ *with the usual metric* $d(x, y) = |x - y|$. *Obviously* $(X, d)$ *is complete. Define* $\varphi : [0, +\infty) \to [0, +\infty)$ *by* $\varphi(t) = t^{\frac{1}{t}}[\frac{1-\ln t}{t^2}]$ *for* $0 < t < e$ *and* $0$ *otherwise. Then, for any* $0 \leq \tau < e$, *we have* $\int_0^{\tau}\varphi(t)dt=\tau^{\frac{1}{\tau}}$ *(because if we define* $\Phi(t)=\begin{cases} t^{\frac{1}{t}} & 0<t<e \\ 0 & t=0 \end{cases}$, *it is easy to check that* $\Phi'(t)=\varphi(t)$ *for all* $0\le t<e$*). Also, define* $T:X\to CB(X)$ *by*

$$Tx=\begin{cases} \{0\} & x=0, \\ [0,\frac{1}{n+1}]\cap X & x=\frac{1}{n}, n\in\mathbb{N}. \end{cases}$$

*If* $x=0$ *and* $y=\frac{1}{n}$ *, then*

$$\begin{aligned}\int_0^{H(Tx,Ty)}\varphi(t)dt &= \int_0^{\frac{1}{n+1}}\varphi(t)dt \\ &= \frac{1}{n+1}^{n+1} \\ &= \frac{1}{n+1}(\frac{1}{n+1})^n \\ &\le \frac{1}{2}(\frac{1}{n})^n \\ &= \phi(\int_0^{d(x,y)}\varphi(t)dt) \\ &\le \phi(\int_0^{M(x,y)}\varphi(t)dt)\end{aligned}$$

*where* $\phi(t)=\frac{1}{2}t$.

*If* $x,y\in\{\frac{1}{n}|n\in\mathbb{N}\}$*, then assume that* $x=\frac{1}{n}$ *and* $y=\frac{1}{m}$*. In this case, we have*

$$\begin{aligned}\int_0^{H(Tx,Ty)}\varphi(t)dt &= \int_0^{|\frac{1}{n+1}-\frac{1}{m+1}|}\varphi(t)dt \\ &= (\frac{|m-n|}{(n+1)(m+1)})^{\frac{(n+1)(m+1)}{|m-n|}} \\ &= (\frac{|m-n|}{(n+1)(m+1)})^{\frac{n+m+1}{|m-n|}}(\frac{|m-n|}{nm})^{\frac{nm}{|m-n|}}(\frac{mn}{(n+1)(m+1)})^{\frac{nm}{|m-n|}} \\ &\le \frac{1}{2}(1)(\frac{|m-n|}{nm})^{\frac{nm}{|m-n|}} \\ &= \phi(\int_0^{d(x,y)}\varphi(t)dt) \\ &\le \phi(\int_0^{M(x,y)}\varphi(t)dt).\end{aligned}$$

*We see that*

$$\int_0^{H(Tx,Ty)}\varphi(t)dt\le\phi(\int_0^{M(x,y)}\varphi(t)dt)\quad \textit{for all}\ \ x,y\in X.$$

*It is easy to see that* $T$ *has the property (HS). Therefore, the mapping* $T$ *defined in this example satisfies conditions of Theorem 2.1, and so by this theorem,* $T$ *has a unique endpoint in* $X$*. Here,* $T0=\{0\}$.

Also, by using [10], we can give some equivalent results for Theorem 2.1. Here, we give only two results in this way as follows.

**Proposition 2.2.** *Let* $(X,d)$ *be a complete metric space and* $T:X\to CB(X)$ *be a multivalued mapping which has the property (HS). Suppose there exist* $\varphi\in\Phi_1$, $\psi:[0,\infty)\to[0,\infty)$ *a lower semicontinuous function* *and* $\eta : [0, \infty) \to [0, \infty)$ *satisfying* $\eta^{-1}\{0\} = \{0\}$ *and* $\liminf_{t\to\infty} \eta(t) > 0$ *such that*

$$\psi(\int_0^{H(Tx,Ty)} \varphi(t)dt) \leq \psi(\int_0^{M(x,y)} \varphi(t)dt) - \eta(\int_0^{M(x,y)} \varphi(t)dt),$$

*for all* $x, y \in X$. *Then* $T$ *has a unique endpoint.*

**Proposition 2.3.** *Let* $(X, d)$ *be a complete metric space and* $T : X \to CB(X)$ *be a multivalued mapping which has the property (HS). Suppose there exist* $\varphi \in \Phi_1$, $\psi : [0, \infty) \to [0, \infty)$ *a nondecreasing function such that* $\Sigma_{n=1}^{\infty} \psi^n(t) < \infty$ *for all* $t > 0$ *and* $\phi : [0, \infty) \to [0, \infty)$ *a right continuous and nondecreasing function satisfying* $\phi(t) < t$ *for all* $t > 0$ *such that*

$$\psi(\int_0^{H(Tx,Ty)} \varphi(t)dt) \leq \phi(\psi(\int_0^{M(x,y)} \varphi(t)dt)),$$

*for all* $x, y \in X$. *Then* $T$ *has a unique endpoint.*

In 2007, Zhang [11] defined a new generalized contractive type condition for a pair of mappings in metric spaces. Let $A \in (0, +\infty]$ and $\mathbb{R}_A^+ = [0, A)$. Denote by $\Im[0, A)$ the collection of all functions $F : \mathbb{R}_A^+ \to \mathbb{R}$ satisfing the following conditions:

(i) $F(0) = 0$ and $F(t) > 0$ for each $t \in (0, A)$,
(ii) $F$ is nondecreasing on $\mathbb{R}_A^+$,
(iii) $F$ is continuous.

From [11], we know that for any $F \in \Im[0, A)$, $\lim_{n\to+\infty} F(\varepsilon_n) = 0$ ($\varepsilon_n \in \mathbb{R}_A^+$) implies $\lim_{n\to+\infty} \varepsilon_n = 0$. Denote by $\Psi[0, A)$ the family of all functions $\psi : \mathbb{R}_A^+ \to \mathbb{R}^+$ which is nondecreasing and right upper semi-continuous such that $\lim_{n\to+\infty} \psi^n(t) = 0$ for each $t \in (0, A)$. It is easy to see that for any $\psi \in \Psi[0, A)$ we have $\psi(0) = 0$ and $\psi(t) < t$ for each $t \in (0, A)$.

Regarding the above notations Zhang [11] proved the following theorem about existing of common fixed point for a pair of mappings.

**Theorem 2.4.** *Let* $(X, d)$ *be a complete metric space and let* $D = \sup\{d(x, y)|x, y \in X\}$. *Set* $A > D$ *if* $D < \infty$ *and* $A = D$ *if* $D = \infty$. *Suppose that* $T, S : X \to X$ *are two mappings,* $F \in \Im[0, A)$ *and* $\psi \in \Psi[0, F(A^-))$ *satisfing*

$$F(d(Tx, Sy)) \leq \psi(F(M(x, y))) \quad \textit{for each} \quad x, y \in X,$$

*where*

$$M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Sy), \frac{1}{2}(d(Tx, y) + d(x, Sy))\}.$$

*Then $T$ and $S$ have a unique common fixed point in $X$. Moreover, for each $x_0 \in X$, the iterated sequence $\{x_n\}$ with $x_{2n+1} = Tx_{2n}$ and $x_{2n+2} = Sx_{2n+1}$ converges to the common fixed point of $T$ and $S$.*

Similar to the proof of Theorem 2.1, one can easily prove the following endpoint result for multivalued mappings, which is a multivalued endpoint version of Theorem 2.4.

**Theorem 2.5.** *Let $(X, d)$ be a complete metric space and $T : X \to CB(X)$ be a multivalued mapping satisfing property (HS), $F \in \Im[0, A)$ and $\psi \in \Psi[0, F(A^-))$ such that*

$$F(H(Tx, Ty)) \leq \psi(F(M(x, y))) \quad \textit{for each} \quad x, y \in X.$$

*Then $T$ has a unique endpoint.*

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