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Mathematical modeling of dynamics behavior of terrorism and control

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Abstract. Terrorism is generally understood to be the use of threat or extra-normal violence to gain ideological reasons and personal benefit. In this paper, a mathematical modelling of terrorism with military strategies and rehabilitation of terrorists was constructed. The model is developed to control the spread of terrorist ideologies in society and suitable to describe a terrorist group. The population is divided into six compartments: $S(t)$, $I(t)$, $T(t)$, $T_L(t)$, $T_S(t)$ and $Q_T(t)$. Furthermore, the basic reproduction number, R_0 is also calculated if $R_0 < 1$ means the terror-organization is nearly eradicated and if $R_0 > 1$ means the number of terrorists is high where the terrorist are endemic to the population. The result of the sensitivity analysis shows that the most sensitive parameters are the recruitment pool of the terrorist from susceptible to moderate (β_1) and terrorist move to detention facilities due to counter-terrorist activities (b). The least parameter is the probability at which terrorist become militancy leaders ($k\alpha$). (β_1) and (b) are parameters counter-terrorist need to be target. The finding shows that the military/dialogue strategies are to be used while military strategies alone should not be used if the number of terrorists is below a certain basic reproduction number.

Keywords: Counter-Terrorist, Modelling,Terrorism, Dynamic and Sensitivity Analysis.
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1. Introduction

The September 11 2001 attack, known as (9/11) in Washington, D.C., New York and Pennsylvania were acts of war against the entire world. This marked the global counterterrorism operation. According to [21, 24, 25] terrorism can be defined as an unlawful organization, attempting or threatening to cause death or serious bodily injury to any person, causing damage to private property including public places or government facilities. Although, the [27] adopted at its 814th and 8496th meetings, stressing the primary responsibility of member states in countering terrorist acts and reiterating their obligation to prevent and suppress the financing of terrorist acts as well as the call upon all states to become party to the international counter-terrorism conventions and protocols as soon as possible. Similarly, [13] stated that there is no single internationally accepted definition of what constitutes terrorism and the terrorism literature abounds with competing definitions. [22] defined *terrorism* as an extra normal violent or threat to gain a political objective and intimidating innocent people to adopt their ideology by all means. Also,the level of violence and terrorist activities in Iraq and Syria have fallen considerably in the last two years. Iraq recorded the biggest fall in 2017, a trend which seems to have continued in 2018. The Islamic State of Iraq and the Levant (ISIL) known as Islamic states of Iraq and Syria (ISIS) has lost most of its territory and sources of revenue in Syria and Iraq. However, affiliate groups in other regions are becoming more active. In the Maghreb and Sahel regions of Northern Africa, there has been a resurgence of terrorist activity in the past two years, most notably of Al-Qaida. As at March 2018, there were more than 9,000 members of terrorist groups active in the region, mostly concentrated in Libya and Algeria [13]. Al-Shabaab, a Salafist militant group is active in East Africa. It emerged first in a battle over Somalias capital in the summer of 2006. As an Al-Qaida affiliated terrorist group based in Somalia and Kenya, Al-Shabaab pursues Islamist statehood aspirations in Somalia [13].

[15] In Nigeria there was unknown issue of terrorism, but became rampant in the late 90's. In the past, previous Nigerian governments have different policies aligned with terrorism. In 2018, there has been a dramatic increase in violence involving Fulani extremists even as deaths committed by Boko Haram are falling due to counter-terrorist operation. ISIL known as ISIS, the Taliban, Al-Shabaab, and Boko Haram were responsible for 10,632 deaths from terrorism in 2017. Jamatu Ahlis Sunna Liddaawati wal-Jihad, more popularly known as Boko Haram and Fulani extremists are among the most deadliest groups in Nigeria. According [13] out of ten countries most impacted by terrorism, Nigeria ranked

third by number of deaths. Terrorist activity in Nigeria is dominated by two groups which are Boko-Haram and Fulani extremists. In 2017, Boko Haram was the deadliest group in Nigeria with both terrorism deaths and attacks increasing over the prior year. Deaths increased from 34 per cent to 1,022 while attacks increased from 62 per cent to 222. The Fulani extremists were less active in 2017 than the prior year with terrorism deaths dropping from 60 per cent to 321, and attacks dropping from 51 per cent to 72. However, preliminary data for 2018 suggests that there has been a significant increase in violence committed by Fulani extremists. Together, Boko Haram and the Fulani extremists are responsible for 63 per cent of terror attacks and 88 per cent of terror-related deaths in Nigeria. The Borno state is home to Boko Haram. It has experienced the highest level of terrorist activity in Nigeria with all deaths caused by Boko Haram. In 2017, 62 per cent of deaths in Nigeria occurred in Borno state. The group is also active in Chad, Cameroon and Niger and has disrupted Foreign Direct investment and humanitarian efforts in Nigeria and its neighbouring countries. In 2016, Boko Haram splintered into two groups. The newer Islamic State West African Province (ISWAP) declared allegiance to ISIL in 2015. Both groups see themselves as affiliates of ISIL [13].

The transmission dynamics and spread of terrorism like a disease infections was that of [4, 5, 8, 14, 19, 21, 22]. [4] formulated a mathematical model of terrorist and fanatic groups with two models in which the population is divided into non-core and core-groups. The first model considered the case of two core populations with no cross interaction between the sub-populations and both core-groups drawn from the same general population. The more efforts the organization implements to recruit and retain its members the more effective it will be in transmitting the ideology to other vulnerable individuals. The model ignored military/police intervention to combat terrorist risk posed to the society.

[23] developed a deterministic Model of optimal human resources allocation in counter-terrorism operation to study the possible strategies for allocating counter-terrorism resource towards a long-term solution of a given terrorist group. They constructed a simple set of coupled differential equations whose variables are internal and external dynamics of the organization.In their model, detention/rehabilitation facility was ignored.

[7] extended the model of [5] by developing a mathematical model of radicalization process, control of contagion and counter-terrorist measures model by incorporating the spread of radical ideologies, fanaticism, recruitment and terrorist activities. In their model, the control strategy

was not specified. Also, [20] developed a deterministic model for radicalization process in Kenya and used the model to assess the impact of rehabilitation centers to radicalization burden but ignored detention facility in the model.
Similarly, [21] built a simple compartmental model suitable to describe de-radicalization programs. The population is divided into four compartments susceptible (S), extremists (E), recruiters (R), and treatment (T) to combat the extremist.Their model failed to incorporate force recruitment after treatment from susceptible to extremists or recruiters.
Also, [8] developed a mathematical models of radicalization and terrorism with two cross section to describe a dynamics transition between the various sub-populations. The first model is and individual who progresses from non-radical towards a hierarchy of susceptible and moved to moderate group before becoming fanatics. The second model is two radical group, which originates from the susceptible before becoming fully fanatic without control strategies.

On the other hand, [3] built a model by incorporating optimal control of counter-terrorism tactics which classified counter terrorism into two categories fire and water control respectively. The model failed to incorporate some parameters like surveillance, detention/rehabilitation facilities, force recruitment etc. Also,[1] developed a model of terrorist that incorporated optimal control using Differential Transformation Method (DTM) and the result show that knowing the best strategy does not imply optimality but accuracy in result compared to other methods. Similarly, [24] built a dynamical model of terrorism by incorporating the effects of direct military/police intervention to reduce the terrorist population. They affirmed that using military/police intervention may reduce the threat of terrorism. Further, [12] built a dynamic model of suicide terrorism and political mobilization which enables predictions to emphasized or de-emphasized by the forces in protecting society. [6] compared what they call fire and water strategies. Further studies on the eradication of terrorism have been given by [18]. The following existing models are that of [7, 8, 22]. The objective of this paper is to modify the existing models including force recruitment rates, which are from susceptible to terrorist and detention facility/rehabilitation centres such that the model will be in the form of (SITQ).

2. Model Formulation

2.1. **Model Description.** In this paper, the populations of interest are Susceptible $S(t)$, Moderate $I(t)$, Terrorist $T(t)$ Terrorist soldiers $T_S(t)$, Terrorist leaders $T_L(t)$ and Detention facility $Q_T(t)$. Susceptible individuals are recruited into the population by input rate Λ. When susceptible individuals have contact(s) with some terrorists, the probability of embracing the ideology β_1 but embracing the ideology did not make them extremist except with a proportion of β_2 chance for which the individual move to terrorist group. The terrorist group is further split into foot-soldiers $T_S(t)$ and leaders $T_L(t)$, alternatively to become extremist in terrorism this depends on the moderate individual increase in relation with either terrorist leader or foot soldiers. Due to counter-terrorist operations π terrorism some individuals enter detention facilities and later fall back for some reasons from quarantine to terrorist at rate δ. Some time, foot soldiers move to detention facilities if surrender or capture by military counterterrorism activities with rate at c and later fall back to susceptible group after certified repentance/rehabilitation or end jail term at rate a, but some time terrorist can move to detention facilities if same operation applied on them. The parameter d is considered to be induced death rate of terror either by suicide attack or counter-terrorist activities, while η is individual death due to torture or life jail in detention facility and μ represents natural death in each compartment.

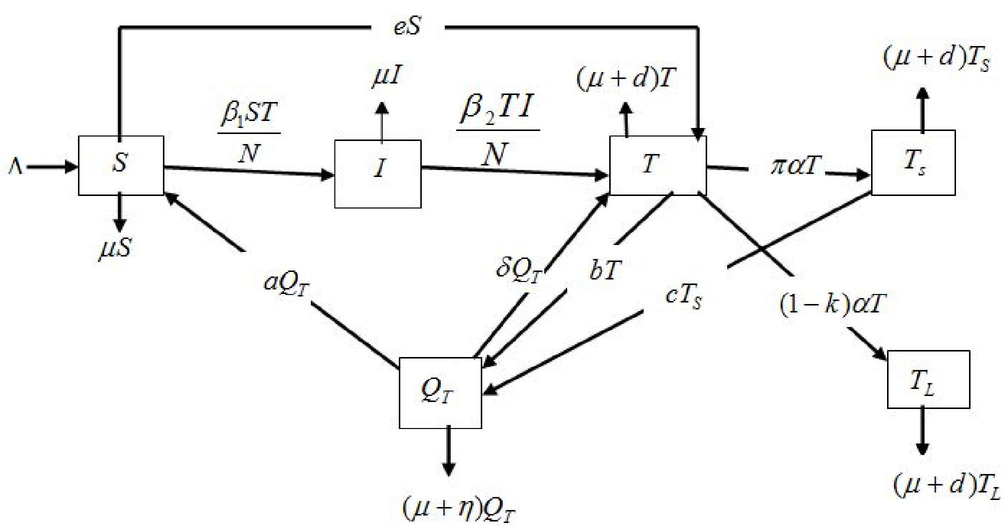

Figure 1. Flow Diagram of Terrorism Network

The model for terrorism network is given by the subsequent deterministic system of non-linear differential equations.

$$\left.\begin{array}{l}
\frac{dS}{dt} = \Lambda + aQ_T - eS - \mu S - \beta_1 \frac{T}{N} S \\
\frac{dI}{dt} = \beta_1 \frac{T}{N} S - \mu I - \beta_2 \frac{T}{N} I \\
\frac{dT}{dt} = \frac{\beta_2 T I}{N} + \delta Q_T + eS - bT - \pi\alpha T - (\mu + d)T - (1-k)\alpha T \\
\frac{dT_S}{dt} = \pi\alpha T - cT_S - (\mu + d)\, T_S \\
\frac{dT_L}{dt} = (1-k)\,\alpha T - (\mu + d)\, T_L \\
\frac{dQ_T}{dt} = bT + cT_S - (a + \delta + \mu + \eta)Q_T \\
N = S + I + T + T_S + T_L + Q_T
\end{array}\right\} \quad (2.1)$$

Assumption of the Model: From the model above, there is a sub-population potentially at risk of adopting the ideology from the general population. The sub-population is divided into six compartments Susceptible S(t), Moderate I(t),Terrorist T(t), Leader $T_L(t)$ and Foot-soldiers $T_S(t)$ and Detention facilities $Q_T(t)$. The contact rate at which susceptible are recruited to adopt the extremist ideology is $\beta_1 TS$, while $\beta_2 TI$ is the conversion rate from moderate to terrorist. The rate at which individuals leave the terrorist compartment to enter foot soldiers is α, where k is the fraction at which terrorist successfully become leaders. It is assumed that the average time individuals spent at the transition rate of militancy is $\frac{1}{\alpha}$. Moderates and terrorists are assumed to have frequent contacts with susceptible individuals. $1-k$ is the probability of some individual terrorists becoming leaders after being fully indoctrinated into the ideology and $k \in [0, 1]$ of terrorist. Since terrorism is not permanently recovered, some individuals may fall back to T(t) from $Q_T(t)$ with the rates δ, while some leave $Q_T(t)$ to S(t) after being certified by counter-terrorist in detention facilities. The rate c enters detention due to counter-terrorist activities. These rates account for individuals that are imprisoned for life. Certified repentance rates from detention facilities to susceptible is a and the individual death rate due to torture is η and not all individuals in the compartmental $Q_T(t)$ are terrorist. It assumed that the populations are susceptible, moderate, terrorist and detention facilities while terrorist group is further sub-divided into foot soldiers and leaders. Finally, it is assumed that everybody in the population has same average natural death rate μ.

The description of variables and Parameters of model formulated are $S(t)$ Susceptible (self identification), $I(t)$ Semi-terrorist (moderate), $T(t)$ Terrorist, $T_L(t)$ Leaders, $T_S(t)$ Foot-soldiers, $Q_T(t)$ Detention facilities, $N(t)$ Total Population, Λ as recruitment Per capital (birth or immigration), d is death due to suicide attack or by counter-terror activities, η

death due to torture and life jail in detention, β_1 rate at which terrorist recruit their members, β_2 Per capital recruitment rate into, μ natural death rates, α rate at which terrorist become foot soldiers or leaders, k is fraction at which terrorist successfully become leaders, $k\alpha$ is the probability at which terrorist become militancy leaders. $(1-k)$ is probability that not all individuals terrorist become leaders, δ rate at which individual fall back to T(t) from $Q_T(t)$, c rate at which terrorist soldiers enter detention due to counterterrorist activities, a certified repentance/end jail term rates from detention facilities to susceptible, b terrorist made to enter the detention facilities due to counter-terrorist operations, e is per capital recruitment from susceptible to terrorist and $\pi\alpha$ is the rate at which military control strategies/dialogue applied to terrorist that become foot soldiers, while α is the rate at which terrorist move to become foot soldiers.

3. Basic Properties of the Terrorism Model

3.1. **Invariant Region.** The population size N can be determined by $N = S+I+T+T_S+T_L+Q_T$ or from the non-linear differential equation of the model formulated

$$\left.\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dT}{dt} + \frac{dT_S}{dt} + \frac{dT_L}{dt} + \frac{dQ_T}{dt}\right\} \tag{3.1}$$

This can be reduced to;

$$\frac{dN}{dt} = \Lambda - \mu(S + T + T_S + T_L + Q_T), \text{which implies that}$$

$$\frac{dN}{dt} = \Lambda - \mu N \tag{3.2}$$

Since $N = S + I + T + T_S + T_L + Q_T$, and equation (3.2) resolved to linear differential equation of the form

$$\frac{dN}{dt} + \mu N = \Lambda \tag{3.3}$$

Lemma 3.1. *The model system of equation (2.1) has solution which is contained in the feasible domain* Ω *, by following the idea in [9].*

Theorem 3.2. *The solution of the system of equation (2.1) is feasible for* $t < 0$ *if they enter the invariant region* Ω.

Proof: Let $(S, I, T, T_S, T_L, Q_T) \in \mathrm{R}^6$ be any solution of the system with non-negative initial conditions.

by using integrating factor (IF), $\exp(\int pdx) = \exp(\int \mu dt) = \exp(\mu t)$. Multiplying both sides of equation (3.2) by IF gives;

$$\exp(\mu t \frac{dN}{dt}) = \Lambda \exp(\int \mu t) \tag{3.4}$$

Integrate equation (3.4) above, which gives:
$N(t)\exp(\mu t) = \frac{\Lambda}{\mu}\exp(\mu t) + K$, where K is constant and divide both sides by $\exp(\mu)$ then applying initial condition at $t = 0$

$$N(t) = \frac{\Lambda}{\mu} + K\exp(-\mu t) \tag{3.5}$$

$N(0) = \frac{\Lambda}{\mu} + K$, this implies that $K = N(0) - \frac{\Lambda}{\mu}$
Therefore, substitute the value of K in equation (3.5) gives;

$$N(t) = N(0)\exp(-\mu t) + \frac{\Lambda}{\mu}(1 - \exp(-\mu t)) \tag{3.6}$$

$$N(0) \leq \frac{\Lambda}{\mu} \tag{3.7}$$

Therefore, as $t \to \infty$ in equation (3.6) the human population N approaches $K = \frac{\Lambda}{\mu}$ i.e. $N \to K = \frac{\Lambda}{\mu}$ The parameter $K = \frac{\Lambda}{\mu}$ is called the caring capacity.

Hence, all feasible solution set of the human population of the model system (2.1) enter the region

$$\Omega = \left\{ (S, I, T, T_S, T_L, Q_T) \in \mathrm{R}_+^6 : S, T, T_S, T_L, Q_T \geq 0; N \leq \frac{\Lambda}{\mu} \right\} \tag{3.8}$$

which is a positively invariant set under the flow induced by the model (2.1). Therefore the model equation (2.1) is mathematical well posed in the domain. Thus, in this domain, it is sufficient to consider the dynamics of the flow generated by the model (2.1). In addition, the usual existence, uniqueness and continuation of results hold for the system in [11].

4. Existence and Positivity of Solutions

Here, the following results guarantee by the terrorist model governed in equation (2.1) is well-posed in a feasible region Ω.

Lemma 4.1. *Let the initial conditions be* $\{S(0) > 0, (I(0), T(0), T_S(0), T_L(0) > 0, Q_T > 0)\} \in \Omega$, *then the solution set* $\{S, I, T, T_S, T_L, Q_T\}(t)$ *of the model system (2.1) is positive* $\forall t > 0$*, using the idea in [11].*

Proof: The first equation of the model (2.1) gives;

$$\frac{dS}{dt} = \Lambda + a\,Q_t - \left(e + \mu + \frac{\beta_1 T}{N}\right) S \geq -\left(e\,S + \mu\,S + \frac{\beta_1 T}{N} S\right) \quad (4.1)$$

Thus,

$$\frac{dS}{dt} \geq -\left(e + \mu + \frac{\beta_1 T}{N}\right) S$$

By using the separation of variable

$$\frac{dS}{S} \geq -\left(e + \mu + \frac{\beta_1 T}{N}\right) dt$$

Integrating both sides of the inequality yields:

$$\left.\begin{aligned} \int \frac{dS}{S} &\geq -\int \left(e + \mu + \frac{\beta_1 T}{N}\right) dt \\ \ln S(t) &\geq -\left(e + \mu + \frac{\beta_1 T}{N}\right) t + K \\ S(t) &\geq K \exp\left(-\left(e + \mu + \frac{\beta_1 T}{N}\right) t\right) \end{aligned}\right\} \quad (4.2)$$

Then at $t = 0$
Applying the initial condition which gives;
'$S(0) \geq 0$ therefore;
Similarly, it can shown that $I(t), T(t), T_S(t), T_L(t)$ and $Q_L(t) > 0$ for all $t \geq 0$.

5. Existence of Terrorist-Free Equilibrium and Basic Reproduction Number

Terrorism-free equilibrium points are steady-state solutions where there is no terrorist. Thus, the terrorism-free equilibrium points E_0, for the terrorist model give $I^* = 0, T^* = 0, T_S^* = 0, T_L^* = 0$ and by solving the model equations using the idea in [29, 30] yields

$$E_0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right) \quad (5.1)$$

$f_i(x)$ is the rate of appearance of new infections in compartment i, and $v_i(x)$ is the net decreasing rate of infections in compartment i due to infective flow inside system of infected compartments.
If $F = (f_1, ...f_{2n})$ and $V = (v_1, ...v_{2n})$, let partition the derivative $DF(E_0)$ and $DV(E_0)$ as

$$DF(E_0) = \begin{bmatrix} F & 0 \\ 0 & 0 \end{bmatrix}, DV(E_0) = \begin{bmatrix} V & 0 \\ J_3 & J_4 \end{bmatrix},$$

where

$$F = \begin{bmatrix} \frac{\partial F_1}{\partial I} & \frac{\partial F_1}{\partial T} & \frac{\partial F_1}{\partial T_S} & \frac{\partial F_1}{\partial T_L} \\ \frac{\partial F_2}{\partial I} & \frac{\partial F_2}{\partial T} & \frac{\partial F_2}{\partial T_S} & \frac{\partial F_2}{\partial T_L} \\ \frac{\partial F_3}{\partial I} & \frac{\partial F_3}{\partial T} & \frac{\partial F_3}{\partial T_S} & \frac{\partial F_3}{\partial T_L} \\ \frac{\partial F_4}{\partial I} & \frac{\partial F_4}{\partial T} & \frac{\partial F_4}{\partial T_S} & \frac{\partial F_4}{\partial T_L} \end{bmatrix} (x_0), V = \begin{bmatrix} \frac{\partial V_1}{\partial I} & \frac{\partial V_1}{\partial T} & \frac{\partial V_1}{\partial T_S} & \frac{\partial V_1}{\partial T_L} \\ \frac{\partial V_2}{\partial I} & \frac{\partial V_2}{\partial T} & \frac{\partial V_2}{\partial T_S} & \frac{\partial V_2}{\partial T_L} \\ \frac{\partial V_3}{\partial I} & \frac{\partial V_3}{\partial T} & \frac{\partial V_3}{\partial T_S} & \frac{\partial V_3}{\partial T_L} \\ \frac{\partial V_4}{\partial I} & \frac{\partial V_4}{\partial T} & \frac{\partial V_4}{\partial T_S} & \frac{\partial V_4}{\partial T_L} \end{bmatrix} (x_0)$$

The next generation matrix: Is the method used to drive R_0, for a compartmental model of the spread of terrorism. This method is given by [10, 30] used to discuss basic reproduction number of the system (2.1). It is clear to see that the system (2.1) has terrorism-free equilibrium.

$$E_0 = (\frac{\Lambda}{\mu}, 0, 0, 0, 0).$$

Let $X = (S, I, T, T_S, T_L, Q_T)^T$, then system (2.1) can be written as $X^{'} = F(X) - V(X)$,
where

$$F = \begin{bmatrix} \frac{\beta_1 \Lambda}{N\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } V = \begin{bmatrix} \mu & 0 & 0 & 0 \\ 0 & A_1 & 0 & 0 \\ 0 & -\pi\alpha & A_2 & 0 \\ 0 & -A_4 & 0 & A_3 \end{bmatrix}$$

$$V^{-1} = \begin{bmatrix} \frac{1}{\mu} & 0 & 0 & 0 \\ 0 & \frac{1}{A_1} & 0 & 0 \\ 0 & \frac{\pi\alpha}{A_1 A_2} & \frac{1}{A_2} & 0 \\ 0 & \frac{-A_4\alpha}{A_1 A_2} & 0 & \frac{1}{A_3} \end{bmatrix}$$

The basic reproduction number, denotes by$R_0 = \Gamma(FV^{-1})$, where Γ denotes as spectral radius, and gives;

$$R_0 = \Gamma(FV^{-1}) = \begin{bmatrix} \frac{\beta_1 \Lambda}{N\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\mu} & 0 & 0 & 0 \\ 0 & \frac{1}{A_1} & 0 & 0 \\ 0 & \frac{\pi\alpha}{A_1 A_2} & \frac{1}{A_2} & 0 \\ 0 & \frac{-A_4\alpha}{A_1 A_2} & 0 & \frac{1}{A_3} \end{bmatrix}$$

$$FV^{-1} = \begin{bmatrix} \frac{\beta_1}{A_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_0 = \frac{\beta_1}{A_1} \tag{5.2}$$

where;

$$\begin{aligned} A_1 &= (b + \alpha\pi + \mu + d + \alpha - \alpha k) \\ A_2 &= (c + \mu + d) \\ A_3 &= (\mu + d) \\ A_4 &= -(1-k)\alpha \end{aligned}$$

The terrorist-free equilibrium, $E_0\left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right)$ of the system (2.1) is locally asymptotically stable if $R_0 < 1$, that is, terrorism dies out and unstable iff $R_0 > 1$ meaning the terrorist activities will persist.
The threshold quantity R_0 is the basic reproduction number of the terrorism. It is the transmission of a new terrorist produced by potential terrorism.

6. Existence of Local stability of Terrorist-Free Equilibrium Point

The Routh-Hurwitz criteria are use to establish asymptotic stability of equilibrium for non-linear system of differential equation. The Routh-Hurwitz criteria provide the necessary and sufficient condition for all roots of the characteristic polynomial to contain negative parts, therefore entails asymptotic stability [17]. The local stability of the equilibrium may be determined from the jacobian matrix gives

$$J(E_0) = \begin{bmatrix} e+\mu & 0 & \beta_1 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 & 0 \\ e & 0 & A_1 & 0 & 0 & 0 \\ 0 & 0 & \pi\alpha & -A_2 & 0 & 0 \\ 0 & 0 & -A_4 & 0 & -A_3 & 0 \\ 0 & 0 & b & -c & 0 & -A_5 \end{bmatrix} \tag{6.1}$$

$$J(E_0 - \lambda I) = \begin{bmatrix} e+\mu-\lambda & 0 & \beta_1 & 0 & 0 & 0 \\ 0 & \mu-\lambda & 0 & 0 & 0 & 0 \\ e & 0 & A_1-\lambda & 0 & 0 & 0 \\ 0 & 0 & \pi\alpha & -A_2-\lambda & 0 & 0 \\ 0 & 0 & -A_4 & 0 & -A_3-\lambda & 0 \\ 0 & 0 & b & -c & 0 & -A_5-\lambda \end{bmatrix} \tag{6.2}$$

According to [17], the Terrorist-free equilibrium E_0 is locally asymptotically stable when there is no terrorist. It follows that the characteristic equation of $J(E_0)$ computed from equation(6.2) is given by solving the determinant with Maple 17 software which gives.

$$\left(-e\lambda + eA_1 - \beta_1 e + \lambda^2 - \lambda\,\mu - \lambda\,A_1 + \mu\,A_1\right)(\mu - \lambda)(-A_2 - \lambda)(-A_3 - \lambda)(-A_5 - \lambda) \tag{6.3}$$

simplify the equation (6.3) gives

$$\begin{aligned}
&\lambda^6 + (-e - 2\,\mu - A_1 + A_2 + A_3 + A_5)\,\lambda^5 + (eA_1 - \beta_1 e + \mu\,A_1 - \\
&(-e - \mu - A_1)\,\mu - (e + 2\,\mu + A_1)\,A_2 + (-e - 2\,\mu - A_1 + A_2)\,A_3 - \\
&(e + 2\,\mu + A_1 - A_2 - A_3)\,A_5)\,\lambda^4 + (-(eA_1 - \beta_1 e + \mu\,A_1)\,\mu - \\
&(-eA_1 + \beta_1 e - \mu\,A_1 + (-e - \mu - A_1)\,\mu)\,A_2 + (eA_1 - \beta_1 e + \mu\,A_1 - \\
&(-e - \mu - A_1)\,\mu - (e + 2\,\mu + A_1)\,A_2)\,A_3 - (-eA_1 + \beta_1 e - \mu\,A_1 + \\
&(-e - \mu - A_1)\,\mu + (e + 2\,\mu + A_1)\,A_2 - (-e - 2\,\mu - A_1 + A_2)\,A_3)\,A_5)\,\lambda^3 + \\
&(-(eA_1 - \beta_1 e + \mu\,A_1)\,\mu\,A_2 + (-(eA_1 - \beta_1 e + \mu\,A_1)\,\mu - (-eA_1 + \beta_1 e - \\
&\mu\,A_1 + (-e - \mu - A_1)\,\mu)\,A_2)\,A_3 - ((eA_1 - \beta_1 e + \mu\,A_1)\,\mu + (-eA_1 + \beta_1 e - \\
&\mu\,A_1 + (-e - \mu - A_1)\,\mu)\,A_2 - (eA_1 - \beta_1 e + \mu\,A_1 - (-e - \mu - A_1)\,\mu - \\
&(e + 2\,\mu + A_1)\,A_2)\,A_3)\,A_5)\,\lambda^2 + (-(eA_1 - \beta_1 e + \mu\,A_1)\,\mu\,A_2 A_3 - \\
&((eA_1 - \beta_1 e + \mu\,A_1)\,\mu\,A_2 - (-(eA_1 - \beta_1 e + \mu\,A_1)\,\mu - (-eA_1 + \beta_1 e - \\
&\mu\,A_1 + (-e - \mu - A_1)\,\mu)\,A_2)\,A_3)\,A_5)\,\lambda - (eA_1 - \beta_1 e + \mu\,A_1)\,\mu\,A_2 A_3 A_5
\end{aligned}$$

Collect the coefficient of the eigenvalues λ and characteristic gives

$$\lambda^6 + a_5\lambda^5 + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0 \tag{6.4}$$

where

$$a = 1$$

$$a_5 = (-e - 2\,\mu - A_1 + A_2 + A_3 + A_5)$$

$$a_4 = (eA_1 - \beta_1 e + \mu\, A_1 - (-e - \mu - A_1)\,\mu - (e + 2\,\mu + A_1)\,A_2 + (-e - 2\,\mu - A_1 + A_2)\,A_3 - (e + 2\,\mu + A_1 - A_2 - A_3)\,A_5)$$

$$a_3 = (-\,(eA_1 - \beta_1 e + \mu\, A_1)\,\mu - (-eA_1 + \beta_1 e - \mu\, A_1 + (-e - \mu - A_1)\,\mu)\,A_2 + (eA_1 - \beta_1 e + \mu\, A_1 - (-e - \mu - A_1)\,\mu - (e + 2\,\mu + A_1)\,A_2)\,A_3 - (-eA_1 + \beta_1 e - \mu\, A_1 + (-e - \mu - A_1)\,\mu + (e + 2\,\mu + A_1)\,A_2 - (-e - 2\,\mu - A_1 + A_2)\,A_3)\,A_5)$$

$$a_2 = (-\,(eA_1 - \beta_1 e + \mu\, A_1)\,\mu\, A_2 + (-\,(eA_1 - \beta_1 e + \mu\, A_1)\,\mu - (-eA_1 + \beta_1 e - \mu\, A_1 + (-e - \mu - A_1)\,\mu)\,A_2)\,A_3 - ((eA_1 - \beta_1 e + \mu\, A_1)\,\mu + (-eA_1 + \beta_1 e - \mu\, A_1 + (-e - \mu - A_1)\,\mu)\,A_2 - (eA_1 - \beta_1 e + \mu\, A_1 - (-e - \mu - A_1)\,\mu - (e + 2\,\mu + A_1)\,A_2)\,A_3)\,A_5)$$

$$a_1 = (-\,(eA_1 - \beta_1 e + \mu\, A_1)\,\mu\, A_2 A_3 - ((eA_1 - \beta_1 e + \mu\, A_1)\,\mu\, A_2 - (-\,(eA_1 - \beta_1 e + \mu\, A_1\Big)\,\mu - (-eA_1 + \beta_1 e - \mu\, A_1 + (-e - \mu - A_1)\,\mu)\,A_2\Big)\,A_3\Big)\,A_5\Big)$$

$$a_0 = (eA_1 - \beta_1 e + \mu\, A_1)\,\mu\, A_2 A_3 A_5$$

Using the Routh-Hurwitz criterion by [17], it can be seen that all the eigenvalues have negative real part and therefore the terrorist-free equilibrium is locally asymptotically stable since there is no terrorist and iff it satisfy equation (6.5)

$$\left.\begin{array}{l} a_1 > 0, a_2 > 0, a_3 > 0, a_4 > 0, a_5 > 0, a_6 > 0, a_1 a_2 a_3 > a_0 a_3^2 - a_1^2 a_4 > 0, \\ (a_1 a_5 - a_0 a_6)(a_1 a_2 a_3 - a_4^2 - a_0 a_1^2 a_5) > 0, a_5(a_1 a_2 - a_3)^2 + a_0 a_1 a_5^2 > 0 \\ a_6(a_2 a_3 a_4 - a_5)^3 + a_0 a_1 a_2 a_6^3 > 0 \end{array}\right\} \quad (6.5)$$

6.1. **Existence of Endemic Equilibrium.** The endemic equilibrium point is a positive steady state solution where the terrorist persists in the population. Therefore, the model equation (2.1) has an endemic equilibrium point E_e. The endemic equilibrium of equation (2.1) is locally asymptotically stable since terrorist exist. However, when $R_0 > 1$, the terrorist recruitment will remain at high [19]

7. Sensitivity Analysis

Sensitivity analysis is a crucial analysis that shows the importance of each parameter to terrorist transmission [2]. The sensitivity index of parameters with respect to the basic reproduction number was calculated to know how crucial each parameter is to the terrorist transmission;

Table 3. Signs of sensitivity index of R_0

Parameter	Parameter value	Sensitivity value	Sensitivity index
β_1	0.00000000056 (Assumed)	1.000000	Positive
b	0.20635 [19]	0.4914449672	Positive
$k\alpha$	0.0792 [19,]	-0.1886234136	Negative
μ	0.000034247 (Assumed)	-0.000081562955	Negative
α	0.12 [20]	-0.2857930510	Negative
d	0.0083 [19]	-0.01976735269	Negative
$\pi\alpha$	0.66 (Assumed)	-0.01428965255	Negative

intervention control strategies that target such parameter should be employed in the control of terrorism. In this way, parameters that are more sensitive to the terrorist are identified and, by reducing the parameters it will also reduce the transmission of the terrorist.
Definition: The normalized forward sensitivity index of a variable ε that depends differentiable on a parameter x is defined as:

$$X_x^{\varepsilon} = \frac{\partial \varepsilon}{\partial x} \times \frac{x}{\varepsilon} \tag{7.1}$$

8. Numerical Simulations

The simulations were carried out using the following variables and parameters for initial conditions. The final time was $t = 6$ years. Computations were run in maple17 software for the analysis.
The rates are e= 0.035 assumed,(β_1= 0.00000000056, c= 0.0008, δ= 0.0016, k= 0.66, a= 0.86, Λ= 600, μ= 0.000034247, d= 0.00083 [20]). b= 0.20635 assumed, β_2= 0.00076 assumed, η= 0.0025 assumed, α= 0.12 [21], π= 0.9 assumed, S(t)= 1000 assumed, I(t)= 300 assumed, T(t)= 700 assumed, $T_S(t)$= 120, $T_L(t)$= 5 [20], $Q_T(t)$= 100 assumed . Secondly, the rates later varies as e= 0.035, 0.000035, 0.0135; k= 0.9, 0.7, 0.5; b= 0.25, 0.50, 0.70; c= 0.8, 0.5, 0.2; α= 0.57, 0.20, 0.10; π= 0.9, 0.4, 0.1; β_2= 0.59, 0.40, 0.21.

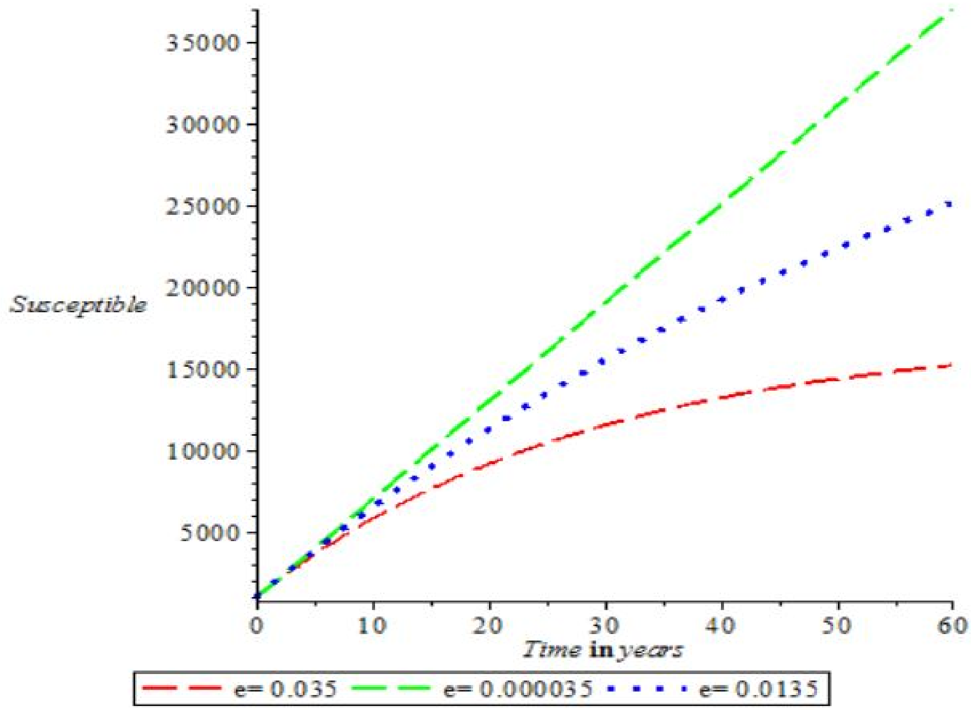

Figure 2. Graph of Susceptible against time with recruitment rate (e) from terrorist group

.

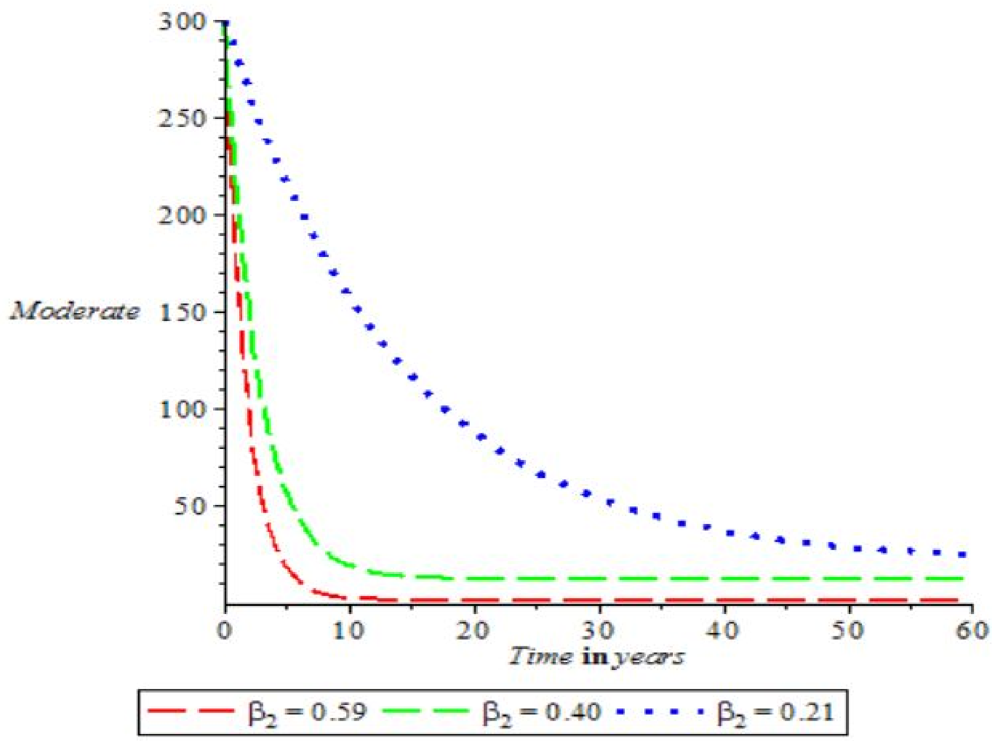

Figure 3. Graph of Moderate group of terrorist against time with different rate of contact β_2

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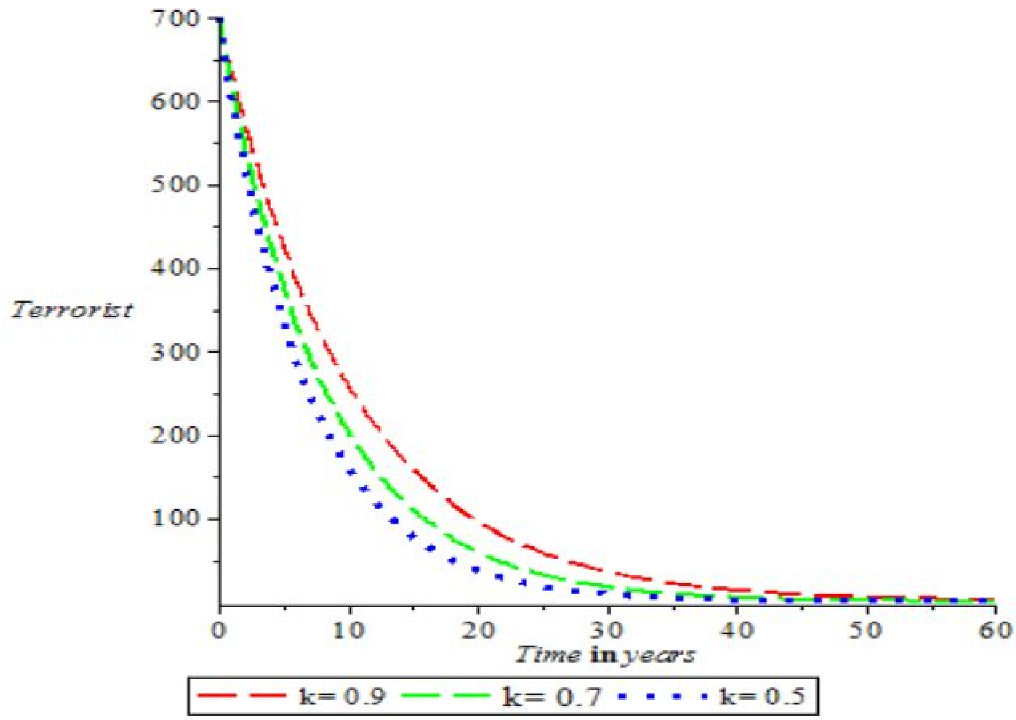

FIGURE 4. Graph of terrorist group against time with different rate (k)

.

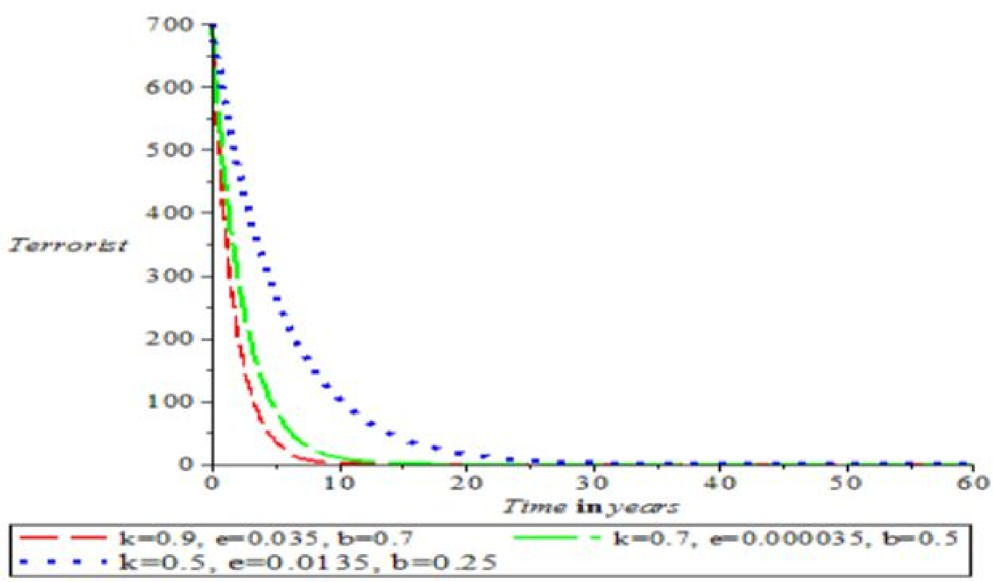

FIGURE 5. Graph of terrorist group against time with different rates

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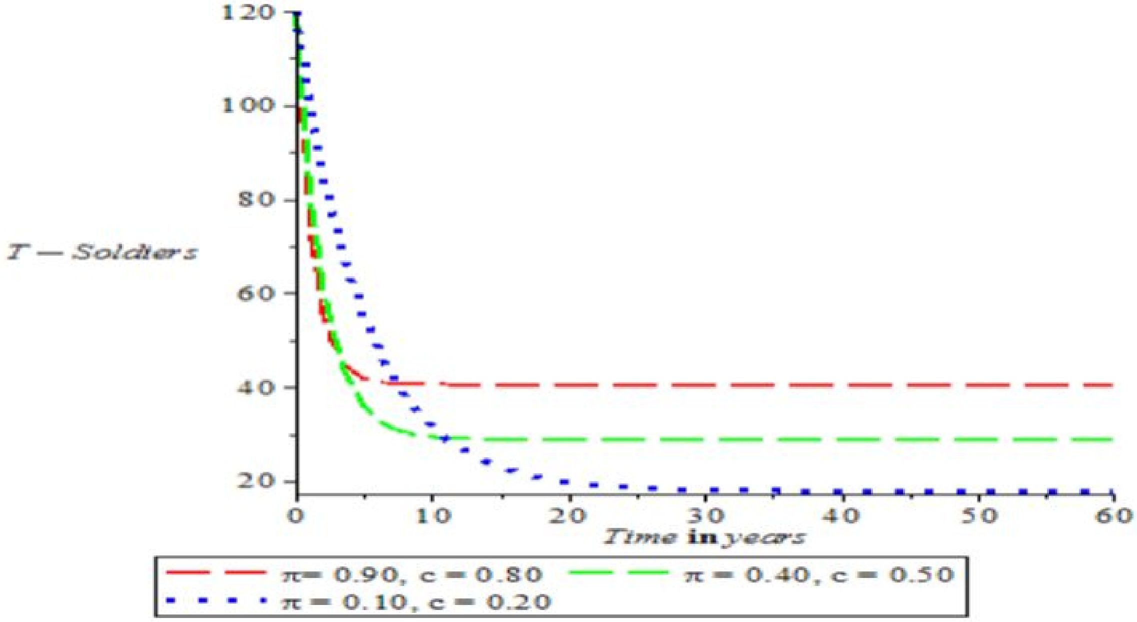

FIGURE 6. Graph of Foot soldiers of terrorist against time with rate of military strategies π

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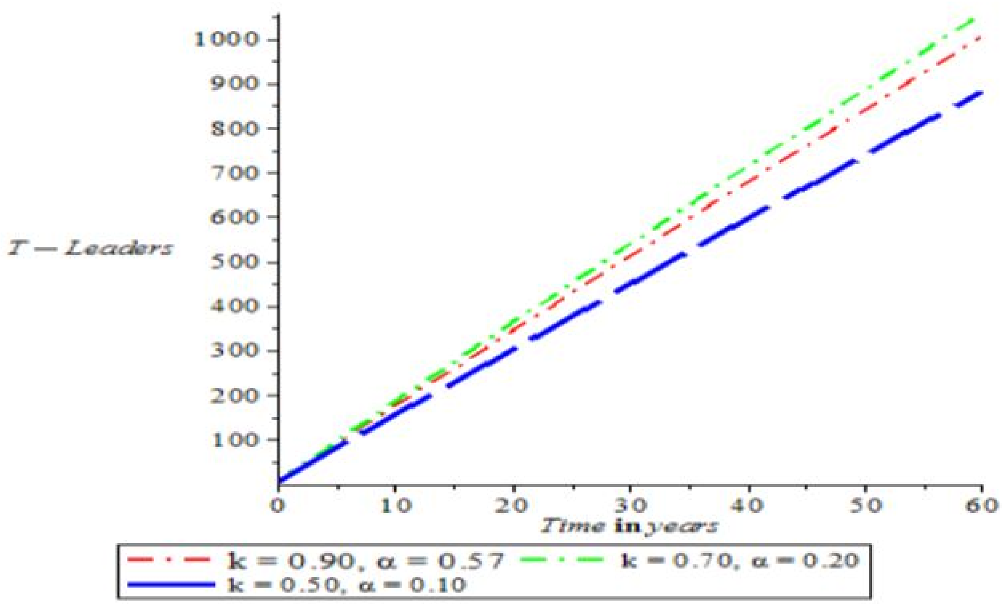

FIGURE 7. Graph of terrorist leaders against time with different rate of contact

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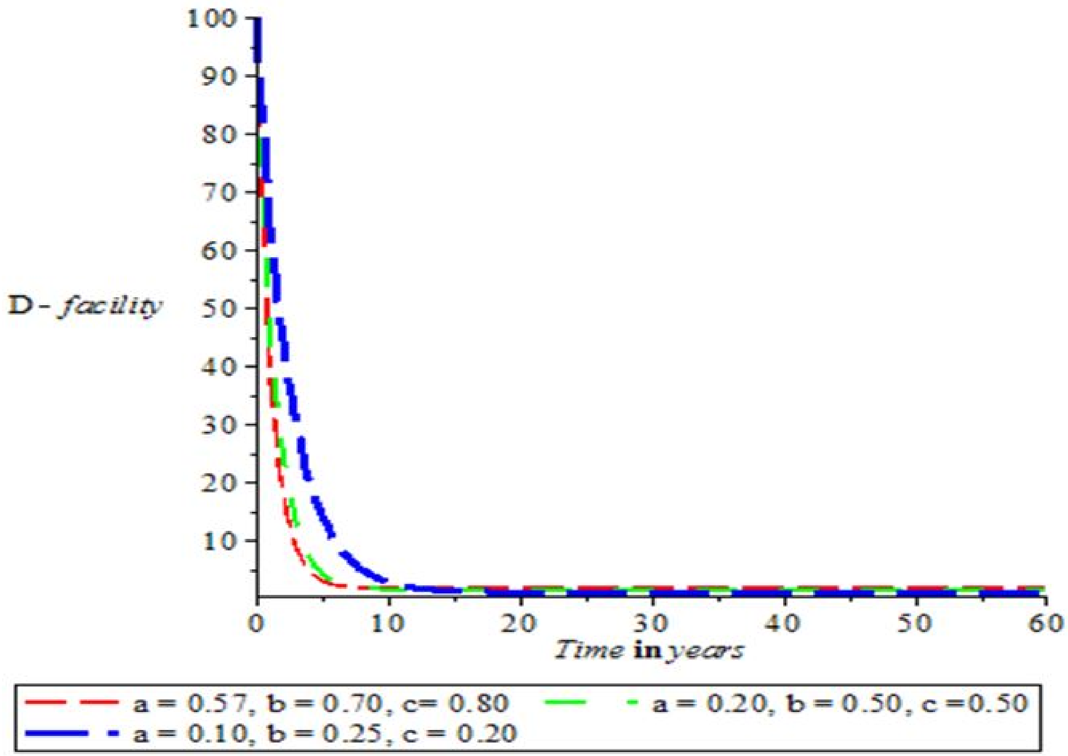

FIGURE 8. Graph of Detention facility against time with different rates

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9. Results and Discussion

The counter-terrorist measures have focused primarily on reducing foot soldiers of terrorist, if possible leaders, but not necessary leaders. Because of decentralized nature of some terrorist organization, these measures have proven unsuccessful. Therefore, this research focuses on military/dialogue strategies on moderates group, terrorist group, terrorist foot-soldiers and terrorist leaders to reduce the way they recruit their new members. But surveillance should also be put in place by counter terrorist operation on susceptible individuals.

The graph in Fig. 2 for a period of six years shows the recruitment pool of terrorists from the susceptible population with contact rate β_1 and increased the population of moderate individuals. This shows that with time, the moderate terrorist will fully convert to terrorist after being indoctrinated. This can be observed from the graph in fig 3-5.

The plots show the impact of counter-terrorist identification rate and self-identification of control mechanism on the removed terrorists. The probability that the interaction leads to militancy is α and proportion rate at which terrorist become leader is k. To combat the insurgency in this model, parameters α and k appear to be dynamics which counter terrorist operation need to check and force them to appoint weak leaders. However, Fig. 4 and 5 shows the effectiveness of counter-terrorism, where large number of terrorist are identified and removed. While Fig. 6 and 7 shows the difficulties in countering the leaders of terrorist organizations such as Al-Qaeda, Boko-Haram, ISIS and other terrorist organisations under counter-terrorist programme. Lastly, Fig. 8 shows a period of six years, detention facilities will be close due to effective intervention/rehabilitation programmed being designed.
Finally, from the simulations above, it can be deduced that The terrorists removed effectively with at least 70%. Therefore, the effectiveness of counter-terrorist measures is shown in figures 2-4 for a period of six years.

10. Conclusion

This paper focused on controlling terrorism using military approach /dialogue to target and strike the terrorist group. However, Boko Haram has suffered significant defeats as a result of efforts by the Multinational Joint Task Force, which includes forces from Benin, Cameroon, Chad, Niger and Nigeria. The task force also receives support from the United States. For instance, Nigeria recorded its second consecutive year of reductions with a 63 per cent drop to 1,832 deaths in 2016.The result of this research can further be extended to other fields of knowledge to study the pattern of terrorism in many parts of the world which pose a significant threat to the public. Therefore, plans are made to address other issues in future studies.

11. Recommendations

1 It is impossible to defeat terrorist organizations without reducing the strength of the organization by using military approach on their footsoldiers. Therefore, security agents should check the parameter k in this study.

2 Security agents should restrict their operations to both local and international media with respect to the video footage of the operations. Only Ministry of Defence has the right to release the video to the media.

3 Government should Propose a practical tool for practitioners dealing with terrorism, counter-terrorism measures and human rights.

4 Military control strategy should not only depends on removing leaders but also concentrate on recruitment poll and terrorism footsoldiers.

5 Military strategies cannot reduce the risk of extremists, radicals, headmen/bandits/cattle-rustlers alone, but law enforcement agencies should also apply dialogue strategy at the same time.

6 Government and NGO should implement policies for youth so as to reduce the level of poverty and unemployment in the society.

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