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Fractional Order Sliding Mode Observer-Based Control in the Presence of Faults

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Abstract:
A successfully validated and precise system model would greatly enhance the performance of the controller, making system identification a major procedure in control system design. The inverted pendulum is a highly nonlinear and open-loop unstable system that makes control more challenging. In this paper, at first, a novel fractional order sliding mode observer (FOSMO) is designed to estimate the state space of the rotary inverted pendulum, and after that, a fractional sliding fault estimation is proposed. The proposed observer had high accuracy and speed in fault and state observation because of the advantages of fractional calculus and the sliding mode observer method. The proposed observer is compared with the classical sliding mode observer.

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1. Introduction

In recent years, the design and development of newer control techniques have emerged to accomplish high performance from a system. In process industries, performance is dependent on many other systems or equipment and, interestingly, becomes nonlinear [1, 2]. During various operating conditions of the system, nonlinearities will present, which deteriorate overall performance. The nonlinearity problem can be overcome by choosing a suitable controller. To verify a new control theory, the inverted pendulum control can be considered a very good example of control engineering. The inverted pendulum is a highly nonlinear and open-loop unstable system, making control more challenging [3]. Recently, intentions to improve fault-diagnosis approaches have been drastically raised. A reliable failure detection process that can monitor sophisticated systems becomes mandatory [4]. Detecting failures, locating fault sources, and identifying each deficiency impact are the diagnosis's main purposes [5]. There are different methods for fault detection, which can generally be used by software analysis redundancy (observer) for fault detection. The most common observers used in studies include full-state observers, reduced-order observers, Parity Space, and so on [6]. In Du et al. [7], the H_∞ fault detection observer is designed for a fractional-order system. An adaptive fuzzy fast terminal sliding mode control is used to create a robust fault-tolerant control system for the care and swing-up control problem of the inverted pendulum-cart system, which is developed in the presence of actuator faults and external disturbances [8]. A radial basis function (RBF) neural network disturbance observer-based fractional order backstepping sliding mode control (SMC) is presented to the controller. This RBF neural network-based disturbance observer estimates unknown disturbances [9]. In Keijzer et al. [10], a sliding mode observer is proposed to estimate the disturbance. The sliding mode observer is one of the most popular methods for detecting and estimating faults and disturbances in different systems. The sliding mode method will be used in the design of the observer due to its advantages, such as inherent robustness against external disturbances, uncertainties, simplicity in design, etc. [11]. Adaptive fault detection technology can enhance fault detection performance by injecting a predesigned auxiliary input signal for a specific fault. Cao et al. [12] investigate a reconciliatory input design problem for both achieving control objectives and improving fault detection performance. An exemplary algorithm for the reconciliatory input design is proposed using a trajectory optimization approach. Today, fractional calculus is one of the most popular methods of controller design and system modeling due to its advantages, such as increasing the stability area, increasing the robustness of the system against disturbing

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factors, reducing the chattering phenomenon caused by the sliding model method, and having a long-term memory [13-16]. In this paper, a novel Fractional Order Sliding Mode Observer (FOSMO) is proposed to estimate the state space and fault of the system. The proposed observer is compared with the classical sliding mode observer.

2. Fractional Calculus

The generalized version of integer computations is called fractional calculus. Basically, different definitions are presented for fractional calculus. Caputo derivative definition of β order is defined as follows [17]:

$$ {}^{C}D_t^{\beta} f(t) = \frac{1}{\Gamma(n-\beta)} \int_{\beta}^{t} (t-\tau)^{\beta-n+1} f^{n}(\tau) d\tau ,\ n-1<\beta<n \tag{1}$$

The following is the definition of a fractional integral:

$$ I_t^{\beta} f(t) = \frac{1}{\Gamma(\beta)} \int_{\beta}^{t} (t-\tau)^{\beta-1} f(\tau) d\tau = D_{0,t}^{-\beta} f(t) \tag{2}$$

3. The Model of Rotary Pendulum

The rotary motion inverted pendulum, which is shown in Figure 1, is driven by a rotary servo motor system.

Figure 1. **Rotary inverted pendulum**

The servo motor drives an independent output gear whose angular position is measured by an encoder. One end of the horizontal rotating arm is connected to and driven by a DC motor, and the other end is connected to the swing rod without additional control between them. The rotary pendulum arm is mounted on the output gear. The pendulum is attached to a hinge instrumented with another encoder at the end of the pendulum arm. The nonlinear and complete model of the rotary inverted pendulum system is given in [3]. After simplifying the dynamic equations of the system as two degrees of freedom, we have state-space equations [18]:

$$\begin{bmatrix} \dot\theta \\ \dot\alpha \\ \ddot\theta \\ \ddot\alpha \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 39.32 & -14.52 & 0 \\ 0 & 81.78 & -13.98 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot\theta \\ \dot\alpha \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 28.54 \\ 24.59 \end{bmatrix} V_m \tag{3}$$

$$ Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot\theta \\ \dot\alpha \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{4}$$

where θ and α are servo load gear angle (radians) and pendulum arm deflection (radians), respectively. $\dot\theta$ and $\dot\alpha$ are the angular velocities of θ and α. V_m is the input voltage. According to Equation 3, there are two sensor faults (f_1, f_2) in this model. In this paper, the goal is to propose a novel fractional sliding mode observer to estimate the faults.

4. Proposed Method

The fault estimation strategy is schematized in Figure 2.

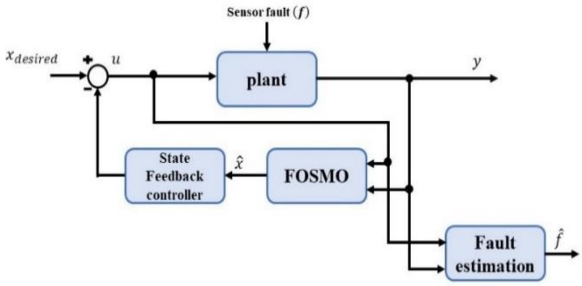

Figure 2. **Scheme of the proposed method**

The class of system model is considered with the following form. It is assumed that only sensor faults occur in the system.

$$\begin{aligned} \dot X(t) &= A\,x(t) + B\,u(t) \\ y(t) &= C\,x(t) + F\,f_s \end{aligned} \tag{5}$$

where $x \in R^n, u \in R^m, y \in R^p$ and $f_s \in R^q$, denote the vector of state variables, inputs, outputs, and sensor faults, respectively. And $A \in R^{n\times n}, B \in R^{n\times m}, C \in R^{p\times n}, F \in R^{p+q}$.

Lemma 1: Consider a coordinate transformation $Z = Tx = \binom{z_1}{z_2}, W = Sy = \binom{w_1}{w_2}$ under assumption 1.

Applying the change of coordinates, the triple (A, B, C) has the form:

$$\begin{aligned} TAT^{-1} &= \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad TB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \\ SCT^{-1} &= \begin{bmatrix} C_1 & 0 \\ 0 & C_4 \end{bmatrix}, \quad SF = \begin{bmatrix} 0 \\ F_2 \end{bmatrix} \end{aligned} \tag{6}$$

where:

$$T = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \in R^{n\times n},\ S = \begin{pmatrix} S_1 \\ S_2 \end{pmatrix} \in R^{p\times p}$$
$$T_1 \in R^{2\times n},\ S_1 \in R^{1\times p},\ Z_1 \in R^2$$
$$B_1 \in R^{2\times m},\ W_1 \in R^2,\ A_1 \in R^{2\times 2} \qquad (7)$$
$$A_4 \in R^{(n-2)\times(n-2)}, \qquad C_1 \in R^{2\times 2}$$
$$C_4 \in R^{p\times(n-2)}, \qquad F_2 \in$$

The coordinate transformation matrices T and S are introduced, and system (5) is converted into the following two subsystems (8) and (9). The first subsystem is as follows:

$$\begin{aligned} \dot{Z}_1 &= A_1 Z_1 + A_2 Z_2 + B_1 U \\ W_1 &= C_1 Z_1 \end{aligned} \qquad (8)$$

The second subsystem is as follows;

$$\begin{aligned} \dot{Z}_2 &= A_3 Z_1 + A_4 Z_2 + B_2 U \\ W_2 &= C_4 Z_2 + F_2 f_S \end{aligned} \qquad (9)$$

4.1. Classical Sliding Mode State Observer

For the system in (3), a sliding mode state observer is designed like (10):

$$\begin{aligned} \dot{\hat{Z}} &= A\hat{Z} + Bu - G_l e_y(t) + G_n v \\ W &= \hat{Y} = C\hat{Z} \end{aligned} \qquad (10)$$

where $e_y = \hat{y} - y$ is the output estimation error. The design freedom is associated with the gains $G_l \in R^{n\times p}$, which are design matrices to be determined. The matrices $G_n \in R^{p\times p}$ determined:

$$G_n = \begin{bmatrix} 0 \\ -I_P \end{bmatrix} \rightarrow I_P = eye(p,p) \qquad (11)$$

The vector v is defined by:

$$v = -\rho \frac{e_y}{\|e_y\|} \qquad \text{if } e_y \neq 0 \qquad (12)$$

where ρ is a positive parameter.

Proposition1. If there exists a matrix, G_l and a Lyapunov matrix P of the form [19]:

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & T^T P_0 T \end{bmatrix} > 0 \qquad (13)$$

where $P_1 \in R^{(n-p)\times(n-p)}$ and $P_o \in R^{p\times p}$ which satisfies.

$$PA_o + A_o^T P < 0 \qquad (14)$$

4.2. Fractional Order Sliding Mode Fault Estimation

The state space equation of the system is defined as (5). Considering the proposed following auxiliary variable as (15):

$$Z_j = \sigma_j + P_j D^\beta e_j \qquad (15)$$

where $j = 0,1$ and P_j indicates the switching gain and other parameters are proposed as follows:

$$\begin{cases} \sigma_j = Z_j - h_j \\ D^\beta h_j = A_j + B_j U + \hat{F}_j \\ D^\beta e_j = sign(\sigma_j) \end{cases} \qquad (16)$$

In which $\hat{F}_j$ is the fault estimated, and e_j is the initial value of $-\sigma_j(0)/P_j e_j$ and, in the finite time, converges to zero. Setting $\sigma_j = 0$, the estimation of the system fault ($\hat{F}_j$) is derived as follows:

$$\hat{F}_j = \theta_{1.j} Z_j + \theta_{2.j} sign(Z_j) + P_j sign(\sigma_j) \qquad (17)$$

In which $\theta_{1.j}, \theta_{2.j}$, and P_j are positive constants. Taking the time-derivative of (18), the following Equation is obtained:

$$\dot{Z}_j = \dot{\sigma_j} + P_j D^\beta e_j \qquad (18)$$

Putting equations, the following equation is deduced.

$$\dot{Z}_j = -\theta_{1.j} Z_j - \theta_{2.j} sign(Z_j) \qquad (19)$$

Stability proof: Choosing the Lyapunov function by (20), the system stability can be proven [20, 21]:

$$\begin{aligned} V_j &= \tfrac{1}{2} Z_j^2 \\ \dot{V}_j &= Z_j \dot{Z}_j \end{aligned} \qquad (20)$$

Putting (19) in (20), (21) is obtained:

$$\dot{V}_j = Z_j\left(-\theta_{1.j} Z_j - \theta_{2.j} sign(Z_j)\right) \le 0 \qquad (21)$$

4.3. Fractional Order Sliding Mode State Observer

The error signal is defined as (22):

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} \theta - \hat{\theta} \\ \alpha - \hat{\alpha} \end{bmatrix} \qquad (22)$$

The fractional sliding surface is as follows:

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} e_1 + k_1 D^{-\beta} e_1 \\ e_2 + k_2 D^{-\beta} e_2 \end{bmatrix} \qquad (23)$$

Taking the time-derivative of $e_{1,2}$ and replacing it in (23) yields:

$$\begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \end{bmatrix} = \begin{bmatrix} \dot{e}_1 + k_1 D^{1-\beta} e_1 \\ \dot{e}_2 + k_2 D^{1-\beta} e_2 \end{bmatrix} \qquad (24)$$

In this paper, to enhance robustness, the active sliding mode control signal is considered as (25). The state observer can be defined as follows:

$$\begin{bmatrix} \hat{\theta} \\ \hat{\alpha} \end{bmatrix} = \begin{bmatrix} k_3 S_1 + k_4\, sign(S_1) + k_1 D^{1-\beta} e_1 \\ k_5 S_2 + k_6\, sign(S_2) + k_2 D^{1-\beta} e_2 \end{bmatrix} \qquad (25)$$

where k_{1-6} are constant and positive. The parameters of fault estimators and state estimators are listed in Table 1.

Table 1. The parameters of estimators

$\boldsymbol{\beta}$	0.9
$\boldsymbol{\theta_{1,j}}$	8,10
$\boldsymbol{\theta_{2,j}}$	6,8
$\boldsymbol{P_j}$	0.1
$\boldsymbol{k_1, k_3, k_4}$	2,10,9
$\boldsymbol{k_2, k_5, k_6}$	4,15,11

4.4. State Feedback Controller

State Feedback is applied to control the load gear angle and pendulum arm deflection. From the model of the system, it is found that the poles are placed at $0, +7.54, -4.93$, and -17.12. As one of the poles is on the right-hand location of the S-plane, the system is considered to be unstable. With definition feedback vector $k = [k_1 \cdots k_n]$ state feedback controller is defined through the following expression [22]:

$$u = -kx \tag{26}$$

where k is chosen so that:

$$\dot{x} = (A - Bk)x \tag{27}$$

All the poles of the closed-loop system should be located in the desired places. Also, in this research, the estimation of state variables from the observers in the controller was used. In this paper, when solving the design requirements, the dominant pole location required is -4-3i and -4+3i, and the other two poles are arbitrarily -2 and -6 feedback gain as per the pole assignment with the nominal model, which is found to be $k = [-0.2674 \quad 7.7110 \quad -0.8324 \quad 0.9247]$.

5. Simulation Results

A fractional order sliding mode fault estimation is designed and proposed in the previous sections. Here for the initial conditions in the simulation are selected as: $\theta(0) = 0, \alpha(0) = \frac{\pi}{6}, \dot{\theta}(0) = 0$ and $\dot{\alpha}(0) = 0$ where the unit of the angles is radians.

5.1. Abrupt Faults

The proposed estimation according to (12) is designed. The performances of proposed estimation is shown in Figure 3 and compared with classical SMO estimation. The f_1 and f_2 are applied to the system. As shown in Figures 3 and 4, the proposed method has been able to estimate faults in the system with high speed and accuracy.

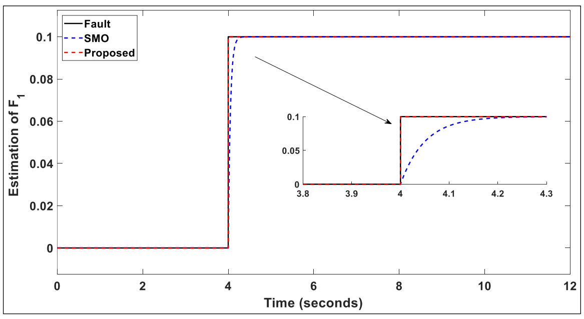

Figure 3. The estimation of f_1

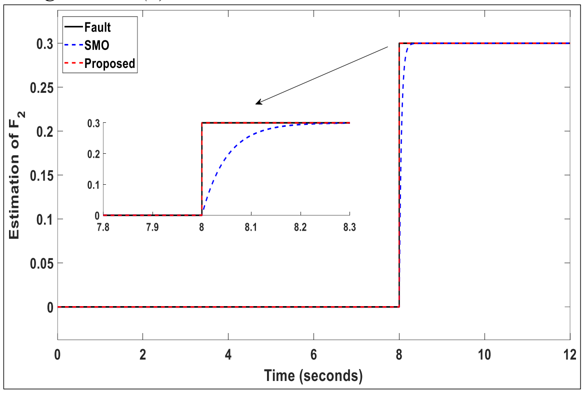

Figure 4. The estimation of f_2

The estimation of θ and α variables is shown in Figures 5 and 6. The proposed method has been able to estimate the variable with high accuracy and speed, while the classical sliding model method has a high estimation error and signal vibration.

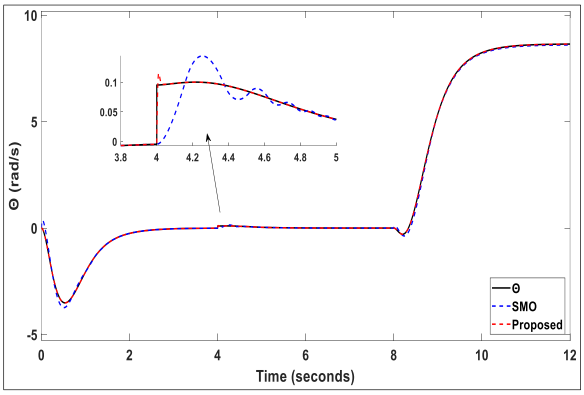

Figure 5. The estimation of θ

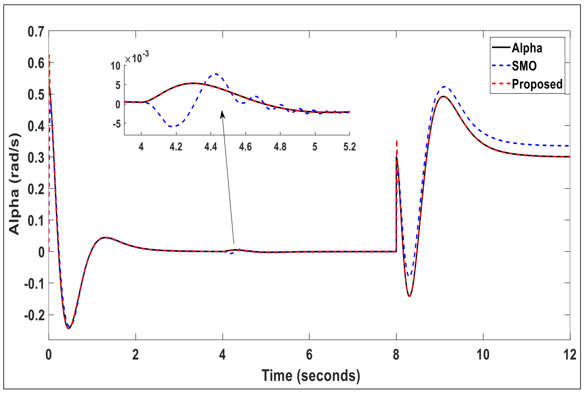

Figure 6. The estimation of α

For a more accurate comparison, the Integral Absolute Error (IAE) between the proposed and classical sliding mode observer is listed in Table 2. Figures 7 and 8 show the error of the state space estimation by the proposed method and SMO.

Table 2. The IAE of Abrupt faults

IAE	$\hat{\alpha}$	$\hat{\theta}$
SMO	0.1398	0.2237
Proposed	$4.424\, e^{-7}$	$1.903\ e^{-5}$

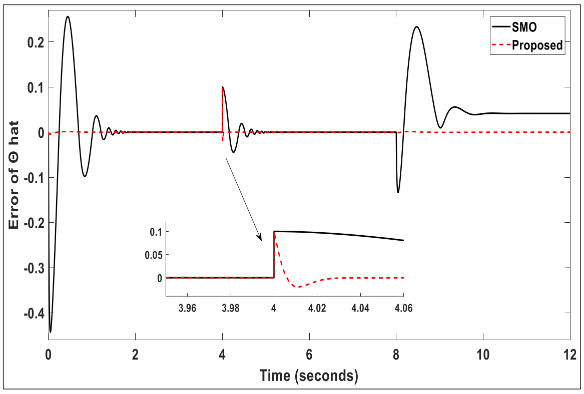

Figure 7. The error of θ estimation

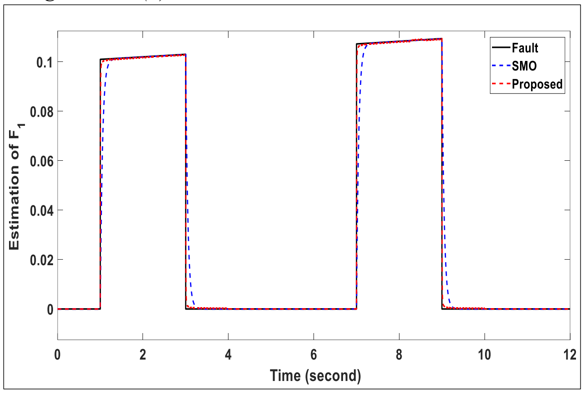

Figure 9. The estimation of intermittent of f_1

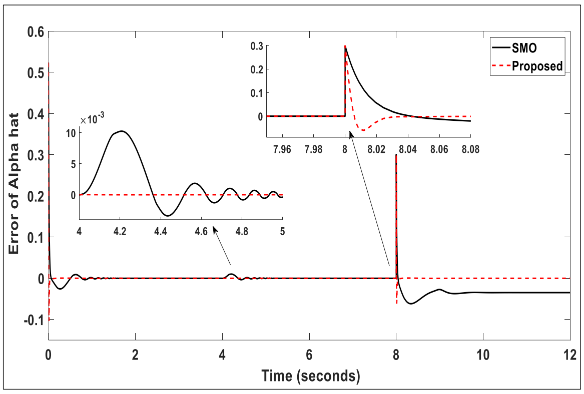

Figure 8. The error of α estimation

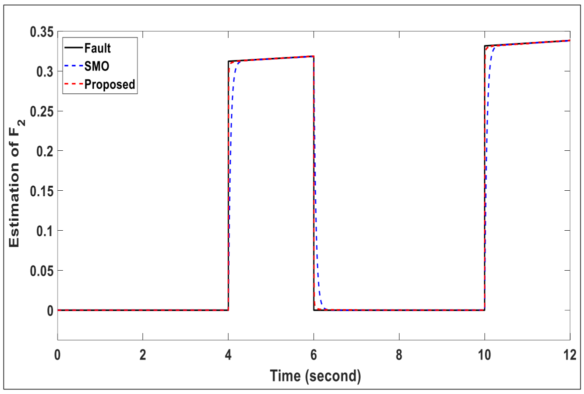

Figure 10. The estimation of intermittent of f_2

The proposed method has less error than the compared method. The proposed method was able to return to the main estimation path after about 0.03 seconds when the fault occurred, which shows the superiority and high robustness of the proposed method in estimation.

5.2. Intermittent Faults

In this section, system faults are considered intermittent faults and applied to the system. The modeled faults are considered according to the following (28) and (29):

$$f_1^{(intermittent)} = \begin{cases} 0 & t < 1 \\ e^{0.01t} & 1 \le t < 3 \\ 0 & 3 \le t < 7 \\ e^{0.01t} & 7 \le t < 9 \\ 0 & t \ge 10 \end{cases} \tag{28}$$

$$f_2^{(intermittent)} = \begin{cases} 0 & t < 4 \\ e^{0.01t} & 4 \le t < 6 \\ 0 & 6 \le t < 10 \\ e^{0.01t} & t \ge 10 \end{cases} \tag{29}$$

The performances of the proposed SMO estimation methods are shown in Figures 9 and 10.

As shown in Figures 9 and 10, the proposed method can estimate the fault applied to the system with a good speed with a low error, which shows the superiority of the proposed method over the classic sliding model observer.

6. Conclusion

In this paper, two fractional order sliding mode observers are proposed to estimate the state space variables and fault estimation. The performance of the proposed method has been studied in the presence of two types of abrupt faults and intermittent faults.

According to Table 2, the proposed method has better performance than the classical sliding mode observer. The estimation error of the proposed method was approximately equal to $1e^{-5}$.

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