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n-fold Commutative Hyper K-ideals

P. Babari [1], M. Pirasghari [2] and M. M. Zahedi [3]
Department of Mathematics, Faculty of Mathematics,
Tarbiat Modares University, Tehran, Iran.
[1] p.babari@modares.ac.ir
[2] m.pirasghari@modraes.ac.ir
[3] zahedi_mm@modares.ac.ir

Abstract. In this paper, we are supposed to introduce the definitions of n-fold commutative, and implicative hyper K-ideals. These definitions are the generalizations of the definitions of commutative, and implicative hyper K-ideals, respectively, which have been defined in [12]. Then we obtain some related results. In particular we determine the relationships between n-fold implicative hyper K-ideal and n-fold commutative hyper K-ideals of a hyper K-algebra of order 3, which satisfy the simple condition. Then, generally we study n-fold commutative hyper K-ideals in simple hyper K-algebras.

Keywords: Hyper K-algebra; Weak hyper K-ideal; Hyper K-ideal; n-fold Commutative, Implicative hyper K-ideals, Simple condition.

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1. Introduction

The theory of hyper compositional structure has been introduced by F. Marty in 1934 during the 8th congress of Scandinavian Mathematicians, where he presented his work [10]. Today the research in the hyper compositional structures field is very vivid. In particular Y. B. Jun, M. M. Zahedi, X. L. Xin and R. A. Borzooei introduced the notions of hyper BCK-algebra and hyper K-algebra in 2000[4,8]. The concepts of an n-fold commutative, and implicative hyper K-ideals are the generalizations of the concepts of commutative, and implicative hyper K-ideals, respectively, which are related to the concepts of commutative, and implicative ideals of a BCK-algebra[15]. The relationships between the types of commutative hyper K-ideals have been studied by M.M. Zahedi and T. Roodbari[13]. They defined 9 types of commutative hyper K-ideals, and an implicative hyper K-ideal. They proved some propositions and theorems in this field. Now in this manuscript we define 9 types of n-fold commutative hyper K-ideals, and we obtain some relationships. Then we define an n-fold implicative hyper K-ideal and we determine the relationships between n-fold implicative hyper K-ideal and n-fold commutative hyper K-ideals of a hyper K-algebra of order 3, which satisfies the simple condition. Finally we study n-fold commutative hyper K-ideals in all simple hyper K-algebras, and we give some results on them.

2. Preliminaries

In this paper we use the definitions of hyper K-algebra and hyper K-ideal as the most important definitions.

Definition 2.1 (4)**.** Let H be a nonempty set, and " $\circ$ " be a hyperoperation on H, that " $\circ$ " is a function from $H \times H$ to $P^*(H)=P(H)$-$\varnothing$. Then H is called a hyper K-algebra if it contains "0" and satisfies the following axioms:

$HK-1$ $\quad (x \circ z) \circ (y \circ z) < x \circ y$;
$HK-2$ $\quad (x \circ y) \circ z = (x \circ z) \circ y$;
$HK-3$ $\quad x < x$;
$HK-4$ $\quad x < y,\, y < x \Rightarrow x = y$;
$HK-5$ $\quad 0 < x$;

for all $x, y, z \in H$, where $x < y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H, A < B$ is defined by $\exists a \in A$, $\exists b \in B$ such that $a < b$.

Note that if $A, B \subseteq H$, then by $A \circ B$ we mean that the subset $\bigcup a \circ b$ of H for all $a \in A$ and $b \in B$.

Theorem 2.2 (2)**.** *Let $(H, \circ, 0)$ be a hyper K-algebra. Then for all $x, y, z \in H$ and for all non-empty subsets A, B and C of H the following relations hold:*

(1) $\quad (x \circ y) < z \Leftrightarrow (x \circ z) < y$;
(2) $\quad (x \circ z) \circ (x \circ y) < (y \circ z)$;
(3) $\quad x \circ (x \circ y) < y$;

(4) $x \circ y < x$;

(5) $A \circ B < A$;

(6) $A \subseteq B \Rightarrow A < B$;

(7) $x \in x \circ 0$;

(8) $(A \circ C) \circ (A \circ B) < (B \circ C)$;

(9) $(A \circ C) \circ (B \circ C) < (A \circ B)$;

(10) $(A \circ B) < C \Leftrightarrow (A \circ C) < B$;

(11) $(A \circ C) \circ B = (A \circ B) \circ C$;

Theorem 2.3 (6). *Let x,y,z be some elements in hyper K-algebra H. Then the following hold:*

(1) $x < y$ implies that $z \circ y < z \circ x$,

(2) $x < y$ implies that $x \circ z < y \circ z$.

Definition 2.4 (2). Let I be a nonempty subset of a hyper K-algebra H and $0 \in I$. Then

(1) I is called a weak hyper K-ideal of H if $x \circ y \subseteq I$ and $y \in I$ imply that $x \in I$ for all $x, y \in H$.

(2) I is called a hyper K-ideal of H if $x \circ y < I$ and $y \in I$ imply that $y \in I$ for all $x, y \in H$.

Note that in any hyper K-algebra H, $\{0\} \subseteq H$ is a hyper K-ideal.

Theorem 2.5 (2). *Any hyper K-ideal of a hyper K-algebra H is a weak hyper K-ideal.*

Definition 2.6 (3). Let I be a nonempty subset of a hyper K-algebra H. Then we say that I is closed, whenever $x < y$, $y \in I$ imply that $x \in I$ for all $x, y \in H$.

Definition 2.7. Definition 1.7.[2] Let H be a hyper K-algebra. An element $a \in H$ is called a left (resp. right) scalar if $|a \circ x| = 1$(resp. $|x \circ a| = 1$) for all $x \in H$.

Theorem 2.8 (13). *Let I be a hyper K-ideal of a hyper K-algebra H. Then the following statements are equivalent:*

(1) $(x \circ y) < I$,

(2) $(x \circ y) \cap I \neq \varnothing$.

Definition 2.9 (12). A hyper K-algebra H is called simple if for all distinct elements $a, b \in H - 0$, $a \nless b$ and $b \nless a$.

Remark: However in [13] there is a definition for simple condition in hyper K-algebra with 3 elements, but regarding to Definition 1.9 we conclude that for a hyper K-algebra H={0,1,2}, we say that H satisfies the simple condition if $1 \nless 2$ and $2 \nless 1$.

Theorem 2.10 (12). *Let H satisfies the simple condition. Then,*

(i) $a \circ 0 = \{a\}$, for all $a \in H - \{0\}$,

(ii) $a \in a \circ b$, for all distinct elements $a, b \in H$,

(iii) $H - \{a\} \subseteq H \circ a$, for all $a \in H$,

(iv) $a \in b \circ c \Longleftrightarrow c \in b \circ a$, for all distinct elements $a, c \in H$, and $b \in H - \{0\}$,

(v) $x < x \circ a \Longleftrightarrow x \in x \circ a$, for all $a, x \in H$,

(vi) $A < A \circ b \Longleftrightarrow A \cap (A \circ b) \neq \varnothing$, for all $b \in H$ and $\varnothing \neq A \subseteq H$,

(vii) $(x \circ y) \circ z < x \circ (y \circ z)$, for all $x, y, z \in H$,

(viii) If $0 \in I \subseteq H$, then $A \circ B < I \Longleftrightarrow (A \circ B) \cap I \neq \varnothing$, for all non-empty subsets A and B of H.

In the rest of this paper, by H we denote a hyper K-algebra.

3. n-fold Commutative and Implicative Hyper K-ideals

In this section we define 9 types of n-fold commutative hyper K-ideals and n-fold implicative hyper K-ideal. Then we determine the relationships between n-fold implicative hyper K-ideal and n-fold commutative hyper K-ideals of a hyper K-algebra of order 3, which satisfies the simple condition.

Definition 3.1. Let I be a nonempty subset of a hyper K-algebra H such that $0 \in I$. If n is a natural number, then I is called an n-fold commutative hyper K-ideal of:

(i) type 1, if for all x,y,$z \in H$, $((x \circ y) \circ z) \cap I \neq \emptyset$, and $z \in I$, imply that $(x \circ (y \circ (y \circ x^n))) \subseteq I$.

(ii) type 2, if for all x,y,$z \in H$, $((x \circ y) \circ z) \cap I \neq \emptyset$, and $z \in I$, imply that $(x \circ (y \circ (y \circ x^n))) \cap I \neq \emptyset$.

(iii) type 3, if for all x,y,$z \in H$, $((x \circ y) \circ z) \cap I \neq \emptyset$, and $z \in I$, imply that $(x \circ (y \circ (y \circ x^n))) < I$.

(iv) type 4, if for all x,y,$z \in H$, $((x \circ y) \circ z) \subseteq I$, and $z \in I$, imply that $(x \circ (y \circ (y \circ x^n))) \subseteq I$.

(v) type 5, if for all x,y,$z$$\in$$H$, $((x\circ y)\circ z)\subseteq I$,and $z\in I$, imply that $(x\circ(y\circ(y\circ x^n)))\cap I\neq\emptyset$.

(vi) type 6, if for all x,y,$z$$\in$$H$, $((x\circ y)\circ z)\subseteq I$, and $z\in I$, imply that $(x\circ(y\circ(y\circ x^n)))<I$.

(vii) type 7, if for all x,y,$z$$\in$$H$, $((x\circ y)\circ z)<I$, and $z\in I$, imply that $(x\circ(y\circ(y\circ x^n)))\subseteq I$.

(viii) type 8, if for all x,y,$z$$\in$$H$, $((x\circ y)\circ z)<I$, and $z\in I$, imply that $(x\circ(y\circ(y\circ x^n)))\cap I\neq\emptyset$.

(ix) type 9, if for all x,y,$z$$\in$$H$, $((x\circ y)\circ z)<I$, and $z\in I$, imply that $(x\circ(y\circ(y\circ x^n)))<I$.

For simplicity of notation we use n-fold CHKI instead of n-fold Commutative Hyper K-Ideal .

Remark: From this definition, we conclude that the notions of 1-fold CHKI of type j and CHKI of type j of H coincide, for any $j = 1, 2, ..., 9$.

Theorem 3.2. *Let I be a hyper K-ideal of hyper K-algebra H. If I is an n-fold CHKI of type 3,5,6,8 or 9, then it is also n+1-fold CHKI of type 3,5,6,8, or 9, respectively.*

Proof: Let I be an n-fold CHKI of type 3. If $((x\circ y)\circ z)\cap I\neq\emptyset$,and $z\in I$, then ,$(x\circ(y\circ(y\circ x^n)))<I$. On the other hand by Theorem 1.2 we have $A\circ B<A$. Hence, we obtain $(y\circ x^n)\circ x = y\circ x^{n+1}<y\circ x^n$. Thus, we have $y\circ(y\circ x^n)<y\circ(y\circ x^{n+1})$, and $x\circ(y\circ(y\circ x^{n+1}))<x\circ(y\circ(y\circ x^n))$. Now $x\circ(y\circ(y\circ x^n))<I$ implies that $x\circ(y\circ(y\circ x^{n+1}))<I$. So I is n+1-fold CHKI of type 3.

The proof of the other types can be obtained by the same way.

Open problem: If I is an n-fold CHKI of type 1,2,...,or 9, then is it an n+1-fold CHKI of type 1,2,...,or 9, respectively?

Example 3.3. Considering the following hyper K-algebra on H={0,1,2}

$\circ$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0}	{0,1}
2	{2}	{2}	{0}

I = {0,1} is an n-fold CHKI of type 1,2,..., and 9, for any $n\in N$.

Theorem 3.4. *Let I be a nonempty subset of a hyper K-algebra H. Then the following statements hold:*

(1) If I is an n-fold CHKI of type 1, then it is of type 4,

(2) If I is an n-fold CHKI of type 2, then it is of type 3,

(3) If I is an n-fold CHKI of type 4, then it is of type 6,

(4) If I is an n-fold CHKI of type 5, then it is of type 6,

(5) If I is an n-fold CHKI of type 7, then it is of types 4, 5, and 9,

(6) If I is an n-fold CHKI of type 8, then it is of types 5 and 9,

(7) If I is an n-fold CHKI of type 9, then it is of type 3.

Proof:(1) Assume that I is an n-fold CHKI of type 1, $((x \circ y) \circ z) \subseteq I$, and $z \in I$. Since $((x \circ y) \circ z) \subseteq I$, so $((x \circ y) \circ z) \cap I \neq \varnothing$. Therefore $x \circ (y \circ (y \circ x^n)) \subseteq I$, because I is of type 1. Thus I is of type 4.

The proofs of other types can be obtained by the same way.

The following example shows that the converse of the statements of Theorem 2.4 are not true in general.

Example 3.5. The following table shows a hyper K-algebra structures on H $= \{0, 1, 2\}$.

$\circ$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0}	{1}
2	{2}	{0,1}	{0,1,2}

(1): Now we can see that: I={0,1} is an n-fold CHKI of type 6, while I is not of type 4, because $(2 \circ 0) \circ 1 \subseteq I$, but $2 \circ (0 \circ (0 \circ 2^n) \not\subseteq I$.

(2) I={0,1} is an n-fold CHKI of type 3 and 9, while I is not of type 2, because $(2 \circ 0) \circ 1 \subseteq I$, but $2 \circ (0 \circ (0 \circ 2^n)) \cap I = \varnothing$.

(3) I={0,1} is an n-fold CHKI of type 6, while I is not of type 5, because $(2 \circ 0) \circ 1 \subseteq I$, but $2 \circ (0 \circ (0 \circ 2^n)) \cap I = \varnothing$.

(4) I={0,2} is an n-fold CHKI of type 4, while I is not of type 1, because $(2 \circ 2) \circ 2 \cap I \neq \varnothing$, but $2 \circ (2 \circ (2 \circ 2^n)) \not\subseteq I$.

(5) I={0,2} is an n-fold CHKI of type 5, while I is not of type 7, because $(2 \circ 2) \circ 2 \cap I \neq \varnothing$, but $2 \circ (2 \circ (2 \circ 2^n)) \not\subseteq I$.

Theorem 3.6. *Let I be an n-fold CHKI of type 1 or 7. Then I is a hyper K-ideal.*

Proof: Let $(x \circ y) \cap I \neq \varnothing$, and $y \in I$. Then there exists $z \in (x \circ y) \cap I$. So we have $z \in ((x \circ 0) \circ y)$, Then $(x \circ 0) \circ y) \cap I \neq \varnothing$. Since I is of type 1, thus $x \circ (0 \circ (0 \circ x^n)) \subseteq I$. Therefore $x \in x \circ (0 \circ (0 \circ x^n))$ implies that $x \in I$. Hence I is a hyper K-ideal.

Theorem 3.7. *Let $0 \in H$ be a left scalar element of a hyper K-algebra H, and I be closed. If I is an n-fold CHKI of type 2 or 3, then I is a hyper K-ideal.*

Proof: Let $(x \circ y) \cap I \neq \varnothing$, $y \in I$. Then $((x \circ 0) \circ y) \cap I \neq \varnothing$. Since I is an n-fold CHKI of type 2, thus $(x \circ (0 \circ (0 \circ x^n))) \cap I \neq \varnothing$, by hypothesis $(x \circ 0) \cap I \neq \varnothing$. Therefore $x \circ 0 < I$. So there exists $k \in x \circ 0$ and $t \in I$ such that $k < t$, i.e. $x \circ t < 0$. Hence, there exists $z \in (x \circ t)$ such that $z < 0$. We have $z = 0$, thus $0 \in x \circ t$. Now since I is closed, we get that $x \in I$. Hence, I is a hyper K-ideal.

The proofs of other types are similar to above.

Theorem 3.8. *Suppose that $0 \in H$ is a right scalar element of a hyper K-algebra H, and I is a closed n-fold CHKI of type 5 or 6. Then I is a hyper K-ideal.*

Proof: The proof is similar to the proof of Theorem 2.7.

Definition 3.9. Let I be a non-empty subset of a hyper K-algebra $(H, \circ, 0)$ such that $0 \in I$ and $n \in N$. Then I is called an n-fold implicative hyper K-ideal if $x \circ (y \circ x^n) < I$ implies that $x \in I$.

Theorem 3.10. *Let $H = \{0, 1, 2\}$ be a hyper K-algebra of order 3 that satisfies the simple condition and $0 \neq I \subseteq H$. If I is an n-fold implicative hyper K-ideal, then I is an n-fold commutative hyper K-ideal of types i, for i = 1, 2,..., 9.*

Proof: Let I be an n-fold implicative hyper K-ideal. Without loss of generality assume that $I = \{0, 1\}$. By the proof of Theorem 4.15 of [1], H has the following hyper structure:

$\circ$	0	1	2
0	{0} or {0,1}	{0} or {0,1}	{0} or {0,1}
1	{1}	{0} or {0,1}	{1}
2	{2}	{2}	{0} or {0,1}

We show that I is an n-fold CHKI of type 1.

Consider the following different cases:

Case (i) . If $x = 0$ or $x = 1$, then for any $y \in H$, we have $0 \circ (y \circ (y \circ 0^n)) \subseteq I$ or $1 \circ (y \circ (y \circ 1^n)) \subseteq I$, and so in this case I is an n-fold CHKI of type 1.

Case (ii) . If x=2, then we consider the following two cases:

Case 1. $y = 0$ or $y = 1$. We can see that $((2 \circ y) \circ z) \cap I = \varnothing$ for any $z \in I$. Thus there is nothing to prove.

Case 2. $y = 2$. An easy computation shows that $2 \circ (2 \circ (2 \circ 2^n)) \subseteq I$. Hence, in this case the proof is complete.

The proof of the other types are similar to above .

Example 3.11. The following table shows a hyper K-algebra structure on H:

$\circ$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{0,1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$

In this example $I = \{0, 2\}$ is an n-fold CHKI of type 9, while it is not n-fold implicative hyper K-ideal, because $1\circ(0\circ 1^n) < I$, while $1 \notin I$.

$I = \{0\}$ is an n-fold CHKI of types 2 and 9, but it is not an n-fold implicative hyper K-ideal, because $1\circ(0\circ 1^n) < I$, while $1 \notin I$.

4. n-fold Commutative Hyper K-ideals in Simple Hyper k-algebras

In this part $(H, \circ, 0)$ is a simple hyper K-algebra, unless otherwise is stated.

Theorem 4.1. *Let $0 \in I \subseteq H$. Then:*

(i) I is an n-fold CHKI of type 1 if and only if I is an n-fold CHKI of type 7,

(ii) I is an n-fold CHKI of type 2 $(3, 8)$ if and only if I is an n-fold CHKI of type 9,

(iii) I is an n-fold CHKI of type 5 if and only if I is an n-fold CHKI of type 6.

Proof: The proof follows from Definition 3.1 and Theorem 1.10.

Theorem 4.2. *Let $a \circ a = \{0\}$, for all $a \in H$. Then $I = \{0\}$ is an n-fold commutative hyper k-ideal of type 4.*

Proof: Let $((x \circ y) \circ z) \subseteq I$, and $z \in H$. Then $x \circ y \subseteq (x \circ y) \circ 0 \subseteq I$, and so $x < y$. Thus $x = 0$ or $x = y$. If $x = 0$, then $(x \circ (y \circ (y \circ x^n))) = (0 \circ (y \circ (y \circ 0^n))) = 0 \circ (y \circ y) = 0 \circ 0 = 0 < I$. If $x = y$, then

$(y \circ (y \circ (y \circ y^n))) < (y \circ (y \circ 0)) = y \circ y = 0 < I$. Therefore I is an n-fold CHKI of type 4.

The following example shows that the condition "$a \circ a = \{0\}$ for all $a \in H$ " in the above theorem is necessary.

Example 4.3. The following table shows a simple hyper K-algebra on H={0,1,2}

$\circ$	0	1	2
0	{0}	{0}	{0,2}
1	{1}	{0,2}	{1,2}
2	{2}	{2}	{0}

We see that $1 \circ 1 \neq \{0\}$. Also $I = \{0\}$ is not an n-fold CHKI of type 4. Because $(0 \circ 1) \circ 0 = I$, while $0 \circ (1 \circ (1 \circ 0^n)) = \{0, 2\} \nsubseteq I$.

Theorem 4.4. *$I = \{0\}$ is an n-fold CHKI of type 7(1) if and only if $a \circ a = \{0\}$, for all $a \in H$.*

Proof: Let $I = \{0\}$ be an n-fold CHKI of type 7. Then $(y \circ y) \circ 0 < I$ and $0 \in I$imply that $y \circ y \subseteq (y \circ y) \circ 0 \subseteq y \circ (y \circ (y \circ y^n)) < I = \{0\}$. Thus $y \circ y = \{0\}$, for all $y \in H$.

The proof of the converse is similar to the proof of Theorem 3.2.

Theorem 4.5. *Let $a \in H - \{0\}$. Then $I = H - \{a\}$ is an n-fold CHKI of type 6(5).*

Proof: Let $(x \circ y) \circ z \subseteq I$ and $z \in I$. If $x = y$, since by Theorem 1.2 $0 \in x \circ x$, we have $0 \in (x \circ (y \circ (y \circ x^n)))$ and so $(x \circ (y \circ (y \circ x^n))) < I$. If $x \neq y$ then $x \in (x \circ 0) \subseteq x \circ (y \circ y) \subseteq x \circ (y \circ (y \circ x^n))$. Now we show that $x \neq a$. On the contrary let $x = a$. Then $x \neq z$ and so by Theorem 1.10(ii) $x \in (x \circ z) \subseteq (x \circ y) \circ z \subseteq I$, which is contradiction. Hence, $x \neq a$ implies $(x \circ (y \circ (y \circ x^n))) < I$. Therefore I is an n-fold CHKI of type 6.

Theorem 4.6. *Let $a \in H - \{0\}$ and $|a \circ x| = 1$, for all $x \in H - \{a\}$. Then $I = H - \{a\}$ is an n-fold CHKI of type 9(2,3,8).*

Proof: Let $(x \circ y) \circ z < I$ and $z \in I$. If $x = y$, then it is clear that $(x \circ (y \circ (y \circ x^n))) < I$. If $x \neq y$, we consider two cases: (i) $x \neq a$, (ii) $x = a$.

(i) In this case we obtain $x \in (x \circ (y \circ (y \circ x^n)))$ and so $(x \circ (y \circ (y \circ x^n))) < I$.

(ii) Since $|a \circ y| = |a \circ z| = 1$, then $\{a\} = a \circ z = (a \circ y) \circ z < I$. Thus there exists $t \in I$ such that $a < t$. So $a = 0$ or $a = t$, which is impossible. Therefore, I is an n-fold CHKI of type 9.

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