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Some Results on Soft Hypervector Spaces

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Abstract. In this paper, some basic properties of soft hypervector spaces are studied with respect to some well-known operations such as intersection, union, and, or, product and sum. Also, the behavior of them is investigated under linear transformations and b-linear transformations.

Keywords: Soft Set, Hypervector Space, Soft Subhyperspace, Linear Transformation.

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1. Introduction

The theory of soft sets, as a mathematical tool for modeling uncertainty, was introduced by Molodtsov [14] in 1999. Next Maji [11, 12] studied soft set theory and its application in a decision making problem. Afterward, it has been studied in various algebraic structures, for instance, soft BCK/BCI algebras by June [9], soft groups by Aktas [2], soft rings by Acar [1] and soft vector spaces by sezgin [18].

The theory of hyperstructures was introduced by Marty [13] in 1934 and has been studied in many branches of algebra (for example see [5], [6], [7] and [20]). A kind of hypervector spaces (there are several types) was presented by Tallini [19] in 1990 and investigated by Ameri [4] and the author [8] and [17].

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Recently, the theory of soft sets is affected algebraic hyperstructures, for example to hypervector spaces ([10], [15] and [16]). In this paper, we study some basic properties of soft hypervector spaces, specially their behavior under some specific operations and linear transformations. Now we present some definitions and simple properties that we shall use in later.

Definition 1.1. [19] Let K be a field and $(V,+)$ be an Abelian group. Then a hypervector space over K is the quadruple $(V,+,\circ,K)$, where "$\circ : K \times V \longrightarrow P_*(V)$" is an external hyperoperation such that for all $a, b \in K$ and $x, y \in V$ the following conditions hold ($P_*(V)$ is the set of all non-empty subset of V):

(1) $a \circ (x+y) \subseteq a \circ x + a \circ y$;
(2) $(a+b) \circ x \subseteq a \circ x + b \circ x$;
(3) $a \circ (b \circ x) = (ab) \circ x$;
(4) $a \circ (-x) = (-a) \circ x = -(a \circ x)$;
(5) $x \in 1 \circ x$.

A non-empty subset W of hypervector space V is called a subhyperspace of V if W is itself a hypervector space with the external hyperoperation on V, i.e. $x - y \in W$ and $a \circ x \subseteq W$, for all $x, y \in W$ and $a \in K$. In this case we write $W \leqslant V$.

Example 1.2. [19] In $(\mathbb{R}^2,+)$ define the product times a scalar in $\mathbb{R}$ by $a \circ x =$ the line pass origin and point x, for all non-zero $x \in \mathbb{R}^2$ and $a \circ \underline{0} = \{\underline{0}\}$. Then $(\mathbb{R}^2,+,\circ,\mathbb{R})$ is a hypervector space.

Example 1.3. [8] $(\mathbb{Z},+,\circ,\mathbb{Q})$ is a hypervector space, where "$+$" is the usual sum and the mapping "$\circ$" is defined by the following:

$$\begin{cases} \circ : \mathbb{Q} \times \mathbb{Z} \longrightarrow P_*(\mathbb{Z}) \\ \dfrac{r}{s} \circ n = \{m(rn) : m \in \mathbb{Z}\}. \end{cases}$$

Definition 1.4. [14] Let U be an universe set, E be a set of parameters, $P(U)$ be the power set of U and $A \subseteq E$. Then a pair (F,A) is called a soft set over U, where F is a mapping defined by $F : A \to P(U)$. The support of (F,A) is defined by $Supp\,(F,A) = \{x \in A : F(x) \neq \emptyset\}$.

Definition 1.5. [11] Let (F,A) and (G,B) be soft sets over U. Then

(1) The intersection of (F,A) and (G,B) is denoted by $(F,A) \sqcap (G,B)$ and defined as the soft set $(H, A \cap B)$, where $H(x) = F(x) \cap G(x)$;
(2) The union of (F,A) and (G,B) is denoted by $(F,A) \sqcup (G,B)$ and defined as the soft set $(H, A \cup B)$, where $H(x) = F(x)$ for $x \in A \setminus B$, $H(x) = G(x)$ for $x \in B \setminus A$ and $H(x) = F(x) \cup G(x)$ for $x \in A \cap B$;

(3) (F, A) AND (G, B) is denoted by $(F, A) \wedge (G, B)$ and defined as the soft set $(H, A \times B)$, where $H(x, y) = F(x) \cap G(y)$;
(4) (F, A) OR (G, B) is denoted by $(F, A) \vee (G, B)$ and defined as the soft set $(H, A \times B)$, where $H(x, y) = F(x) \cup G(y)$.
(5) (F, A) is called a soft subset of (G, B) and denoted by $(F, A) \sqsubseteq (G, B)$, if $A \subseteq B$ and for all $x \in A$, $F(x)$ and $G(x)$ are identical approximations.

Definition 1.6. [3] Let (F, A) and (G, B) be soft sets over U. Then
(1) The extended intersection of (F, A) and (G, B) is denoted by $(F, A) \sqcap_{\varepsilon} (G, B)$ and defined as the soft set $(H, A \cup B)$, where $H(x) = F(x)$ for $x \in A \setminus B$, $H(x) = G(x)$ for $x \in B \setminus A$ and $H(x) = F(x) \cap G(x)$ for $x \in A \cap B$;
(2) The restricted union of (F, A) and (G, B) is denoted by $(F, A) \sqcup_{\Re} (G, B)$ and defined as the soft set $(H, A \cap B)$, where $A \cap B \neq \emptyset$ and $H(x) = F(x) \cup G(x)$.

2. Basic Operations on Soft Hypervector Spaces

In this section we investigate the behavior of soft hypervector spaces under basic operations such as intersection, union, AND, OR, product and sum.

Definition 2.1. [16] Let (F, A) be a soft set over a hypervector space $(V, +, \circ, K)$. Then (F, A) is called a soft hypervector space over V if $F(x)$ is a subhyperspace of V, for all $x \in A$.

Example 2.2. Consider the hypervector space $V = (\mathbb{R}^2, +, \circ, \mathbb{R})$ in Example 1.2. Define $F : \mathbb{R}^2 \to P(V)$ by $F(x) = a \circ x$, for fixed $a \in \mathbb{R}$. Then $(F, \mathbb{R}^2)$ is a soft hypervector space over $\mathbb{R}^2$. But if define $G : \mathbb{R}^2 \to P(\mathbb{R}^2)$ by $G(x, y) = \{(x, y) \in mathbbR^2; \sqrt{x^2 + y^2} \leqslant 1\}$, then the soft set $(G, \mathbb{R}^2)$ is not a soft hypervector space over $\mathbb{R}^2$.

Example 2.3. Consider the hypervector space $(\mathbb{Z}, +, \circ, \mathbb{Q})$ in Example 1.3. Then $(F, \mathbb{N})$ is a soft hypervector space over $\mathbb{Z}$, where $F : \mathbb{N} \to P(\mathbb{Z})$ defined by $F(n) = n\mathbb{Z}$.

Proposition 2.4. [16] *Let (F, A) and (G, B) be soft hypervector spaces over V. Then*
(1) *if $A \cap B \neq \emptyset$ and $F(x) \cap G(x) \neq \emptyset$, for all $x \in A \cap B$, then $(F, A) \sqcap (G, B)$ is a soft hypervector space of V;*
(2) *if $(F, A) \sqsubseteq (G, B)$, then $(F, A) \sqcup (G, B)$ is a soft hypervector space of V;*
(3) *$(F, A) \wedge (G, B)$ is a soft hypervector space of V.*

Proposition 2.5. *Let (F, A) and (G, B) be soft hypervector spaces over V. Then*

(1) $(F,A) \sqcap_{\varepsilon} (G,B)$ *is a soft hypervector space of* V*;*
(2) *if* A *and* B *are disjoint, then* $(F,A) \sqcup (G,B)$ *is a soft hypervector space of* V*;*
(3) *if* $F(x)$ *and* $G(x)$ *are ordered by inclusion, for all* $x \in A \cap B$*, then* $(F,A) \sqcup_{\Re} (G,B)$ *is a soft hypervector space of* V*;*
(4) *if* $F(x)$ *and* $G(y)$ *are ordered by inclusion, for all* $(x,y) \in A \times B$*, then* $(F,A) \vee (G,B)$ *is a soft hypervector space of* V*.*

Proof. (1) Let $(F,A) \sqcap_{\varepsilon} (G,B) = (M, A \cup B)$, where

$$M(x) = \begin{cases} F(x) & \text{if } x \in A \setminus B, \\ G(x) & \text{if } x \in B \setminus A, \\ F(x) \cap G(x) & \text{if } x \in A \cap B. \end{cases}$$

If $x \in A \setminus B$, then $M(x) = F(x) \leqslant V$. If $x \in B \setminus A$, then $M(x) = G(x) \leqslant V$ and if $x \in A \cap B$, then $M(x) = F(x) \cap G(x) \leqslant V$. Thus $M(x) \leqslant V$ for all $x \in A \cup B$. Hence $(F,A) \sqcap_{\varepsilon} (G,B)$ is a soft hypervector space over V.

(2) Let $(F,A) \sqcup (G,B) = (T, A \cup B)$, where

$$M(x) = \begin{cases} F(x) & \text{if } x \in A \setminus B, \\ G(x) & \text{if } x \in B \setminus A, \\ F(x) \cup G(x) & \text{if } x \in A \cap B. \end{cases}$$

Note that since $A \cap B = \emptyset$, either $x \in A \setminus B$ or $x \in B \setminus A$, for all $x \in A \cup B$. If $x \in A \setminus B$, then $T(x) = F(x)$ is a subhyperspace of V and if $x \in B \setminus A$, then $T(x) = G(x)$ is a subhyperspace of V. Thus $(T, A \cup B)$ is a soft hypervector space over V.

(3) Let $(F,A) \sqcup_{\Re} (G,B) = (S, A \cap B)$, where $S(x) = F(x) \cup G(x)$ for all $x \in A \cap B \neq \emptyset$. If $x \in A \cap B$, then $S(x) = F(x) \leqslant V$ or $S(x) = G(x) \leqslant V$. Hence $(S, A \cap B)$ is a soft hypervector space over V.

(4) Let $(F,A) \vee (G,B) = (N, A \times B)$, where $N(x,y) = F(x) \cup G(y)$ for all $(x,y) \in A \times B$. Then $F(x) \cup G(y) = F(x) \leqslant V$ or $F(x) \cup G(y) = G(x) \leqslant V$. Thus $(N, A \times B)$ is a soft hypervector space over V.

□

Note that the condition "disjoint" is necessary in item (2) of above Proposition. For see this, consider the hypervector space $V = (\mathbb{R}^2, +, \circ, \mathbb{R})$ in Example 1.2. Suppose $A = B = \mathbb{R}^2$ and $F : \mathbb{R}^2 \to P(\mathbb{R}^2)$ defined by $F(x,y) = x - axis$ and $G : \mathbb{R}^2 \to P(\mathbb{R}^2)$ defined by $G(x,y) = y - axis$. Then $(F,A) \sqcup (G,B)$ is not a soft hypervector space of V, because for all $(x,y) \in \mathbb{R}^2$, $F(x,y) \cup G(x,y) \nleqslant \mathbb{R}^2$.

Definition 2.6. Let (F, A) and (G, B) be soft hypervector spaces over V and W, respectively. Then the product of (F, A) and (G, B) is defined as $(F, A) \times (G, B) = (U, A \times B)$, where $U(x, y) = F(x) \times G(y)$, for all $(x, y) \in A \times B$.

Proposition 2.7. *If $(V, +_1, \circ_1, K)$ and $(W, +_2, \circ_2, K)$ are hypervector spaces over K, then $(V \times W, +, \circ, K)$ is a hypervector space over K, where $(v, w) + (v', w') = (v +_1 v', w +_2 w')$ and $a \circ (v, w) = \{(x, y) : x \in a \circ_1 v,\ y \in a \circ_2 w\}$.*

Proof. We show that $(V \times W, +, \circ, K)$ satisfies in the conditions of Definition 1.1.

1)

$$\begin{aligned} a \circ ((v, w) + (v', w')) &= a \circ (v +_1 v', w +_2 w') \\ &= \{(x, y) : x \in a \circ_1 (v +_1 v'), y \in a \circ_2 (w +_2 w')\} \\ &\subseteq \{(x, y) : x \in a \circ_1 v +_1 a \circ_1 v', \\ &\quad\ y \in a \circ_2 w +_2 a \circ_2 w'\} \\ &= \{(r +_1 r', s +_2 s') : r \in a \circ_1 v, r' \in a \circ_1 v', \\ &\quad\ s \in a \circ_2 w, s' \in a \circ_2 w'\} \\ &= \{(r, s) : r \in a \circ_1 v, s \in a \circ_2 w\} \\ &\quad +\{(r', s') : r' \in a \circ_1 v', s' \in a \circ_2 w'\} \\ &= a \circ (v, w) + a \circ (v', w'). \end{aligned}$$

2) It is similar to (1).
3)

$$\begin{aligned} a \circ (b \circ (v, w)) &= \bigcup_{(x,y) \in b \circ (v,w)} (a \circ (x, y)) \\ &= \bigcup_{x \in b \circ_1 v, y \in b \circ_2 w} \{(r, s) : r \in a \circ_1 x, s \in a \circ_2 y\} \\ &= \{(r, s) : r \in a \circ_1 (b \circ_1 v), s \in a \circ_2 (b \circ_2 w)\} \\ &= \{(r, s) : r \in (ab) \circ_1 v, s \in (ab) \circ_2 w\} \\ &= (ab) \circ (v, w). \end{aligned}$$

4)

$$\begin{aligned}a\circ(-(v,w)) &= a\circ(-v,-w)\\ &= \{(x,y): x\in a\circ_1(-v), y\in a\circ_2(-w)\}\\ &= \{(x,y): x\in(-a)\circ_1 v, y\in(-a)\circ_2 w\}\\ &= (-a)\circ(v,w)\\ &= \{(x,y): x\in -(a\circ_1 v), y\in -(a\circ_2 w)\}\\ &= -(a\circ(v,w)).\end{aligned}$$

5) $(v,w)\in 1\circ(v,w)=\{(x,y): x\in 1\circ_1 v, y\in 1\circ_2 w\}$.

□

Lemma 2.8. *If $(V,+_1,\circ_1,K)$ and $(W,+_2,\circ_2,K)$ are hypervector spaces over K, $V'\leqslant V$, and $W'\leqslant W$, then $V'\times W'\leqslant V\times W$.*

Proof. Let $(x,y),(r,s)\in V'\times W'$ and $a\in K$. Then $(x,y)-(r,s)=(x-_1 r, y-_2 s)\in V'\times W'$ and $a\circ(x,y)=\{(v,w): v\in a\circ_1 x, w\in a\circ_2 y\}\subseteq V'\times W'$. □

Theorem 2.9. *Let (F,A) and (G,B) be soft hypervector spaces over V and W, respectively. Then $(F,A)\times(G,B)$ is a soft hypervector space over $V\times W$.*

Proof. Let $(F,A)\times(G,B)=(U,A\times B)$, where $U(x,y)=F(x)\times G(y)$ for all $(x,y)\in A\times B$. Then $F(x)\leqslant V$ and $G(y)\leqslant W$. Thus by Lemma 2.8, $F(x)\times G(y)\leqslant V\times W$. Hence $(U,A\times B)$ is a soft hypervector space over $V\times W$. □

Definition 2.10. Let (F,A) and (G,B) be soft hypervector spaces over V. Then the sum of (F,A) and (G,B) is defined as $(F,A)+(G,B)=(H,A\times B)$, where $H(x,y)=F(x)+G(y)$, for all $(x,y)\in A\times B$.

Theorem 2.11. *If (F,A) and (G,B) are soft hypervector spaces over V, then $(F,A)+(G,B)$ is a soft hypervector space over V.*

Proof. Let $(F,A)+(G,B)=(H,A\times B)$, where $H(x,y)=F(x)+G(y)$, for all $(x,y)\in A\times B$. If $(x,y)\in A\times B$, then $F(x)\leqslant V$ and $G(y)\leqslant V$. It is easy to verify that the sum of two arbitrary subhyperspaces of V is a subhyperspace of V. Thus $F(x)+G(y)\leqslant V$ and so $(F,A)+(G,B)$ is a soft hypervector space over V. □

The Propositions 2.4 and 2.5 and the Theorem 2.11 are satisfied for any non-empty family $\{(F_i,A_i)\}_{i\in I}$ of soft hypervector spaces over V. Also the Theorem 2.9 is hold for any non-empty family $\{(F_i,A_i)\}_{i\in I}$ of soft hypervector spaces over $\{V_i\}_{i\in I}$.

Definition 2.12. Let (F, A) and (G, B) be soft hypervector spaces over V. Then (F, A) is called a soft subhyperspace of (G, B) if $A \subseteq B$ and $F(x) \leqslant G(x)$, for all $x \in A$.

Example 2.13. Consider the hypervector space $V = (\mathbb{R}^2, +, \circ, \mathbb{R})$ in Example 1.2. Suppose $A = \mathbb{R}^2$ and $B = \{(x, y) \in \mathbb{R}^2 : xy \geq 0\}$. Define $F : A \to P(V)$ by $F(x, y) = a \circ (x, y)$ and $G : B \to P(V)$ by $G(x, y) = a \circ (x, y)$. Then (G, B) is a soft subhyperspace of (F, A).

Theorem 2.14. *Let (F, A), (G, B) and (H, A) be soft hypervector spaces over V. Then*

(1) *if $A \subseteq B$ and $F(x) \subseteq G(x)$ for all $x \in A$, then (F, A) is a soft subhyperspace of (G, B);*
(2) *$(F, A) \sqcap (G, B)$ is a soft subhyperspace of (F, A) and (G, B);*
(3) *$(F, A) \sqcap_\varepsilon (H, A)$ is a soft subhyperspace of (F, A) and (H, A);*
(4) *$(F, A) \sqcup (H, A)$ is a soft subhyperspace of (H, A), if $F(x) \subseteq H(x)$, for all $x \in A$;*
(5) *$(F, A) \sqcup_{\mathfrak{R}} (G, B)$ is a soft subhyperspace of (G, B), if $F(x) \subseteq G(x)$, for all $x \in A \cap B$;*
(6) *if $A \subseteq B$ and $F(x) \subseteq G(x)$, for all $x \in A$, then $(F, A) \wedge (G, B)$ is a soft subhyperspace of $(G, B) \wedge (G, B)$;*
(7) *if $A \subseteq B$, $F(x)$ and $G(y)$ are ordered by inclusion, for all $(x, y) \in A \times B$, $G(x) \subseteq G(y)$, for all $x, y \in B$ and $F(x) \subseteq G(x)$, for all $x \in A$, then $(F, A) \vee (G, B)$ is a soft subhyperspace of $(G, B) \vee (G, B)$;*
(8) *if $A \subseteq B$ and $F(x) \subseteq G(x)$, for all $x \in A$, then $(F, A) \times (G, B)$ is a soft subhyperspace of $(G, B) \times (G, B)$;*
(9) *if $A \subseteq B$ and $F(x) \subseteq G(x)$, for all $x \in A$, then $(F, A) + (G, B)$ is a soft subhyperspace of $(G, B) + (G, B)$.*

Proof. (1) If $F(x) \subseteq G(x)$, then $F(x) \leqslant G(x)$, for all $x \in A$. Thus (F, A) is a soft subhyperspace of (G, B);
(2) It follows from (1), since the intersection of subhyperspaces of V is a subhyperspace of V;
(3) Let $(F, A) \sqcap_\varepsilon (H, A) = (Q, A)$ where $Q(x) = F(x) \cap H(x)$ for all $x \in A$. Then $Q(x) \subseteq F(x)$ and $Q(x) \subseteq H(x)$ for all $x \in A$, so by (1) the result is hold;
(4) Let $(F, A) \sqcup (H, A) = (S, A)$ where $S(x) = F(x) \cup H(x)$ for all $x \in A$. Then $S(x) \subseteq H(x)$ for all $x \in A$ and so by (1) the result is hold;
(5) Let $(F, A) \sqcup_{\mathfrak{R}} (G, B) = (N, A \cap B)$ where $N(x) = F(x) \cup G(x)$ for all $x \in A \cap B \neq \emptyset$. Then $N(x) \subseteq G(x)$ for all $x \in A \cap B$ and so by (1) the result is hold;

(6) Let $(F, A) \wedge (G, B) = (Q, C)$ where $C = A \times B$ and $Q(x, y) = F(x) \cap G(y)$. Then $A \times B \subseteq B \times B$ and $F(x) \cap G(y) \subseteq G(x) \cap G(y)$. Thus by (1) the result is hold;
(7) Let $(F, A) \vee (G, B) = (Q, C)$ where $C = A \times B$ and $Q(x, y) = F(x) \cup G(y)$. Then $A \times B \subseteq B \times B$, $F(x) \cup G(y) \leqslant V$ and $G(x) \cup G(y) \leqslant V$ such that $F(x) \cup G(y) \subseteq G(x) \cup G(y)$. Thus $F(x) \cup G(y) \leqslant G(x) \cup G(y)$ and so $(F, A) \vee (G, B)$ is a soft subhyperspace of $(G, B) \vee (G, B)$;
(8) Let $(F, A) \times (G, B) = (H, A \times B)$ where $H(x, y) = F(x) \times G(y)$. Then $A \times B \subseteq B \times B$ and $F(x) \times G(y) \subseteq G(x) \times G(y)$. Thus by (1) the result is hold;
(9) Let $(F, A) + (G, B) = (H, A \times B)$ where $H(x, y) = F(x) + G(y)$. Then $A \times B \subseteq B \times B$ and $F(x) + G(y) \leqslant G(x) + G(y)$. Thus $(F, A) + (G, B)$ is a soft subhyperspace $(G, B) + (G, B)$.

□

3. Soft Hypervector Spaces under Linear Transformations

Let V and W be hypervector spaces, (F, A) and (G, B) be soft sets over V and W, respectively and $f : V \to W$ be a mapping. Then the soft set $(f(F), Supp(F, A))$ over W is defined by $f(F) : Supp(F, A) \to P(W)$ where $f(F)(x) = f(F(x))$ for all $x \in Supp(F, A)$. It is clear that $Supp(F, A) = Supp(f(F), Supp(F, A))$. Moreover, if f is an onto mapping, then $(f^{-1}(G), Supp(G, B))$ is a soft set over V such that $f^{-1}(G) : Supp(G, B) \to P(V)$ is given by $f^{-1}(G)(y) = f^{-1}(G(y))$ for all $y \in Supp(G, B)$. Similarly, $Supp(G, B) = Supp(f^{-1}(G), Supp(G, B))$.

Proposition 3.1. *Let $T : V \to W$ be a good linear transformation between hypervector spaces (i.e. $T(x + y) = T(x) + T(y)$ and $T(a \circ x) = a \circ T(x)$). If (F, A) is a soft hypervector space over V, then $(T(F), Supp(F, A))$ is a soft hypervector space over W.*

Proof. For any $x \in Supp(F, A)$, $F(x) \neq \emptyset$, so $T(F)(x) = T(F(x)) \neq \emptyset$. It is easy to see that the image of any subhyperspace of V under T is a subhyperspace of W. Thus $T(F(x))$ is a subhyperspace of W. Hence $(T(F), Supp(F, A))$ is a soft hypervector space over W. □

Proposition 3.2. *Let $T : V \to W$ be an onto good linear transformation between hypervector spaces. If (G, B) is a soft hypervector space over W, then $(T^{-1}(G), Supp(G, B))$ is a soft hypervector space over V.*

Proof. For any $y \in Supp(G, B)$, $G(y) \neq \emptyset$, so $T^{-1}(G)(y) = T^{-1}(G(y)) \neq \emptyset$. It is easy to verify that the inverse image of any subhyperspace of W under T is a subhyperspace of V. Thus $T^{-1}(G(y))$ is a subhyperspace of V. Hence $(T^{-1}(G), Supp(G, B))$ is a soft hypervector space over V. □

Theorem 3.3. *Let $T: V \to W$ be a good linear transformation between hypervector spaces. If (F, A) and (G, B) are soft hypervector spaces over V such that (G, B) is a soft subhyperspace of (F, A), then $(T(G), Supp(G, B))$ is a soft subhyperspace of $(T(F), Supp(F, A))$.*

Proof. For any $x \in Supp(G, B)$, $G(x)$ is a subhyperspace of $F(x)$. Then $T(G)(x) = T(G(x))$ is a subhyperspace of $T(F)(x) = T(F(x))$. Hence $(T(G), Supp(G, B)$ is a soft subhyperspace of $(T(F), Supp(F, A))$. □

Theorem 3.4. *Let $T: V \to W$ be an onto good linear transformation between hypervector spaces. If (F, A) and (G, B) are soft hypervector spaces over W such that (G, B) is a soft subhyperspace of (F, A), then $(T^{-1}(G), Supp(G, B))$ is a soft subhyperspace of $(T^{-1}(F), Supp(F, A))$.*

Proof. For all $y \in Supp(G, B))$, $G(y)$ is a subhyperspace of $F(y)$. Thus $T^{-1}(G)(y) = T^{-1}(G(y))$ is a subhyperspace of $T^{-1}(F)(y) = T^{-1}(F(y))$. Hence $(T^{-1}(G), Supp(G, B))$ is a soft subhyperspace of $(T^{-1}(F), Supp(F, A))$. □

Definition 3.5. Let (F, A) and (G, B) be soft hypervector spaces over V and W, respectively. Then the pair (T, f) of mappings $T: V \to W$ and $f: A \to B$ is called a soft mapping from (F, A) to (G, B).
A soft mapping (T, f) is called a soft linear transformation if T is a good linear transformation and $T(F(x)) = G(f(x))$, for all $x \in A$.
If T and f are bijection, then (T, f) is said to be a soft b-linear transformation and denoted by $(F, A) \simeq_b (G, B)$.

Theorem 3.6. *Let (F, A), (G, B) and (H, C) be soft hypervector spaces over U, V and W, respectively. If (T, f) is a soft linear transformation from (F, A) to (G, B) and (T^*, f^*) is a soft linear transformation from (G, B) to (H, C), then $(T^* \circ T, f^* \circ f)$ is a soft linear transformation from (F, A) to (H, C).*

Proof. By the hypothesis $T: U \to V$ and $T^*: V \to W$ are good linear transformations and $f: A \to B$ and $f^*: B \to C$ are mappings such that $T(F(x)) = G(f(x))$ and $T^*(G(y)) = H(f^*(y))$, for all $x \in A$ and $y \in B$. Then $T^* \circ T: U \to W$ is a good linear transformation such that

$$\begin{aligned}(T^* \circ T)(F(x)) &= T^*(T(F(x))) \\ &= T^*(G(f(x))) \\ &= H(f^*(f(x))) \\ &= H((f^* \circ f)(x)).\end{aligned}$$

for all $x \in A$. Thus $(T^* \circ T, f^* \circ f)$ is a soft linear transformation from (F, A) to (H, C). □

Theorem 3.7. *The relation $\simeq_b$ is an equivalence relation on soft hypervector spaces.*

Proof. Let (F,A) be a soft hypervector space over V. Then $(F,A) \simeq_b (F,A)$ under the soft b-linear transformation (I_V, I_A), where I_V and I_A are the identity functions of V and A, respectively. Thus $\simeq_b$ is reflexive. Now if (F,A) and (G,B) are soft hypervector spaces over V_1 and V_2, respectively, such that $(F,A) \simeq_b (G,B)$, then there exist a bijection linear transformation $T : V_1 \to V_2$, and a bijection $f : A \to B$ which satisfy $T(f(x)) = G(f(x))$, for all $x \in A$. Hence $T^{-1} : V_2 \to V_1$ is a bijection linear transformation and $f^{-1} : B \to A$ is a bijections such that $T(F(f^{-1}(y))) = G(f(f^{-1}(y))) = G(y)$ and so $T^{-1}(G(y)) = T^{-1}(T(F(f^{-1}(y)))) = F(f^{-1}(y))$, for all $y \in B$. Therefore $(G,B) \simeq_b (F,A)$ and $\simeq_b$ is symmetric. Also by Theorem 3.6, $\simeq_b$ is transitive. Consequently, $\simeq_b$ is an equivalent relation. □

Theorem 3.8. *Let (F,A) and (G,B) be soft sets over hypervector spaces V and W, respectively, such that $(F,A) \simeq_b (G,B)$. If (F,A) is a soft hypervector space over V, then (G,B) is a soft hypervector space over W.*

Proof. For any $y \in Supp(G,B)$, there exists $x \in A$ such that $y = f(x)$. Then $F(x) \leqslant V$ and so $G(y) = G(f(x)) = T(F(x)) \leqslant W$. Thus (G,B) is a soft hypervector space over W. □

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