

Single-Valued Intuitionistic Neutrosophic Sets

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ABSTRACT. In this paper, the notions of single-valued intuitionistic neutrosophic sets and single-valued intuitionistic neutrosophic graphs are introduced. Also, the concepts of degree and total degree of a node in single-valued intuitionistic neutrosophic graphs are presented. Then the concepts of irregular single-valued intuitionistic neutrosophic graphs, highly irregular neutrosophic intuitionistic single-valued intuitionistic neutrosophic graphs and neighborly irregular single-valued intuitionistic neutrosophic graphs are expressed.

Keywords: Neutrosophic sets, Intuitionistic fuzzy sets, Single-valued intuitionistic neutrosophic sets, Single-valued intuitionistic neutrosophic graphs, Irregular single-valued intuitionistic neutrosophic graphs, Highly irregular single-valued intuitionistic neutrosophic graphs, Neighborly irregular single-valued intuitionistic neutrosophic graphs.

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1. INTRODUCTION

Cantor believed that values of elements in a set are either zero or one. This cannot manage ambiguity and uncertainty. In 1965, Zadeh [38] instituted fuzzy theory because he believed that values of elements in a fuzzy set are between zero and one. Human life has been greatly impacted by graphs in various contexts. The theory of fuzzy graphs by Rosenfeld in 1975, [21] have numerous applications in various fields of human life, including computer science, expert systems, medical diagnosis, etc. In 1983, Atanassov [1, 2, 3] generalized fuzzy sets and presented intuitionistic fuzzy sets. Rashmanlou et al. [18] studied on interval valued intuitionistic (s, t)-fuzzy graphs. F. Smarandache [29, 30, 31, 32, 33, 34] introduced notion of neutrosophic sets and gave a mathematical tool for managing problems including ambiguous, uncertainty and inconsistent data. Shi et al. [27] investigated the main energies of picture fuzzy graphs. M Shoaib et al. [28] studied on complex pythagorean fuzzy graphs. Rashmanlou et al. [36, 20, 26] studied on vague graphs and their properties. Shao et al. [25] studied on fuzzy decision making in medical diagnosis using vague sets. Kosari et al. [9] studied on topological indices in fuzzy graphs. Kaviyarasu et al. [8] discussed the use of t-neutrosophic fuzzy graphs to address circular economy strategies. Chen et al. [6] discussed on elementary abelian covers of the wreath graph $W(3, 2)$ and the foster graph. Broumi et al. [5] defined the notion of single valued neutrosophic graph. Gnaana Bhargava et al. [7] presented neighbourly irregular graphs. Nagoor Gani and Radha [12, 13] introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. Kosari et al. [10] studied on perfectly regular fuzzy graphs. Nagoor Gani and Latha [11] studied neighbourly, highly and totally irregular fuzzy graphs. Nagoor Gani et al. [14] studied some types of irregular intuitionistic fuzzy graphs. The Nandhinis [15] introduced strongly irregular fuzzy graphs and strongly total irregular fuzzy graphs also they established some of their properties. Rashmanlou et al. [19] studied on highly irregular bipolar fuzzy graphs. Poulik et al. [16] investigated on certain indices of graphs under bipolar fuzzy environment. poulik et al. [17] established wiener index in bipolar fuzzy graphs. Talebi et al. [35] investigated some types of irregular intuitionistic fuzzy graphs. Santhi Maheswari et al. [22, 23, 24] researched for neighbourly and strongly edge irregular fuzzy graphs.

2. PRELIMINARIES

Herein, we review some definitions and notions for the next sections.

Definition 2.1. [38] Assume that $\mathfrak{U}$ represent the nonempty set. A *fuzzy set*(FS) A is symbolized as $\{\langle u, \delta_A(u) \rangle \mid u \in \mathfrak{U}\}$, such that δ_A is a membership function from $\mathfrak{U}$ to the interval $[0, 1]$.

Definition 2.2. [1, 2] Consider a collection $\mathfrak{U}$ made up of different objects. An *intuitionistic fuzzy set* (IFS), is represented by

$$B = \{\langle u, \alpha_B(u), \beta_B(u) \rangle \mid u \in \mathfrak{U}\}$$

such that $\alpha_B : \mathfrak{U} \rightarrow [0, 1]$ and $\beta_B : \mathfrak{U} \rightarrow [0, 1]$ referred the degree of membership(DM) and the degree of non-membership(DNM) of u in A . It is important to note that for any $u \in \mathfrak{U}$, the relation $0 \leq \alpha_A(u) + \beta_A(u) \leq 1$ must hold true. The class of *IFSs* on the universe $\mathfrak{U}$ will be referred to as $IFS(\mathfrak{U})$.

Definition 2.3. [34] Consider a collection $\mathfrak{U}$ made up of different objects, then the *neutrosophic set* (NS) A can be presented by $A = \{\langle u, \mathfrak{T}_A(u), \mathfrak{I}_A(u), \mathfrak{F}_A(u) \rangle \mid u \in \mathfrak{U}\}$. Here, the functions $\mathfrak{T}, \mathfrak{I}, \mathfrak{F} : \mathfrak{U} \rightarrow]0^-, 1^+[$ assigned by the DM, the degree of indeterminacy(DI), and the DNM of the element $u \in \mathfrak{U}$ to the set A . It is important to note that the condition for these functions is

$$0^- \leq \mathfrak{T}_A(u) + \mathfrak{I}_A(u) + \mathfrak{F}_A(u) \leq 3^+.$$

These functions $\mathfrak{T}_A(u), \mathfrak{I}_A(u)$ and $\mathfrak{F}_A(u)$ are subsets of $]0^-, 1^+[$ that can be either real standard or nonstandard. For practical issues, the concept of SVNS instead of NS was presented by Wang et al.

Definition 2.4. [37] Consider a collection $\mathfrak{U}$ made up of different objects. A SVNS A is presented by $A = \{\langle u, (\mathfrak{T}_A(u), \mathfrak{I}_A(u), \mathfrak{F}_A(u)) \rangle; u \in \mathfrak{U}\}$. These functions give values between 0 and 1.

Definition 2.5. [4] An ordered pair $\mathfrak{G}^* = (\mathfrak{V}, \mathfrak{E})$ is referred to as a graph. $\mathfrak{V}$ and $\mathfrak{E}$ represents the set of nodes and edges of $\mathfrak{G}^*$, respectively. If both $\mathfrak{V}$ and $\mathfrak{E}$ be finite, then $\mathfrak{G}^* = (\mathfrak{V}, \mathfrak{E})$ is finite.

Definition 2.6. The number of edges in $\mathfrak{G}^*$ that is coincident with node v is called the degree of v , or simply $d_{\mathfrak{G}^*}(v)$.

Definition 2.7. A fuzzy graph (FG) represented by $\mathfrak{G} = (\rho, \nu)$. where ρ is a fuzzy subset of the nonempty set $\mathfrak{V}$ and ν is a symmetric fuzzy relation on ρ so that

$$\nu(a, b) = \nu(ab) \leq \min[\rho(a), \rho(b)] \quad \text{for every} \quad a, b \in \mathfrak{V}.$$

Notice that these functions ρ, ν are subsets of $[0, 1]$.

Definition 2.8. [3] An intuitionistic fuzzy graph (IFG) can be represented as $\mathfrak{G} = (\rho, \nu)$ where $\rho = (\rho_1, \rho_2)$ and $\nu = (\nu_1, \nu_2)$ such that

- (1) ρ_1 and ρ_2 are functions from $\mathfrak{V}$ to $[0, 1]$ that express the DM and the DNM of the element $a \in \mathfrak{V}$, respectively and $0 \leq \rho_1(a) + \rho_2(a) \leq 1$ for each $a \in \mathfrak{V}$;
- (2) ν_1 and ν_2 are functions from $\mathfrak{V} \times \mathfrak{V}$ to $[0, 1]$ that express the DM and the DNM of an edge $ab \in \mathfrak{E}$, respectively. So that $\nu_1(ab) \leq \min[\rho_1(a), \rho_2(b)]$ and $\nu_2(ab) \leq \max[\rho_2(a), \rho_2(b)]$ and $0 \leq \nu_1(ab) + \nu_2(ab) \leq 1$ for each ab in $\mathfrak{E}$.

Definition 2.9. [5] The form of a single valued neutrosophic graph (SVNG) is $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ where $\mathfrak{M} = (\mathfrak{T}_{\mathfrak{M}}, \mathfrak{I}_{\mathfrak{M}}, \mathfrak{F}_{\mathfrak{M}})$ and $\mathfrak{N} = (\mathfrak{T}_{\mathfrak{N}}, \mathfrak{I}_{\mathfrak{N}}, \mathfrak{F}_{\mathfrak{N}})$ such that

- (1) $\mathfrak{T}_{\mathfrak{M}}, \mathfrak{I}_{\mathfrak{M}}$ and $\mathfrak{F}_{\mathfrak{M}}$ are functions from $\mathfrak{V}$ to $[0, 1]$ that signify the degree of truth-membership, the degree of indeterminacy-membership and the degree of falsity-membership of the element $a \in \mathfrak{V}$, respectively, $0 \leq \mathfrak{T}_{\mathfrak{M}}(a) + \mathfrak{I}_{\mathfrak{M}}(a) + \mathfrak{F}_{\mathfrak{M}}(a) \leq 3$ for each $a \in \mathfrak{V}$;

$$\mathfrak{T}_{\mathfrak{N}}(ab) \leq \min[\mathfrak{T}_{\mathfrak{M}}(a), \mathfrak{T}_{\mathfrak{M}}(b)]$$

$$\mathfrak{I}_{\mathfrak{N}}(ab) \geq \max[\mathfrak{I}_{\mathfrak{M}}(a), \mathfrak{I}_{\mathfrak{M}}(b)]$$

$$\mathfrak{F}_{\mathfrak{N}}(ab) \geq \max[\mathfrak{F}_{\mathfrak{M}}(a), \mathfrak{F}_{\mathfrak{M}}(b)]$$

and $0 \leq \mathfrak{T}_{\mathfrak{N}}(ab) + \mathfrak{I}_{\mathfrak{N}}(ab) + \mathfrak{F}_{\mathfrak{N}}(ab) \leq 3$ for every ab in $\mathfrak{E}$.

3. SINGLE-VALUED INTUITIONISTIC NEUTROSOPHIC SET

We define the notion of Single-Valued Intuitionistic Neutrosophic Set (SVINS) and express some of its properties.

Definition 3.1. Consider a set $\mathfrak{U}$ be fixed. A SVINS A on $\mathfrak{U}$ is presented by

$$\begin{aligned} A &= \{(u, \chi, \Psi) | u \in \mathfrak{U}\} \\ \chi &= \langle \mathfrak{T}^\chi, \mathfrak{I}^\chi, \mathfrak{F}^\chi \rangle ; \mathfrak{T}^\chi, \mathfrak{I}^\chi, \mathfrak{F}^\chi : \mathfrak{U} \longrightarrow [0, 1] \\ \psi &= \langle \mathfrak{T}^\Psi, \mathfrak{I}^\Psi, \mathfrak{F}^\Psi \rangle ; \mathfrak{T}^\Psi, \mathfrak{I}^\Psi, \mathfrak{F}^\Psi : \mathfrak{U} \longrightarrow [0, 1] \end{aligned}$$

where

$$\mathfrak{T}^\chi(u) + \mathfrak{T}^\Psi(u) \leq 1,$$

$$\mathfrak{I}^\chi(u) + \mathfrak{I}^\Psi(u) \leq 1,$$

$$\mathfrak{F}^\chi(u) + \mathfrak{F}^\Psi(u) \leq 1.$$

A SVINS A can be written as

$$A = \{(u, (\mathfrak{T}^\chi(u), \mathfrak{I}^\chi(u), \mathfrak{F}^\chi(u), \mathfrak{T}^\Psi(u), \mathfrak{I}^\Psi(u), \mathfrak{F}^\Psi(u)))\}.$$

Example 3.2. Let $\mathfrak{U} = \{u_1, u_2\}$ and $A = \{\langle u_1, (0.5, 0.3, 0.1, 0.4, 0.2, 0.5) \rangle, \langle u_2, (0.1, 0.7, 0.2, 0.5, 0.1, 0.6) \rangle\}$ is a SVINS.

Definition 3.3. Let a set $\mathfrak{U}$ be fixed and $A_1 = \{\langle a, (\mathfrak{T}_1^\chi(a), \mathfrak{I}_1^\chi(a), \mathfrak{F}_1^\chi(a), \mathfrak{T}_1^\Psi(a), \mathfrak{I}_1^\Psi(a), \mathfrak{F}_1^\Psi(a)) \rangle\}$ and $A_2 = \{\langle a, (\mathfrak{T}_2^\chi(a), \mathfrak{I}_2^\chi(a), \mathfrak{F}_2^\chi(a), \mathfrak{T}_2^\Psi(a), \mathfrak{I}_2^\Psi(a), \mathfrak{F}_2^\Psi(a)) \rangle\}$ be two SVINSs. Then $A_1 \subseteq A_2$ if and only if

$$\begin{aligned} \mathfrak{T}_1^\chi(a) &\leq \mathfrak{T}_2^\chi(a) \quad , \quad \mathfrak{I}_1^\chi(a) \leq \mathfrak{I}_2^\chi(a) \quad , \quad \mathfrak{F}_1^\chi(a) \geq \mathfrak{F}_2^\chi(a) \\ \mathfrak{T}_1^\Psi(a) &\geq \mathfrak{T}_2^\Psi(a) \quad , \quad \mathfrak{I}_1^\Psi(a) \geq \mathfrak{I}_2^\Psi(a) \quad , \quad \mathfrak{F}_1^\Psi(a) \leq \mathfrak{F}_2^\Psi(a) \end{aligned} \quad \text{for all } a \in \mathfrak{U}.$$

Example 3.4. Let $\mathfrak{U} = \{a_1, a_2\}$ and $A_1 = \{\langle a_1, (0.2, 0.2, 0.6, 0.6, 0.5, 0.3) \rangle, \langle a_2, (0.1, 0.2, 0.5, 0.5, 0.4, 0.2) \rangle\}$ and $A_2 = \{\langle a_1, (0.3, 0.4, 0.2, 0.5, 0.3, 0.4) \rangle, \langle a_2, (0.3, 0.4, 0.3, 0.2, 0.2, 0.6) \rangle\}$ be two SVINSs. We can see that $A_1 \subseteq A_2$.

Definition 3.5. Assume that set $\mathfrak{U}$ be fixed. Consider $A_1 = \{\langle a, (\mathfrak{T}_1^\chi(a), \mathfrak{I}_1^\chi(a), \mathfrak{F}_1^\chi(a), \mathfrak{T}_1^\Psi(a), \mathfrak{I}_1^\Psi(a), \mathfrak{F}_1^\Psi(a)) \rangle\}$ and $A_2 = \{\langle a, (\mathfrak{T}_2^\chi(a), \mathfrak{I}_2^\chi(a), \mathfrak{F}_2^\chi(a), \mathfrak{T}_2^\Psi(a), \mathfrak{I}_2^\Psi(a), \mathfrak{F}_2^\Psi(a)) \rangle\}$ be two SVINSs. Then $A_1 = A_2$ if and only if

$$\begin{aligned} \mathfrak{T}_1^\chi(a) &= \mathfrak{T}_2^\chi(a) \quad , \quad \mathfrak{I}_1^\chi(a) = \mathfrak{I}_2^\chi(a) \quad , \quad \mathfrak{F}_1^\chi(a) = \mathfrak{F}_2^\chi(a) \\ \mathfrak{T}_1^\Psi(a) &= \mathfrak{T}_2^\Psi(a) \quad , \quad \mathfrak{I}_1^\Psi(a) = \mathfrak{I}_2^\Psi(a) \quad , \quad \mathfrak{F}_1^\Psi(a) = \mathfrak{F}_2^\Psi(a) \end{aligned} \quad \text{for all } a \in \mathfrak{U}.$$

4. SINGLE-VALUED INTUITIONISTIC NEUTROSOPHIC GRAPH

Herein, we present the notion of *single-valued intuitionistic neutrosophic relation*. Then we investigate the concept of *single-valued intuitionistic neutrosophic graph*(SVING). Also, we prove the results of it.

Definition 4.1. Let $\mathfrak{U}$ and $\mathfrak{V}$ be ordinary finite nonempty sets. Then a *single-valued intuitionistic neutrosophic relation* is called to be single-valued intuitionistic neutrosophic subset of $\mathfrak{U} \times \mathfrak{V}$ and it is denoted by $\mathfrak{R}$:

$$\mathfrak{R} = \{((u, v), (\mathfrak{T}_\mathfrak{R}^\chi(u, v), \mathfrak{I}_\mathfrak{R}^\chi(u, v), \mathfrak{F}_\mathfrak{R}^\chi(u, v), \mathfrak{T}_\mathfrak{R}^\Psi(u, v), \mathfrak{I}_\mathfrak{R}^\Psi(u, v), \mathfrak{F}_\mathfrak{R}^\Psi(u, v))) \mid (u, v) \in \mathfrak{U} \times \mathfrak{V}\}.$$

Where

$$\begin{aligned} \mathfrak{T}_\mathfrak{R}^\chi(u, v), \mathfrak{T}_\mathfrak{R}^\Psi(u, v) &: \mathfrak{U} \times \mathfrak{V} \longrightarrow [0, 1] \times [0, 1] \\ \mathfrak{I}_\mathfrak{R}^\chi(u, v), \mathfrak{I}_\mathfrak{R}^\Psi(u, v) &: \mathfrak{U} \times \mathfrak{V} \longrightarrow [0, 1] \times [0, 1] \\ \mathfrak{F}_\mathfrak{R}^\chi(u, v), \mathfrak{F}_\mathfrak{R}^\Psi(u, v) &: \mathfrak{U} \times \mathfrak{V} \longrightarrow [0, 1] \times [0, 1]. \end{aligned}$$

and

$$\begin{aligned}\mathfrak{T}^{\chi}(\mathfrak{u}, \mathfrak{v}) + \mathfrak{T}^{\psi}(\mathfrak{u}, \mathfrak{v}) &\leq 1, \\ \mathfrak{J}^{\chi}(\mathfrak{u}, \mathfrak{v}) + \mathfrak{J}^{\psi}(\mathfrak{u}, \mathfrak{v}) &\leq 1, \\ \mathfrak{F}^{\chi}(\mathfrak{u}, \mathfrak{v}) + \mathfrak{F}^{\psi}(\mathfrak{u}, \mathfrak{v}) &\leq 1 \quad \text{for every } (\mathfrak{u}, \mathfrak{v}) \in \mathfrak{U} \times \mathfrak{V}\end{aligned}$$

Definition 4.2. $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ is a SVING, If

$A = (\mathfrak{T}_{\mathfrak{M}}^{\chi}(a), \mathfrak{J}_{\mathfrak{M}}^{\chi}(a), \mathfrak{F}_{\mathfrak{M}}^{\chi}(a), \mathfrak{T}_{\mathfrak{M}}^{\psi}(a), \mathfrak{J}_{\mathfrak{M}}^{\psi}(a), \mathfrak{F}_{\mathfrak{M}}^{\psi}(a))$ be a SVINS on $\mathfrak{U}$ where

$$\begin{aligned}\mathfrak{T}_{\mathfrak{M}}^{\chi}, \mathfrak{J}_{\mathfrak{M}}^{\chi}, \mathfrak{F}_{\mathfrak{M}}^{\chi} : \mathfrak{U} &\longrightarrow [0, 1] \\ \mathfrak{T}_{\mathfrak{M}}^{\psi}, \mathfrak{J}_{\mathfrak{M}}^{\psi}, \mathfrak{F}_{\mathfrak{M}}^{\psi} : \mathfrak{U} &\longrightarrow [0, 1]\end{aligned}$$

with condition

$$\begin{aligned}\mathfrak{T}_{\mathfrak{M}}^{\chi}(a) + \mathfrak{T}_{\mathfrak{M}}^{\psi}(a) &\leq 1, \\ \mathfrak{J}_{\mathfrak{M}}^{\chi}(a) + \mathfrak{J}_{\mathfrak{M}}^{\psi}(a) &\leq 1, \\ \mathfrak{F}_{\mathfrak{M}}^{\chi}(a) + \mathfrak{F}_{\mathfrak{M}}^{\psi}(a) &\leq 1.\end{aligned}$$

and $\mathfrak{N} = (\mathfrak{T}_{\mathfrak{N}}^{\chi}(a), \mathfrak{J}_{\mathfrak{N}}^{\chi}(a), \mathfrak{F}_{\mathfrak{N}}^{\chi}(a), \mathfrak{T}_{\mathfrak{N}}^{\psi}(a), \mathfrak{J}_{\mathfrak{N}}^{\psi}(a), \mathfrak{F}_{\mathfrak{N}}^{\psi}(a))$ be a SVINS on $\mathfrak{U} \times \mathfrak{V}$ where

$$\begin{aligned}\mathfrak{T}_{\mathfrak{N}}^{\chi}, \mathfrak{J}_{\mathfrak{N}}^{\chi}, \mathfrak{F}_{\mathfrak{N}}^{\chi} : \mathfrak{U} \times \mathfrak{V} &\longrightarrow [0, 1] \\ \mathfrak{T}_{\mathfrak{N}}^{\psi}, \mathfrak{J}_{\mathfrak{N}}^{\psi}, \mathfrak{F}_{\mathfrak{N}}^{\psi} : \mathfrak{U} \times \mathfrak{V} &\longrightarrow [0, 1]\end{aligned}$$

with condition

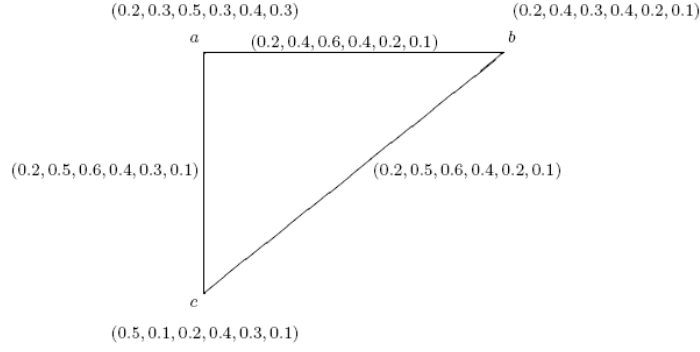
$$\begin{aligned}\mathfrak{T}_{\mathfrak{N}}^{\chi}(a, b) + \mathfrak{T}_{\mathfrak{N}}^{\psi}(a, b) &\leq 1, \\ \mathfrak{J}_{\mathfrak{N}}^{\chi}(a, b) + \mathfrak{J}_{\mathfrak{N}}^{\psi}(a, b) &\leq 1, \\ \mathfrak{F}_{\mathfrak{N}}^{\chi}(a, b) + \mathfrak{F}_{\mathfrak{N}}^{\psi}(a, b) &\leq 1.\end{aligned}$$

such that

$$\begin{aligned}\mathfrak{T}_{\mathfrak{N}}^{\chi}(a, b) &\leq \min(\mathfrak{T}_{\mathfrak{M}}^{\chi}(a), \mathfrak{T}_{\mathfrak{M}}^{\chi}(b)), \quad \mathfrak{T}_{\mathfrak{N}}^{\psi}(a, b) \leq \max(\mathfrak{T}_{\mathfrak{M}}^{\psi}(a), \mathfrak{T}_{\mathfrak{M}}^{\psi}(b)) \\ \mathfrak{J}_{\mathfrak{N}}^{\chi}(a, b) &\geq \max(\mathfrak{J}_{\mathfrak{M}}^{\chi}(a), \mathfrak{J}_{\mathfrak{M}}^{\chi}(b)), \quad \mathfrak{J}_{\mathfrak{N}}^{\psi}(a, b) \geq \min(\mathfrak{J}_{\mathfrak{M}}^{\psi}(a), \mathfrak{J}_{\mathfrak{M}}^{\psi}(b)) \quad \text{for all } a, b \in \mathfrak{U} \\ \mathfrak{F}_{\mathfrak{N}}^{\chi}(a, b) &\geq \max(\mathfrak{F}_{\mathfrak{M}}^{\chi}(a), \mathfrak{F}_{\mathfrak{M}}^{\chi}(b)), \quad \mathfrak{F}_{\mathfrak{N}}^{\psi}(a, b) \geq \min(\mathfrak{F}_{\mathfrak{M}}^{\psi}(a), \mathfrak{F}_{\mathfrak{M}}^{\psi}(b)).\end{aligned}$$

Example 4.3. The SVING $\mathfrak{G}$ is shown in this figure.

Definition 4.4. Let $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ be a SVING where $\mathfrak{M} = (\mathfrak{T}_{\mathfrak{M}}^{\chi}, \mathfrak{J}_{\mathfrak{M}}^{\chi}, \mathfrak{F}_{\mathfrak{M}}^{\chi}, \mathfrak{T}_{\mathfrak{M}}^{\psi}, \mathfrak{J}_{\mathfrak{M}}^{\psi}, \mathfrak{F}_{\mathfrak{M}}^{\psi})$ and $\mathfrak{N} = (\mathfrak{T}_{\mathfrak{N}}^{\chi}, \mathfrak{J}_{\mathfrak{N}}^{\chi}, \mathfrak{F}_{\mathfrak{N}}^{\chi}, \mathfrak{T}_{\mathfrak{N}}^{\psi}, \mathfrak{J}_{\mathfrak{N}}^{\psi}, \mathfrak{F}_{\mathfrak{N}}^{\psi})$ be two SVINSs on a non-empty finite set $\mathfrak{V}$ and $\mathfrak{C} \subseteq \mathfrak{V} \times \mathfrak{V}$ respectively. The *complement* of a SVING $\mathfrak{G}$ is $\overline{\mathfrak{G}} = (\overline{\mathfrak{M}}, \overline{\mathfrak{N}})$ where $\overline{\mathfrak{M}} = (\overline{\mathfrak{T}_{\mathfrak{M}}^{\chi}}, \overline{\mathfrak{J}_{\mathfrak{M}}^{\chi}}, \overline{\mathfrak{F}_{\mathfrak{M}}^{\chi}}, \overline{\mathfrak{T}_{\mathfrak{M}}^{\psi}}, \overline{\mathfrak{J}_{\mathfrak{M}}^{\psi}}, \overline{\mathfrak{F}_{\mathfrak{M}}^{\psi}})$ is a

FIGURE 1. SVING $\mathfrak{G}$.

SVINS on $\overline{\mathfrak{V}}$ and $\overline{\mathfrak{N}} = (\overline{\mathfrak{T}}_{\mathfrak{N}}^{\chi}, \overline{\mathfrak{I}}_{\mathfrak{N}}^{\chi}, \overline{\mathfrak{F}}_{\mathfrak{N}}^{\chi}, \overline{\mathfrak{T}}_{\mathfrak{N}}^{\Psi}, \overline{\mathfrak{I}}_{\mathfrak{N}}^{\Psi}, \overline{\mathfrak{F}}_{\mathfrak{N}}^{\Psi})$ is a SVINS on $\overline{\mathfrak{E}} \subseteq \overline{\mathfrak{V}} \times \overline{\mathfrak{V}}$ such that

(1) $\overline{\mathfrak{V}} = \mathfrak{V}$

(2) $\overline{\mathfrak{M}} = \mathfrak{M}$

and

$$\overline{\mathfrak{T}}_{\mathfrak{N}}^{\chi}(\mathfrak{u}, \mathfrak{v}) = \min(\mathfrak{T}_{\mathfrak{M}}^{\chi}(\mathfrak{u}), \mathfrak{T}_{\mathfrak{M}}^{\chi}(\mathfrak{v})) - \mathfrak{T}_{\mathfrak{N}}^{\chi}(\mathfrak{u}, \mathfrak{v})$$

$$\overline{\mathfrak{I}}_{\mathfrak{N}}^{\chi}(\mathfrak{u}, \mathfrak{v}) = \mathfrak{I}_{\mathfrak{N}}^{\chi}(\mathfrak{u}, \mathfrak{v}) - \max(\mathfrak{I}_{\mathfrak{M}}^{\chi}(\mathfrak{u}), \mathfrak{I}_{\mathfrak{M}}^{\chi}(\mathfrak{v}))$$

$$\overline{\mathfrak{F}}_{\mathfrak{N}}^{\chi}(\mathfrak{u}, \mathfrak{v}) = \mathfrak{F}_{\mathfrak{N}}^{\chi}(\mathfrak{u}, \mathfrak{v}) - \max(\mathfrak{F}_{\mathfrak{M}}^{\chi}(\mathfrak{u}), \mathfrak{F}_{\mathfrak{M}}^{\chi}(\mathfrak{v}))$$

$$\overline{\mathfrak{T}}_{\mathfrak{N}}^{\Psi}(\mathfrak{u}, \mathfrak{v}) = \max(\mathfrak{T}_{\mathfrak{M}}^{\Psi}(\mathfrak{u}), \mathfrak{T}_{\mathfrak{M}}^{\Psi}(\mathfrak{v})) - \mathfrak{T}_{\mathfrak{N}}^{\Psi}(\mathfrak{u}, \mathfrak{v}) \quad \text{for all } \mathfrak{u}, \mathfrak{v} \in \mathfrak{U}.$$

$$\overline{\mathfrak{I}}_{\mathfrak{N}}^{\Psi}(\mathfrak{u}, \mathfrak{v}) = \mathfrak{I}_{\mathfrak{N}}^{\Psi}(\mathfrak{u}, \mathfrak{v}) - \min(\mathfrak{I}_{\mathfrak{M}}^{\Psi}(\mathfrak{u}), \mathfrak{I}_{\mathfrak{M}}^{\Psi}(\mathfrak{v}))$$

$$\overline{\mathfrak{F}}_{\mathfrak{N}}^{\Psi}(\mathfrak{u}, \mathfrak{v}) = \mathfrak{F}_{\mathfrak{N}}^{\Psi}(\mathfrak{u}, \mathfrak{v}) - \min(\mathfrak{F}_{\mathfrak{M}}^{\Psi}(\mathfrak{u}), \mathfrak{F}_{\mathfrak{M}}^{\Psi}(\mathfrak{v}))$$

Definition 4.5. A SVING $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ is said *complete* if

$$\mathfrak{T}_{\mathfrak{N}}^{\chi}(\mathfrak{u}, \mathfrak{v}) = \min(\mathfrak{T}_{\mathfrak{M}}^{\chi}(\mathfrak{u}), \mathfrak{T}_{\mathfrak{M}}^{\chi}(\mathfrak{v}))$$

$$\mathfrak{I}_{\mathfrak{N}}^{\chi}(\mathfrak{u}, \mathfrak{v}) = \max(\mathfrak{I}_{\mathfrak{M}}^{\chi}(\mathfrak{u}), \mathfrak{I}_{\mathfrak{M}}^{\chi}(\mathfrak{v}))$$

$$\mathfrak{F}_{\mathfrak{N}}^{\chi}(\mathfrak{u}, \mathfrak{v}) = \max(\mathfrak{F}_{\mathfrak{M}}^{\chi}(\mathfrak{u}), \mathfrak{F}_{\mathfrak{M}}^{\chi}(\mathfrak{v}))$$

$$\mathfrak{T}_{\mathfrak{N}}^{\Psi}(\mathfrak{u}, \mathfrak{v}) = \max(\mathfrak{T}_{\mathfrak{M}}^{\Psi}(\mathfrak{u}), \mathfrak{T}_{\mathfrak{M}}^{\Psi}(\mathfrak{v})) \quad \text{for all } \mathfrak{u}, \mathfrak{v} \in \mathfrak{U}$$

$$\mathfrak{I}_{\mathfrak{N}}^{\Psi}(\mathfrak{u}, \mathfrak{v}) = \min(\mathfrak{I}_{\mathfrak{M}}^{\Psi}(\mathfrak{u}), \mathfrak{I}_{\mathfrak{M}}^{\Psi}(\mathfrak{v}))$$

$$\mathfrak{F}_{\mathfrak{N}}^{\Psi}(\mathfrak{u}, \mathfrak{v}) = \min(\mathfrak{F}_{\mathfrak{M}}^{\Psi}(\mathfrak{u}), \mathfrak{F}_{\mathfrak{M}}^{\Psi}(\mathfrak{v})).$$

Definition 4.6. A SVING $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ is said *strong* if

$$\begin{aligned} \mathfrak{T}_{\mathfrak{N}}^{\chi}(uv) &= \min(\mathfrak{T}_{\mathfrak{M}}^{\chi}(u), \mathfrak{T}_{\mathfrak{M}}^{\chi}(v)) \\ \mathfrak{J}_{\mathfrak{N}}^{\chi}(uv) &= \max(\mathfrak{J}_{\mathfrak{M}}^{\chi}(u), \mathfrak{J}_{\mathfrak{M}}^{\chi}(v)) \\ \mathfrak{F}_{\mathfrak{N}}^{\chi}(uv) &= \max(\mathfrak{F}_{\mathfrak{M}}^{\chi}(u), \mathfrak{F}_{\mathfrak{M}}^{\chi}(v)) \\ \mathfrak{T}_{\mathfrak{N}}^{\Psi}(uv) &= \max(\mathfrak{T}_{\mathfrak{M}}^{\Psi}(u), \mathfrak{T}_{\mathfrak{M}}^{\Psi}(v)) && \text{for all } uv \in \mathfrak{E} \\ \mathfrak{J}_{\mathfrak{N}}^{\Psi}(uv) &= \min(\mathfrak{J}_{\mathfrak{M}}^{\Psi}(u), \mathfrak{J}_{\mathfrak{M}}^{\Psi}(v)) \\ \mathfrak{F}_{\mathfrak{N}}^{\Psi}(uv) &= \min(\mathfrak{F}_{\mathfrak{M}}^{\Psi}(u), \mathfrak{F}_{\mathfrak{M}}^{\Psi}(v)). \end{aligned}$$

Remark 4.7. A complete SVING $\mathfrak{G}$ is strong. But the converse is not correct.

Definition 4.8. The *degree* of any node a in the SVING $\mathfrak{G}$ is determined by

$$d_{\mathfrak{G}}(a) = (d_{\mathfrak{T}_{\mathfrak{M}}}^{\chi}(a), d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(a), d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(a), d_{\mathfrak{T}_{\mathfrak{M}}}^{\Psi}(a), d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(a), d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(a))$$

where

$$\begin{aligned} d_{\mathfrak{T}_{\mathfrak{M}}}^{\chi}(a) &= \sum_{a \neq b} \mathfrak{T}_{\mathfrak{M}}^{\chi}(a, b) \quad \mathfrak{T}^{\chi} - \text{degree of } a \\ d_{\mathfrak{T}_{\mathfrak{M}}}^{\Psi}(a) &= \sum_{a \neq b} \mathfrak{T}_{\mathfrak{M}}^{\Psi}(a, b) \quad \mathfrak{T}^{\Psi} - \text{degree of } a \\ d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(a) &= \sum_{a \neq b} \mathfrak{J}_{\mathfrak{M}}^{\chi}(a, b) \quad \mathfrak{J}^{\chi} - \text{degree of } a \\ d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(a) &= \sum_{a \neq b} \mathfrak{J}_{\mathfrak{M}}^{\Psi}(a, b) \quad \mathfrak{J}^{\Psi} - \text{degree of } a \\ d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(a) &= \sum_{a \neq b} \mathfrak{F}_{\mathfrak{M}}^{\chi}(a, b) \quad \mathfrak{F}^{\chi} - \text{degree of } a \\ d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(a) &= \sum_{a \neq b} \mathfrak{F}_{\mathfrak{M}}^{\Psi}(a, b) \quad \mathfrak{F}^{\Psi} - \text{degree of } a \end{aligned}$$

Definition 4.9. The *total degree* (TD) of a node a in the SVING $\mathfrak{G}$ is assigned by

$$td_G(a) = (td_{\mathfrak{T}_{\mathfrak{M}}}^{\chi}(a), td_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(a), td_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(a), td_{\mathfrak{T}_{\mathfrak{M}}}^{\Psi}(a), td_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(a), td_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(a))$$

where

$$\begin{aligned}
td_{\mathfrak{T}_{\mathfrak{M}}}^{\chi}(a) &= \sum_{a \neq b} \mathfrak{T}_{\mathfrak{M}}^{\chi}(a, b) + \mathfrak{T}_{\mathfrak{M}}^{\chi}(a) && \text{total } \mathfrak{T}^{\chi} - \text{degree of } a \\
td_{\mathfrak{T}_{\mathfrak{M}}}^{\Psi}(a) &= \sum_{a \neq b} \mathfrak{T}_{\mathfrak{M}}^{\Psi}(a, b) + \mathfrak{T}_{\mathfrak{M}^{\Psi}(a)} && \text{total } \mathfrak{T}^{\Psi} - \text{degree of } a \\
td_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(a) &= \sum_{a \neq b} \mathfrak{J}_{\mathfrak{M}}^{\chi}(a, b) + \mathfrak{J}_{\mathfrak{M}}^{\chi}(a) && \text{total } \mathfrak{J}^{\chi} - \text{degree of } a \\
td_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(a) &= \sum_{a \neq b} \mathfrak{J}_{\mathfrak{M}}^{\Psi}(a, b) + \mathfrak{J}_{\mathfrak{M}}^{\Psi}(a) && \text{total } \mathfrak{J}^{\Psi} - \text{degree of } a \\
td_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(a) &= \sum_{a \neq b} F_{\mathfrak{M}}^{\chi}(a, b) + \mathfrak{F}_{\mathfrak{M}}^{\chi}(a) && \text{total } \mathfrak{F}^{\chi} - \text{degree of } a \\
td_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(a) &= \sum_{a \neq b} F_{\mathfrak{M}}^{\Psi}(a, b) + \mathfrak{F}_{\mathfrak{M}}^{\Psi}(a) && \text{total } \mathfrak{F}^{\Psi} - \text{degree of } a.
\end{aligned}$$

Definition 4.10. The *minimum degree* of any node a in SVING $\mathfrak{G}$ is determined by

$$\delta(\mathfrak{G}) = (\delta_{\mathfrak{T}}^{\chi}, \delta_{\mathfrak{J}}^{\chi}, \delta_{\mathfrak{F}}^{\chi}, \delta_{\mathfrak{T}}^{\Psi}, \delta_{\mathfrak{J}}^{\Psi}, \delta_{\mathfrak{F}}^{\Psi})$$

where

$$\begin{aligned}
\delta_{\mathfrak{T}}^{\chi}(\mathfrak{G}) &= \bigwedge \{d_{\mathfrak{T}}^{\chi}(a) | a \in \mathfrak{V}\} && \text{minimum } \mathfrak{T}^{\chi} - \text{degree of } a \\
\delta_{\mathfrak{J}}^{\chi}(\mathfrak{G}) &= \bigwedge \{d_{\mathfrak{J}}^{\chi}(a) | a \in \mathfrak{V}\} && \text{minimum } \mathfrak{J}^{\chi} - \text{degree of } a \\
\delta_{\mathfrak{F}}^{\chi}(\mathfrak{G}) &= \bigwedge \{d_{\mathfrak{F}}^{\chi}(a) | a \in \mathfrak{V}\} && \text{minimum } \mathfrak{F}^{\chi} - \text{degree of } a \\
\delta_{\mathfrak{T}}^{\Psi}(\mathfrak{G}) &= \bigwedge \{d_{\mathfrak{T}}^{\Psi}(a) | a \in \mathfrak{V}\} && \text{minimum } \mathfrak{T}^{\Psi} - \text{degree of } a \\
\delta_{\mathfrak{J}}^{\Psi}(\mathfrak{G}) &= \bigwedge \{d_{\mathfrak{J}}^{\Psi}(a) | a \in \mathfrak{V}\} && \text{minimum } \mathfrak{J}^{\Psi} - \text{degree of } a \\
\delta_{\mathfrak{F}}^{\Psi}(\mathfrak{G}) &= \bigwedge \{d_{\mathfrak{F}}^{\Psi}(a) | a \in \mathfrak{V}\} && \text{minimum } \mathfrak{F}^{\Psi} - \text{degree of } a.
\end{aligned}$$

Definition 4.11. The *maximum degree* of any node a in SVING $\mathfrak{G}$ is determined by

$$\Delta(\mathfrak{G}) = (\Delta_{\mathfrak{T}}^{\chi}, \Delta_{\mathfrak{J}}^{\chi}, \Delta_{\mathfrak{F}}^{\chi}, \Delta_{\mathfrak{T}}^{\Psi}, \Delta_{\mathfrak{J}}^{\Psi}, \Delta_{\mathfrak{F}}^{\Psi})$$

where

$$\begin{aligned}
\Delta_{\mathfrak{I}}^{\chi}(\mathfrak{G}) &= \bigvee \{d_{\mathfrak{I}}^{\chi}(a) | a \in \mathfrak{V}\} && \text{maximum } \mathfrak{I}^{\chi} - \text{degree of } a \\
\Delta_{\mathfrak{J}}^{\chi}(\mathfrak{G}) &= \bigvee \{d_{\mathfrak{J}}^{\chi}(a) | a \in \mathfrak{V}\} && \text{maximum } \mathfrak{J}^{\chi} - \text{degree of } a \\
\Delta_{\mathfrak{F}}^{\chi}(\mathfrak{G}) &= \bigvee \{d_{\mathfrak{F}}^{\chi}(a) | a \in \mathfrak{V}\} && \text{maximum } \mathfrak{F}^{\chi} - \text{degree of } a \\
\Delta_{\mathfrak{I}}^{\Psi}(\mathfrak{G}) &= \bigvee \{d_{\mathfrak{I}}^{\Psi}(a) | a \in \mathfrak{V}\} && \text{maximum } \mathfrak{I}^{\Psi} - \text{degree of } a \\
\Delta_{\mathfrak{J}}^{\Psi}(\mathfrak{G}) &= \bigvee \{d_{\mathfrak{J}}^{\Psi}(a) | a \in \mathfrak{V}\} && \text{maximum } \mathfrak{J}^{\Psi} - \text{degree of } a \\
\Delta_{\mathfrak{F}}^{\Psi}(\mathfrak{G}) &= \bigvee \{d_{\mathfrak{F}}^{\Psi}(a) | a \in \mathfrak{V}\} && \text{maximum } \mathfrak{F}^{\Psi} - \text{degree of } a.
\end{aligned}$$

Definition 4.12. The *order* of SVING $\mathfrak{G}$ is presented by

$$O(\mathfrak{G}) = (O_{\mathfrak{I}}^{\chi}(\mathfrak{G}), O_{\mathfrak{J}}^{\chi}(\mathfrak{G}), O_{\mathfrak{F}}^{\chi}(\mathfrak{G}), O_{\mathfrak{I}}^{\Psi}(\mathfrak{G}), O_{\mathfrak{J}}^{\Psi}(\mathfrak{G}), O_{\mathfrak{F}}^{\Psi}(\mathfrak{G}))$$

where

$$\begin{aligned}
O_{\mathfrak{I}}^{\chi}(\mathfrak{G}) &= \sum_{a \in \mathfrak{V}} \mathfrak{I}^{\chi}(a) && \mathfrak{I}^{\chi} - \text{order of } a \\
O_{\mathfrak{I}}^{\Psi}(\mathfrak{G}) &= \sum_{a \in \mathfrak{V}} \mathfrak{I}^{\Psi}(a) && \mathfrak{I}^{\Psi} - \text{order of } a \\
O_{\mathfrak{J}}^{\chi}(\mathfrak{G}) &= \sum_{a \in \mathfrak{V}} \mathfrak{J}^{\chi}(a) && \mathfrak{J}^{\chi} - \text{order of } a \\
O_{\mathfrak{J}}^{\Psi}(\mathfrak{G}) &= \sum_{a \in \mathfrak{V}} \mathfrak{J}^{\Psi}(a) && \mathfrak{J}^{\Psi} - \text{order of } a \\
O_{\mathfrak{F}}^{\chi}(\mathfrak{G}) &= \sum_{a \in \mathfrak{V}} \mathfrak{F}^{\chi}(a) && \mathfrak{F}^{\chi} - \text{order of } a \\
O_{\mathfrak{F}}^{\Psi}(\mathfrak{G}) &= \sum_{a \in \mathfrak{V}} \mathfrak{F}^{\Psi}(a) && \mathfrak{F}^{\Psi} - \text{order of } a.
\end{aligned}$$

Definition 4.13. The *size* of SVING $\mathfrak{G}$ is determined by

$$S(\mathfrak{G}) = (S_{\mathfrak{I}}^{\chi}(\mathfrak{G}), S_{\mathfrak{J}}^{\chi}(\mathfrak{G}), S_{\mathfrak{F}}^{\chi}(\mathfrak{G}), S_{\mathfrak{I}}^{\Psi}(\mathfrak{G}), S_{\mathfrak{J}}^{\Psi}(\mathfrak{G}), S_{\mathfrak{F}}^{\Psi}(\mathfrak{G}))$$

where

$$\begin{aligned}
S_{\mathfrak{I}}^{\chi}(\mathfrak{G}) &= \sum_{a \neq b} \mathfrak{I}^{\chi}(a, b) && \mathfrak{I}^{\chi} - \text{size of } a && , && S_{\mathfrak{I}}^{\Psi}(\mathfrak{G}) = \sum_{a \neq b} \mathfrak{I}^{\Psi}(a, b) && \mathfrak{I}^{\Psi} - \text{size of } a \\
S_{\mathfrak{J}}^{\chi}(\mathfrak{G}) &= \sum_{a \neq b} \mathfrak{J}^{\chi}(a, b) && \mathfrak{J}^{\chi} - \text{size of } a && , && S_{\mathfrak{J}}^{\Psi}(\mathfrak{G}) = \sum_{a \neq b} \mathfrak{J}^{\Psi}(a, b) && \mathfrak{J}^{\Psi} - \text{size of } a \\
S_{\mathfrak{F}}^{\chi}(\mathfrak{G}) &= \sum_{a \neq b} \mathfrak{F}^{\chi}(a, b) && \mathfrak{F}^{\chi} - \text{size of } a && , && S_{\mathfrak{F}}^{\Psi}(\mathfrak{G}) = \sum_{a \neq b} \mathfrak{F}^{\Psi}(a, b) && \mathfrak{F}^{\Psi} - \text{size of } a
\end{aligned}$$

Example 4.14. Consider SVING $\mathfrak{G}$ in Figure 1

The degree of all nodes of $\mathfrak{G}$, $\delta(\mathfrak{G})$, $S(\mathfrak{G})$, $O(\mathfrak{G})$ and $\Delta(\mathfrak{G})$ are computed.

$$\begin{aligned} d_{\mathfrak{G}}(a) &= (0.4, 0.9, 1.2, 0.8, 0.5, 0.2) \\ d_{\mathfrak{G}}(b) &= (0.4, 0.9, 1.2, 0.8, 0.4, 0.2) \\ d_{\mathfrak{G}}(c) &= (0.4, 1, 1.2, 0.8, 0.5, 0.2) \\ \delta(\mathfrak{G}) &= (0.4, 0.9, 1.2, 0.8, 0.4, 0.2) \\ \Delta(\mathfrak{G}) &= (0.4, 1, 1.2, 0.8, 0.5, 0.2) \\ O(\mathfrak{G}) &= (0.9, 0.8, 1, 1.1, 0.9, 0.5) \\ S(\mathfrak{G}) &= (0.6, 1.4, 1.8, 1.2, 0.7, 0.3). \end{aligned}$$

Definition 4.15. If the degree of any node in SVING $\mathfrak{G}$ be $\mathfrak{k} = (\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \mathfrak{k}_4, \mathfrak{k}_5, \mathfrak{k}_6)$.

Then $\mathfrak{G}$ is called to be a $\mathfrak{k}$ -regular SVING (RSVING).

If the TD of any node in $\mathfrak{G}$ is the same. Then $\mathfrak{G}$ is called a totally regular SVING (TRSVING).

Theorem 4.16. Let $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ be a SVING. Then $\mathfrak{M}$ is a constant function (CF) if and only if the following are equivalent:

- (1) $\mathfrak{G}$ is a RSVING.
- (2) $\mathfrak{G}$ is a TRSVING.

Proof. Since $\mathfrak{M}$ is a CF and $\mathfrak{G}$ is a RSVING, the rest follow.

$$\begin{aligned} td_{\mathfrak{G}}(a) &= d_{\mathfrak{G}}(a) + \mathfrak{M}(a) & \text{for all } a \in \mathfrak{V} \\ \implies td_G(a) &= (\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \mathfrak{k}_4, \mathfrak{k}_5, \mathfrak{k}_6) + (\mathfrak{c}_1, \mathfrak{c}_2, \mathfrak{c}_3, \mathfrak{c}_4, \mathfrak{c}_5, \mathfrak{c}_6) \end{aligned}$$

Hence $\mathfrak{G}$ is a TRSVING.

Then (1) $\Rightarrow$ (2) is proved.

Now, suppose that $\mathfrak{G}$ is a $\mathfrak{k}'$ -TRSVING.

Then

$$\begin{aligned} td_{\mathfrak{G}}(a) &= (\mathfrak{k}'_1, \mathfrak{k}'_2, \mathfrak{k}'_3, \mathfrak{k}'_4, \mathfrak{k}'_5, \mathfrak{k}'_6) & \text{for all } a \in \mathfrak{V} \\ d_{\mathfrak{G}}(u) + \mathfrak{M}(a) &= (\mathfrak{k}'_1, \mathfrak{k}'_2, \mathfrak{k}'_3, \mathfrak{k}'_4, \mathfrak{k}'_5, \mathfrak{k}'_6) & \text{for all } a \in \mathfrak{V} \\ \Rightarrow d_{\mathfrak{G}}(a) + (\mathfrak{c}_1, \mathfrak{c}_2, \mathfrak{c}_3, \mathfrak{c}_4, \mathfrak{c}_5, \mathfrak{c}_6) &= (\mathfrak{k}'_1, \mathfrak{k}'_2, \mathfrak{k}'_3, \mathfrak{k}'_4, \mathfrak{k}'_5, \mathfrak{k}'_6) & \text{for all } a \in \mathfrak{V} \\ \Rightarrow d_{\mathfrak{G}}(a) &= (\mathfrak{k}'_1 - \mathfrak{c}_1, \mathfrak{k}'_2 - \mathfrak{c}_2, \mathfrak{k}'_3 - \mathfrak{c}_3, \mathfrak{k}'_4 - \mathfrak{c}_4, \mathfrak{k}'_5 - \mathfrak{c}_5, \mathfrak{k}'_6 - \mathfrak{c}_6) \end{aligned}$$

So $\mathfrak{G}$ is a RSVING.

Thus (2) $\Rightarrow$ (1) is proved.

Hence (1) and (2) are equivalent.

On the contrary, suppose (1) and (2) are equivalent and $\mathfrak{M}$ is not a CF.

Therefore $\mathfrak{M}(a) \neq \mathfrak{M}(b)$ for atleast one pair of vertices $a, b \in \mathfrak{V}$.

Let $\mathfrak{G}$ be a RSVING. Then $d_{\mathfrak{G}}(a) = d_{\mathfrak{G}}(b) = \mathfrak{k}$.
So

$$td_{\mathfrak{G}}(a) = \mathfrak{k} + \mathfrak{M}(a)$$

and

$$td_{\mathfrak{G}}(b) = \mathfrak{k} + \mathfrak{M}(b)$$

Since $\mathfrak{M}(a) \neq \mathfrak{M}(b)$, we have $td_{\mathfrak{G}}(a) \neq td_{\mathfrak{G}}(b)$. So $\mathfrak{G}$ is not TRSVING which is a contradiction to our assumption.
Now, let $\mathfrak{G}$ be a TRSVING. Then

$$\begin{aligned} td_{\mathfrak{G}}(a) &= td_{\mathfrak{G}}(b) \\ d_{\mathfrak{G}}(a) + \mathfrak{M}(a) &= d_{\mathfrak{G}}(b) + \mathfrak{M}(b) \\ d_{\mathfrak{G}}(a) - d_{\mathfrak{G}}(b) &= \mathfrak{M}(b) - \mathfrak{M}(a) \\ \Rightarrow d(a) &\neq d(b) \end{aligned}$$

So $\mathfrak{G}$ is not RSVING which is a contradiction to our assumption. Hence $\mathfrak{M}$ is a CF. $\square$

Definition 4.17. If $\mathfrak{G}$ be an SVING on a regular graph $\mathfrak{G}^*$. Then $\mathfrak{G}$ is called a partially regular SVING (PRSVING).

Definition 4.18. A SVING $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ is called a fully regular SVING (FRSVING), If $\mathfrak{G}$ is both RSVING and PRSVING.

Theorem 4.19. The size of a $(\mathfrak{k}_1, \dots, \mathfrak{k}_6)$ -regular SVING $\mathfrak{G}$ is obtained by

$$S(\mathfrak{G}) = \left(\frac{\mathfrak{p}\mathfrak{k}_1}{2}, \dots, \frac{\mathfrak{p}\mathfrak{k}_6}{2} \right) \quad \text{where } \mathfrak{p} = |\mathfrak{V}|.$$

Proof. The size of $\mathfrak{G}$ is

$$S(\mathfrak{G}) = \left(\sum_{a \neq b} \mathfrak{T}_{\mathfrak{N}}^{\chi}(a, b), \sum_{a \neq b} \mathfrak{J}_{\mathfrak{N}}^{\chi}(a, b), \sum_{a \neq b} \mathfrak{F}_{\mathfrak{N}}^{\chi}(a, b), \sum_{a \neq b} \mathfrak{T}_{\mathfrak{N}}^{\Psi}(a, b), \sum_{a \neq b} \mathfrak{J}_{\mathfrak{N}}^{\Psi}(a, b), \sum_{a \neq b} \mathfrak{F}_{\mathfrak{N}}^{\Psi}(a, b) \right)$$

we have

$$\begin{aligned} \sum_{a \in \mathfrak{V}} d_{\mathfrak{G}}(a) &= 2 \left(\sum_{a \neq b} \mathfrak{T}_{\mathfrak{N}}^{\chi}(a, b), \sum_{a \neq b} \mathfrak{J}_{\mathfrak{N}}^{\chi}(a, b), \sum_{a \neq b} \mathfrak{F}_{\mathfrak{N}}^{\chi}(a, b), \sum_{a \neq b} \mathfrak{T}_{\mathfrak{N}}^{\Psi}(a, b), \sum_{a \neq b} \mathfrak{J}_{\mathfrak{N}}^{\Psi}(a, b), \sum_{a \neq b} \mathfrak{F}_{\mathfrak{N}}^{\Psi}(a, b) \right) \\ &= 2S(\mathfrak{G}) \end{aligned}$$

Since $\mathfrak{G}$ is $(\mathfrak{k}_1, \dots, \mathfrak{k}_6)$ -regular, $d_{\mathfrak{G}}(a) = (\mathfrak{k}_1, \dots, \mathfrak{k}_6)$ for every $a \in \mathfrak{V}$.
So

$$\begin{aligned} 2S(\mathfrak{G}) &= \sum_{a \in \mathfrak{V}} d_{\mathfrak{G}}(a) \\ &= \sum_{a \in \mathfrak{V}} (\mathfrak{k}_1, \dots, \mathfrak{k}_6) \\ &= (\mathfrak{p}\mathfrak{k}_1, \dots, \mathfrak{p}\mathfrak{k}_6). \end{aligned}$$

Hence $S(\mathfrak{G}) = (\frac{\mathfrak{p}\mathfrak{k}_1}{2}, \dots, \frac{\mathfrak{p}\mathfrak{k}_6}{2})$. $\square$

Theorem 4.20. In the $(r_1, \dots, r_6)$ -TRSVING $\mathfrak{G}$, the following statement is satisfied.

$$2S(\mathfrak{G}) + O(\mathfrak{G}) = (\mathfrak{p}r_1, \dots, \mathfrak{p}r_6) \quad \text{where } \mathfrak{p} = |\mathfrak{V}|.$$

Corollary 4.21. Assume that $\mathfrak{G}$ be a $(\mathfrak{r}_1, \dots, \mathfrak{r}_6)$ -RSVING and $(\mathfrak{t}_1, \dots, \mathfrak{t}_6)$ -TRSVING. Then

$$O(\mathfrak{G}) = (\mathfrak{p}(\mathfrak{t}_1 - \mathfrak{r}_1), \dots, \mathfrak{p}(\mathfrak{t}_6 - \mathfrak{r}_6))$$

Proof. from 4.19 and 4.20

$$\begin{aligned} O(\mathfrak{G}) &= (\mathfrak{p}\mathfrak{t}_1, \dots, \mathfrak{p}\mathfrak{t}_6) - 2S(\mathfrak{G}) \\ &= (\mathfrak{p}\mathfrak{t}_1, \dots, \mathfrak{p}\mathfrak{t}_6) - (\mathfrak{p}\mathfrak{r}_1, \dots, \mathfrak{p}\mathfrak{r}_6) \\ &= (\mathfrak{p}(\mathfrak{t}_1 - \mathfrak{r}_1), \dots, \mathfrak{p}(\mathfrak{t}_6 - \mathfrak{r}_6)) \end{aligned}$$

$\square$

Definition 4.22. The edge degree of an edge $ab(\text{ED})$ in the SVING $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ is expressed by

$$d_{\mathfrak{G}}(ab) = (d_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(ab), d_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(ab), d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(ab), d_{\mathfrak{T}_{\mathfrak{N}}}^{\Psi}(ab), d_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(ab), d_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(ab))$$

where

$$\begin{aligned} d_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(ab) &= d_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(a) + d_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(b) - 2\mathfrak{T}_{\mathfrak{N}}^{\chi}(ab) \\ d_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(ab) &= d_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(a) + d_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(b) - 2\mathfrak{J}_{\mathfrak{N}}^{\chi}(ab) \\ d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(ab) &= d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(a) + d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(b) - 2\mathfrak{F}_{\mathfrak{N}}^{\chi}(ab) \\ d_{\mathfrak{T}_{\mathfrak{N}}}^{\Psi}(ab) &= d_{\mathfrak{T}_{\mathfrak{N}}}^{\Psi}(a) + d_{\mathfrak{T}_{\mathfrak{N}}}^{\Psi}(b) - 2\mathfrak{T}_{\mathfrak{N}}^{\Psi}(ab) \\ d_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(ab) &= d_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(a) + d_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(b) - 2\mathfrak{J}_{\mathfrak{N}}^{\Psi}(ab) \\ d_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(ab) &= d_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(a) + d_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(b) - 2\mathfrak{F}_{\mathfrak{N}}^{\Psi}(ab). \end{aligned}$$

Definition 4.23. The total edge degree of an edge ab (TED) in the SVING $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ is presented by

$$td_{\mathfrak{G}}(ab) = (td_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(ab), td_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(ab), td_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(ab), td_{\mathfrak{I}_{\mathfrak{N}}}^{\Psi}(ab), td_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(ab), td_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(ab))$$

where

$$\begin{aligned} td_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(ab) &= d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(ab) + \mathfrak{I}_{\mathfrak{N}}^{\chi}(ab) \\ &= d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(a) + d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(b) - 2\mathfrak{I}_{\mathfrak{N}}^{\chi}(ab) + \mathfrak{I}_{\mathfrak{N}}^{\chi}(ab) \\ &= d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(a) + d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(b) - \mathfrak{I}_{\mathfrak{N}}^{\chi}(ab) \end{aligned}$$

$$\begin{aligned} td_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(ab) &= d_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(ab) + \mathfrak{J}_{\mathfrak{N}}^{\chi}(ab) \\ &= d_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(a) + d_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(b) - 2\mathfrak{J}_{\mathfrak{N}}^{\chi}(ab) + \mathfrak{J}_{\mathfrak{N}}^{\chi}(ab) \\ &= d_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(a) + d_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(b) - \mathfrak{J}_{\mathfrak{N}}^{\chi}(ab) \end{aligned}$$

$$\begin{aligned} td_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(ab) &= d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(ab) + \mathfrak{F}_{\mathfrak{N}}^{\chi}(ab) \\ &= d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(a) + d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(b) - 2\mathfrak{F}_{\mathfrak{N}}^{\chi}(ab) + \mathfrak{F}_{\mathfrak{N}}^{\chi}(ab) \\ &= d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(a) + d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(b) - \mathfrak{F}_{\mathfrak{N}}^{\chi}(ab) \end{aligned}$$

$$\begin{aligned} td_{\mathfrak{I}_{\mathfrak{N}}}^{\Psi}(ab) &= d_{\mathfrak{I}_{\mathfrak{N}}}^{\Psi}(ab) + \mathfrak{I}_{\mathfrak{N}}^{\Psi}(ab) \\ &= d_{\mathfrak{I}_{\mathfrak{N}}}^{\Psi}(a) + d_{\mathfrak{I}_{\mathfrak{N}}}^{\Psi}(b) - 2\mathfrak{I}_{\mathfrak{N}}^{\Psi}(ab) + \mathfrak{I}_{\mathfrak{N}}^{\Psi}(ab) \\ &= d_{\mathfrak{I}_{\mathfrak{N}}}^{\Psi}(a) + d_{\mathfrak{I}_{\mathfrak{N}}}^{\Psi}(b) - \mathfrak{I}_{\mathfrak{N}}^{\Psi}(ab) \end{aligned}$$

$$\begin{aligned} td_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(ab) &= d_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(ab) + \mathfrak{J}_{\mathfrak{N}}^{\Psi}(ab) \\ &= d_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(a) + d_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(b) - 2\mathfrak{J}_{\mathfrak{N}}^{\Psi}(ab) + \mathfrak{J}_{\mathfrak{N}}^{\Psi}(ab) \\ &= d_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(a) + d_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(b) - \mathfrak{J}_{\mathfrak{N}}^{\Psi}(ab) \end{aligned}$$

$$\begin{aligned} td_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(ab) &= d_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(ab) + \mathfrak{F}_{\mathfrak{N}}^{\Psi}(ab) \\ &= d_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(a) + d_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(b) - 2\mathfrak{F}_{\mathfrak{N}}^{\Psi}(ab) + \mathfrak{F}_{\mathfrak{N}}^{\Psi}(ab) \\ &= d_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(a) + d_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(b) - \mathfrak{F}_{\mathfrak{N}}^{\Psi}(ab). \end{aligned}$$

Definition 4.24. If the ED of any edge in SVING $\mathfrak{G}$ be $\mathfrak{l} = (\mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3, \mathfrak{l}_4, \mathfrak{l}_5, \mathfrak{l}_6)$, then $\mathfrak{G}$ is said to be an edge regular SVING of degree $\mathfrak{l}$ (ERSVING).

If TED of any edge in $\mathfrak{G}$ be the same $\mathfrak{l}$, then $\mathfrak{G}$ is said to be a totally edge regular SVING (TERSVING).

5. IRREGULAR SVING

Finally, we discuss the concept of irregularity in SVING, and establish the corresponding theorems.

Definition 5.1. If in the SVING $\mathfrak{G}$ there is a node that is adjacent to nodes with distinct degrees, then $\mathfrak{G}$ is called to be *Irregular SVING* (ISVING).

Definition 5.2. If in the SVING $\mathfrak{G}$ there is a node that is adjacent to nodes with distinct total degrees, then $\mathfrak{G}$ is said to be *Totally Irregular SVING* (TISVING).

Example 5.3. Consider a SVING $\mathfrak{G}$ such that $\mathfrak{V} = \{a, b, c, d\}$, $\mathfrak{E} = \{ab, bc, ac\}$. From figure 2;

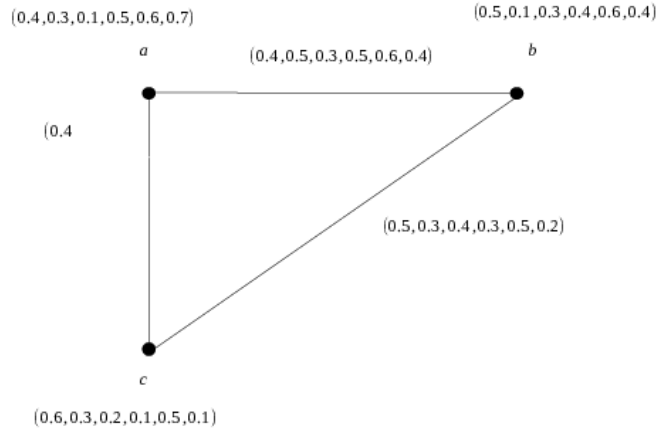


FIGURE 2. Example of Irregular SVING $\mathfrak{G}$.

$$d_{\mathfrak{G}}(a) = (0.8, 1.0, 0.5, 0.9, 1.1, 0.5)$$

$$d_{\mathfrak{G}}(b) = (0.9, 0.8, 0.7, 0.8, 1.1, 0.6)$$

$$d_{\mathfrak{G}}(c) = (0.9, 0.8, 0.6, 0.7, 1.0, 0.3)$$

$$td_{\mathfrak{G}}(a) = (1.2, 1.3, 0.6, 1.4, 1.7, 1.2)$$

$$td_{\mathfrak{G}}(b) = (1.4, 0.9, 1.0, 1.2, 1.7, 1.0)$$

$$td_{\mathfrak{G}}(c) = (1.5, 1.1, 0.8, 0.8, 1.5, 0.4)$$

Hence, $\mathfrak{G}$ is an ISVING.

Definition 5.4. If in the connected SVING $\mathfrak{G}$ the degree of each pair of adjacent nodes be different, then $\mathfrak{G}$ is called to be a *neighbourly irregular* SVING (NISVING).

Definition 5.5. If in the connected SVING $\mathfrak{G}$ the TD of each pair of adjacent nodes be distinct, then $\mathfrak{G}$ is called to be a *neighbourly totally irregular* SVING (NTISVING).

Definition 5.6. A connected SVING $\mathfrak{G}$ is called a *strongly irregular* SVING (SISVING), If the degree of every each pair of nodes be distinct.

Definition 5.7. A connected SVING $\mathfrak{G}$ is called a *strongly totally irregular* SVING (STISVING), If the TD of every each pair of nodes be distinct.

Definition 5.8. A connected SVING $\mathfrak{G}$ is called a *highly irregular* SVING (HISVING) If each node is next to the nodes with different degrees.

Definition 5.9. A connected SVING $\mathfrak{G}$ is called a *highly totally irregular* SVING (HTISVING) If each node is next to the nodes with different TD.

Definition 5.10. If in the connected SVING $\mathfrak{G}$ the ED of each pair of adjacent edges be different, then $\mathfrak{G}$ is said to be a *neighbourly edge irregular* SVING (NEISVING).

Definition 5.11. If in the connected SVING $\mathfrak{G}$ the TED of each pair of adjacent edges be different, then $\mathfrak{G}$ is said to be a *neighbourly edge totally irregular* SVING (NETISVING).

Example 5.12. Consider SVING $\mathfrak{G}$ with $\mathfrak{V} = \{a, b, c, d\}$ and $\mathfrak{E} = \{ab, bc, cd, ad\}$ in figure 3

$$\begin{aligned} d_{\mathfrak{G}}(a) &= (0.3, 0.8, 1, 0.8, 0.4, 0.2) \\ d_{\mathfrak{G}}(b) &= (0.3, 0.8, 1, 0.8, 0.4, 0.2) \\ d_{\mathfrak{G}}(c) &= (0.3, 0.8, 1, 0.8, 0.4, 0.2) \\ d_{\mathfrak{G}}(d) &= (0.3, 0.8, 1, 0.8, 0.4, 0.2) \\ d_{\mathfrak{G}}(ab) &= (0.5, 1.2, 1.5, 1.2, 0.6, 0.3) \\ d_{\mathfrak{G}}(bc) &= (0.2, 0.8, 1, 0.8, 0.4, 0.2) \\ d_{\mathfrak{G}}(cd) &= (0.4, 0.8, 1, 0.8, 0.4, 0.2) \\ d_{\mathfrak{G}}(ad) &= (0.4, 0.8, 1, 0.8, 0.4, 0.2). \end{aligned}$$

Here, $\mathfrak{G}$ is a NEISVING. TED of the edges as follows:

$$\begin{aligned} td_{\mathfrak{G}}(ab) &= (0.6, 1.6, 2, 1.6, 0.8, 0.4) \\ td_{\mathfrak{G}}(bc) &= (0.4, 1.2, 1.5, 1.2, 0.6, 0.3) \\ td_{\mathfrak{G}}(cd) &= (0.5, 1.2, 1.5, 1.2, 0.6, 0.3) \\ td_{\mathfrak{G}}(ad) &= (0.6, 1.2, 1.5, 1.2, 0.6, 0.3). \end{aligned}$$

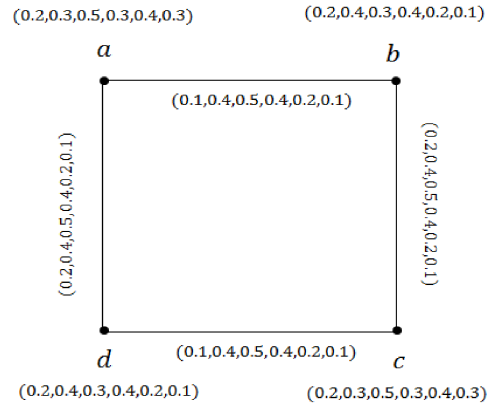
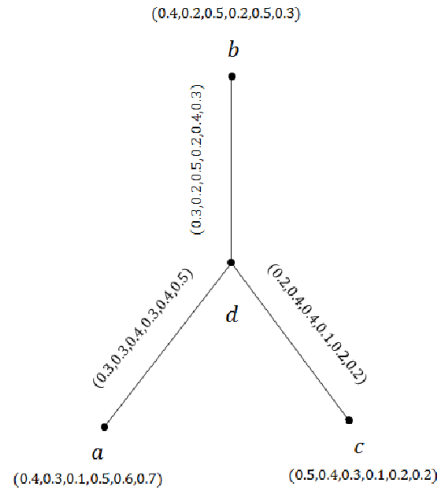


FIGURE 3. Both NEISVING and NETISVING

We can see that $\mathfrak{G}$ is NETISVING.

Hence, $\mathfrak{G}$ is both NEISVING and NETISVING.

Example 5.13. NEISVING do not need to be NETISVING. see figure 4

FIGURE 4. $\mathfrak{G}$ is a NISVING, but $\mathfrak{G}$ is not a NTISVING

$$\begin{aligned}
 d_{\mathfrak{G}}(d) &= (0.8, 0.9, 1.3, 0.8, 1.0, 1) \\
 d_{\mathfrak{G}}(a) &= (0.3, 0.3, 0.4, 0.5, 0.4, 0.5) \\
 d_{\mathfrak{G}}(b) &= (0.3, 0.2, 0.5, 0.2, 0.4, 0.3) \\
 d_{\mathfrak{G}}(c) &= (0.2, 0.4, 0.4, 0.1, 0.2, 0.2) \\
 d_{\mathfrak{G}}(bd) &= (0.5, 0.7, 0.8, 0.6, 0.6, 0.7) \\
 d_{\mathfrak{G}}(ad) &= (0.5, 0.6, 0.9, 0.3, 0.6, 0.5) \\
 d_{\mathfrak{G}}(cd) &= (0.6, 0.5, 0.9, 0.7, 0.8, 0.8). \\
 \\
 td_{\mathfrak{G}}(bd) &= (0.8, 0.9, 1.3, 0.8, 1, 1) \\
 td_{\mathfrak{G}}(ad) &= (0.8, 0.9, 1.3, 0.8, 1, 1) \\
 td_{\mathfrak{G}}(cd) &= (0.8, 0.9, 1.3, 0.8, 1, 1).
 \end{aligned}$$

Here,

$$d_{\mathfrak{G}}(ad) \neq d_{\mathfrak{G}}(bd) \neq d_{\mathfrak{G}}(cd).$$

Hence, $\mathfrak{G}$ is a NEISVING. But it is not a NETISVING.

Example 5.14. Consider $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ be a SVING such that $\mathfrak{V} = \{a, b, c, d\}$. see figure 5

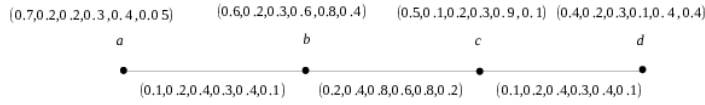


FIGURE 5. $\mathfrak{G}$ is a NETISVING, but $\mathfrak{G}$ is not a NEISVING

$$\begin{aligned}
 d_{\mathfrak{G}}(a) &= (0.1, 0.2, 0.4, 0.3, 0.4, 0.1) \\
 d_{\mathfrak{G}}(b) &= (0.3, 0.6, 1.2, 0.9, 1.2, 0.3) \\
 d_{\mathfrak{G}}(c) &= (0.3, 0.6, 1.2, 0.9, 1.2, 0.3) \\
 d_{\mathfrak{G}}(d) &= (0.1, 0.2, 0.4, 0.3, 0.4, 0.1) \\
 d_{\mathfrak{G}}(ab) &= (0.2, 0.4, 0.8, 0.6, 0.8, 0.2) \\
 d_{\mathfrak{G}}(bc) &= (0.2, 0.4, 0.8, 0.6, 0.8, 0.2) \\
 d_{\mathfrak{G}}(cd) &= (0.2, 0.4, 0.8, 0.6, 0.8, 0.2) \\
 td_{\mathfrak{G}}(ab) &= (0.3, 0.6, 1.2, 0.9, 1.2, 0.3) \\
 td_{\mathfrak{G}}(bc) &= (0.4, 0.8, 1.6, 1.2, 1.6, 0.4) \\
 td_{\mathfrak{G}}(cd) &= (0.3, 0.6, 1.2, 0.9, 1.2, 0.3).
 \end{aligned}$$

Here, $d_{\mathfrak{G}}(ab) = d_{\mathfrak{G}}(bc) = d_{\mathfrak{G}}(cd)$. Hence, $\mathfrak{G}$ is not a NEISVING. But $\mathfrak{G}$ is a NETISVING.

Theorem 5.15. *Let $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ be a connected SVING and $\mathfrak{N}$ is a CF. Then $\mathfrak{G}$ is a NEISVING if and only if $\mathfrak{G}$ is a NETISVING.*

Proof. Assume that $\mathfrak{N}$ is a CF. Let $\mathfrak{N}(ab) = (c_1, \dots, c_6)$, for all $ab \in \mathfrak{E}$, where $c_1, \dots, c_6$ are constant. Let $\mathfrak{G}$ be a NEISVING. Let ab and bc be a pair of adjacent edges, then

$$\begin{aligned}
d_{\mathfrak{G}}(ab) \neq d_{\mathfrak{G}}(bc) &\iff d_{\mathfrak{G}}(ab) + (c_1, \dots, c_6) \neq d_{\mathfrak{G}}(bc) + (c_1, \dots, c_6) \\
&(d_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(ab), d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(ab), d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(ab), d_{\mathfrak{T}_{\mathfrak{N}}}^{\psi}(ab), d_{\mathfrak{I}_{\mathfrak{N}}}^{\psi}(ab), d_{\mathfrak{F}_{\mathfrak{N}}}^{\psi}(ab)) + (c_1, \dots, c_6) \\
&\neq (d_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(bc), d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(bc), d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(bc), d_{\mathfrak{T}_{\mathfrak{N}}}^{\psi}(bc), d_{\mathfrak{I}_{\mathfrak{N}}}^{\psi}(bc), d_{\mathfrak{F}_{\mathfrak{N}}}^{\psi}(bc)) + (c_1, \dots, c_6) \\
&\iff (d_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(ab) + c_1, d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(ab) + c_2, d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(ab) + c_3, d_{\mathfrak{T}_{\mathfrak{N}}}^{\psi}(ab) + c_4, d_{\mathfrak{I}_{\mathfrak{N}}}^{\psi}(ab) + c_5, d_{\mathfrak{F}_{\mathfrak{N}}}^{\psi}(ab) + c_6) \\
&\neq (d_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(bc) + c_1, d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(bc) + c_2, d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(bc) + c_3, d_{\mathfrak{T}_{\mathfrak{N}}}^{\psi}(bc) + c_4, d_{\mathfrak{I}_{\mathfrak{N}}}^{\psi}(bc) + c_5, d_{\mathfrak{F}_{\mathfrak{N}}}^{\psi}(bc) + c_6) \\
&\iff (d_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(ab) + \mathfrak{T}_{\mathfrak{N}}^{\chi}(ab), d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(ab) + \mathfrak{I}_{\mathfrak{N}}^{\chi}(ab), d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(ab) + \mathfrak{F}_{\mathfrak{N}}^{\chi}(ab), \\
&d_{\mathfrak{T}_{\mathfrak{N}}}^{\psi}(ab) + \mathfrak{T}_{\mathfrak{N}}^{\psi}(ab), d_{\mathfrak{I}_{\mathfrak{N}}}^{\psi}(ab) + \mathfrak{I}_{\mathfrak{N}}^{\psi}(ab), d_{\mathfrak{F}_{\mathfrak{N}}}^{\psi}(ab) + \mathfrak{F}_{\mathfrak{N}}^{\psi}(ab)) \\
&\neq (d_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(bc) + \mathfrak{T}_{\mathfrak{N}}^{\chi}(bc), d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(bc) + \mathfrak{I}_{\mathfrak{N}}^{\chi}(bc), d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(bc) + \mathfrak{F}_{\mathfrak{N}}^{\chi}(bc), \\
&d_{\mathfrak{T}_{\mathfrak{N}}}^{\psi}(bc) + \mathfrak{T}_{\mathfrak{N}}^{\psi}(bc), d_{\mathfrak{I}_{\mathfrak{N}}}^{\psi}(bc) + \mathfrak{I}_{\mathfrak{N}}^{\psi}(bc), d_{\mathfrak{F}_{\mathfrak{N}}}^{\psi}(bc) + \mathfrak{F}_{\mathfrak{N}}^{\psi}(bc)) \\
&\iff (td_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(ab), td_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(ab), td_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(ab), td_{\mathfrak{T}_{\mathfrak{N}}}^{\psi}(ab), td_{\mathfrak{I}_{\mathfrak{N}}}^{\psi}(ab), td_{\mathfrak{F}_{\mathfrak{N}}}^{\psi}(ab)) \\
&\neq (td_{\mathfrak{T}_{\mathfrak{N}}}^{\chi}(bc), td_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(bc), td_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(bc), td_{\mathfrak{T}_{\mathfrak{N}}}^{\psi}(bc), td_{\mathfrak{I}_{\mathfrak{N}}}^{\psi}(bc), td_{\mathfrak{F}_{\mathfrak{N}}}^{\psi}(bc)) \\
&\iff td_{\mathfrak{G}}(ab) \neq td_{\mathfrak{G}}(bc).
\end{aligned}$$

Therefore ED of every pair of adjacent edges is different iff TED of every pair of adjacent edges is different. Hence, $\mathfrak{G}$ is a NEISVING iff $\mathfrak{G}$ is a NETISVING. $\square$

Remark 5.16. If a connected SVING $\mathfrak{G}$ be both NEISVING and NETISVING, then it is not necessary for $\mathfrak{N}$ to be a CF.

Example 5.17. Consider $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ be an SVING as shown in figure 6. We have

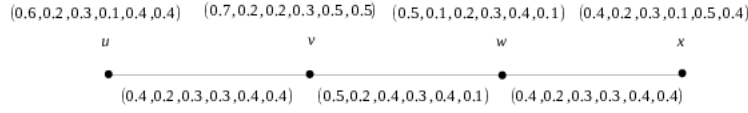


FIGURE 6. $\mathfrak{N}$ is not a constant function.

$$\begin{aligned}
 d_{\mathfrak{G}}(u) &= (0.4, 0.2, 0.3, 0.3, 0.4, 0.4) \\
 d_{\mathfrak{G}}(x) &= (0.4, 0.2, 0.3, 0.3, 0.4, 0.4) \\
 d_{\mathfrak{G}}(v) &= (0.9, 0.4, 0.7, 0.6, 0.8, 0.5) \\
 d_{\mathfrak{G}}(w) &= (0.9, 0.4, 0.7, 0.6, 0.8, 0.5) \\
 d_{\mathfrak{G}}(uv) &= (0.5, 0.2, 0.4, 0.3, 0.4, 0.1) \\
 d_{\mathfrak{G}}(vw) &= (0.8, 0.4, 0.6, 0.6, 0.8, 0.8) \\
 d_{\mathfrak{G}}(wx) &= (0.5, 0.2, 0.4, 0.3, 0.4, 0.1) \\
 td_{\mathfrak{G}}(uv) &= (0.9, 0.4, 0.7, 0.6, 0.8, 0.5) \\
 td_{\mathfrak{G}}(vw) &= (1.3, 0.6, 1, 0.9, 1.2, 0.9) \\
 d_{\mathfrak{G}}(wx) &= (0.9, 0.4, 0.7, 0.6, 0.8, 0.5).
 \end{aligned}$$

Here, $d_{\mathfrak{G}}(uv) \neq d_{\mathfrak{G}}(vw)$ and $d_{\mathfrak{G}}(vw) \neq d_{\mathfrak{G}}(wx)$. Hence, $\mathfrak{G}$ is a NEISVING. Also, $td_{\mathfrak{G}}(uv) \neq td_{\mathfrak{G}}(vw)$ and $td_{\mathfrak{G}}(vw) \neq td_{\mathfrak{G}}(wx)$. Hence, $\mathfrak{G}$ is a NETISVING. But $\mathfrak{N}$ is not CF.

Theorem 5.18. *Let $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ be a connected SVING and $\mathfrak{N}$ is a CF. If $\mathfrak{G}$ is a SISVING, then $\mathfrak{G}$ is a NEISVING.*

Proof. Let $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ be a connected SVING. Assume that $\mathfrak{N}$ is CF. Let $\mathfrak{N}(\mathfrak{uv}) = (c_1, \dots, c_6)$, for each $\mathfrak{uv} \in \mathfrak{E}$, such that $c_1, \dots, c_6$ are constants. Let $\mathfrak{uv}$ and $\mathfrak{vw}$ be any two adjacent edges in $\mathfrak{G}$. Also let $\mathfrak{G}$ is SISVING. Then the degree of each pair of nodes in $\mathfrak{G}$ is different.

Therefore

$$\begin{aligned}
& d_{\mathfrak{G}}(u) \neq d_{\mathfrak{G}}(v) \neq d_{\mathfrak{G}}(w) \\
& \implies (d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(u), d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(u), d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(u), d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(u), d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(u), d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(u)) \\
& \neq (d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(v), d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(v), d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(v), d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(v), d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(v), d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(v)) \\
& \neq (d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(w), d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(w), d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(w), d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(w), d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(w), d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(w)) \\
& \implies (d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(u), d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(u), d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(u), d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(u), d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(u), d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(u)) \\
& + (d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(v), d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(v), d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(v), d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(v), d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(v), d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(v)) - 2(c_1, \dots, c_6) \\
& \neq (d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(v), d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(v), d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(v), d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(v), d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(v), d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(v)) \\
& + (d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(w), d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(w), d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(w), d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(w), d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(w), d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(w)) - 2(c_1, \dots, c_6) \\
& \implies (d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(u) + d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(v) - 2c_1, d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(u) + d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(v) - 2c_2, d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(u) + d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(v) - 2c_3, \\
& d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(u) + d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(v) - 2c_4, d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(u) + d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(v) - 2c_5, d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(u) + d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(v) - 2c_6) \\
& \neq (d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(v) + d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(w) - 2c_1, d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(v) + d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(w) - 2c_2, d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(v) + d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(w) - 2c_3, \\
& d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(v) + d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(w) - 2c_4, d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(v) + d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(w) - 2c_5, d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(v) + d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(w) - 2c_6) \\
& \implies (d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(u) + d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(v) - 2\mathfrak{I}_{\mathfrak{N}}^{\chi}(uv), d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(u) + d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(v) - 2\mathfrak{J}_{\mathfrak{N}}^{\chi}(uv), d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(u) + d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(v) - 2\mathfrak{F}_{\mathfrak{N}}^{\chi}(uv), \\
& d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(u) + d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(v) - 2\mathfrak{I}_{\mathfrak{N}}^{\Psi}(uv), d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(u) + d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(v) - 2\mathfrak{J}_{\mathfrak{N}}^{\Psi}(uv), d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(u) + d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(v) - 2\mathfrak{F}_{\mathfrak{N}}^{\Psi}(uv)) \\
& \neq (d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(v) + d_{\mathfrak{I}_{\mathfrak{M}}}^{\chi}(w) - 2\mathfrak{I}_{\mathfrak{N}}^{\chi}(vw), d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(v) + d_{\mathfrak{J}_{\mathfrak{M}}}^{\chi}(w) - 2\mathfrak{J}_{\mathfrak{N}}^{\chi}(vw), d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(v) + d_{\mathfrak{F}_{\mathfrak{M}}}^{\chi}(w) - 2\mathfrak{F}_{\mathfrak{N}}^{\chi}(vw), \\
& d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(v) + d_{\mathfrak{I}_{\mathfrak{M}}}^{\Psi}(w) - 2\mathfrak{I}_{\mathfrak{N}}^{\Psi}(vw), d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(v) + d_{\mathfrak{J}_{\mathfrak{M}}}^{\Psi}(w) - 2\mathfrak{J}_{\mathfrak{N}}^{\Psi}(vw), d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(v) + d_{\mathfrak{F}_{\mathfrak{M}}}^{\Psi}(w) - 2\mathfrak{F}_{\mathfrak{N}}^{\Psi}(vw)) \\
& \implies (d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(uv), d_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(uv), d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(uv), d_{\mathfrak{I}_{\mathfrak{N}}}^{\Psi}(uv), d_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(uv), d_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(uv)) \\
& \neq (d_{\mathfrak{I}_{\mathfrak{N}}}^{\chi}(vw), d_{\mathfrak{J}_{\mathfrak{N}}}^{\chi}(vw), d_{\mathfrak{F}_{\mathfrak{N}}}^{\chi}(vw), d_{\mathfrak{I}_{\mathfrak{N}}}^{\Psi}(vw), d_{\mathfrak{J}_{\mathfrak{N}}}^{\Psi}(vw), d_{\mathfrak{F}_{\mathfrak{N}}}^{\Psi}(vw)). \\
& d_{\mathfrak{G}}(uv) \neq d_{\mathfrak{G}}(vw).
\end{aligned}$$

Therefore the ED of each pair of adjacent edges is different. Hence, $\mathfrak{G}$ is a NEISVING. $\square$

Theorem 5.19. Let $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ be a connected SVING and $\mathfrak{N}$ is a CF. If $\mathfrak{G}$ is a SISVING, then $\mathfrak{G}$ is a NETISVING.

Theorem 5.20. Let $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ be a connected SVING and $\mathfrak{N}(uv) = (c_1, \dots, c_6)$ be CF. If $\mathfrak{G}$ is RSVING, then $\mathfrak{G}$ is TERSVING.

Proof. Let $\mathfrak{G} = (\mathfrak{M}, \mathfrak{N})$ be a $(\mathfrak{k}_1, \dots, \mathfrak{k}_6)$ -regular SVING and $\mathfrak{N}(uv) = (c_1, \dots, c_6)$ for all $uv \in \mathfrak{E}$, where $c_1, \dots, c_6$ are constants. We have to prove that $\mathfrak{G}$ is TERSVING. Therefore,

$$td_{\mathfrak{G}}(uv) = (2\mathfrak{k}_1 - c_1, 2\mathfrak{k}_2 - c_2, 2\mathfrak{k}_3 - c_3, 2\mathfrak{k}_4 - c_4, 2\mathfrak{k}_5 - c_5, 2\mathfrak{k}_6 - c_6).$$

Hence, $\mathfrak{G}$ is TERSVING. $\square$

Remark 5.21. The converse of the theorem 5.20 is not true.

6. CONCLUSION

As a powerful tool for dealing with ambiguous data, neutrosophic sets have many applications in computer science, medicine, etc. In this paper, the notion of SVINS is propounded. Then, by combining this concept with graph theory, SVING is introduced. SVING has many applications in various fields. For example it can be referred to its various applications in Computer Science etc. Also, the concepts of degree, total degree of each node in SVINGS are presented. Then, irregularity and neighbourly irregularity on the SVING were expressed and the issues are investigated.

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