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Green Mixed-Model Assembly Lines Sequencing Problems

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Abstract:

The focus of this paper is on sequencing problems in green MMALs, with specific consideration of line efficiency and environmental impact. By applying green supply chain management principles, we aim to minimize vehicle routes for product delivery and specifically reduce carbon dioxide emissions. The specific objectives of the paper include reducing total setup costs, minimizing work overload situations, and lowering total earliness and tardiness costs according to order priorities. To achieve these goals, we define a mathematical model and present new sustainability goals. Because of the NP-hardness of this problem, we employ Benders decomposition to develop complex algorithms. The Benders decomposition algorithm is improved through incorporating acceleration techniques and adding optimality cuts. In addition, we use the Multi-Objective Particle Swarm Optimization (MOPSO) algorithm to solve the proposed model and compare the results across different objectives. The comparison demonstrates that the Benders decomposition with additional optimality cuts outperforms the MOPSO algorithm. The practical efficiency of the proposed Improved Benders Decomposition Algorithm (BDA) is demonstrated through experimental computations, considering both small- and large-scale cases. For small-sized problems, the algorithm obtains optimal solutions, improves objective function values by up to 3.9 percent compared to the MOPSO algorithm, and has a reasonable computing time. Between the improved BDA and the MOPSO, the former was observed to have superior solution quality, both in actual solutions and in diversity measures, in large-scale test cases, with much smaller optimality gaps (zero in most cases), and was computationally efficient.

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1. Introduction

Mixed-model assembly lines have become an important aspect of contemporary manufacturing systems, especially in industries that produce a variety of products in small lots to cater to customers' different needs. Nevertheless, conventional sequencing techniques tend to ignore the environmental impact of production work. Due to growing global concerns about resource depletion and climate change, it has become necessary to incorporate green strategies in manufacturing. Based on the discussions of the principles of Green Supply Chain Management (GSCM) and sustainable operations management, this research intends to fill this gap by addressing the environmental sustainability of scheduling decisions in mixed-model assembly lines.

This study will develop an optimization framework that considers the environment (i.e., fuel consumption and greenhouse gas emissions) together with efficiency in product sequencing and planned deliveries. The research question is as follows: How could the sequencing of a mixed-model assembly line be optimized to increase customer satisfaction, minimize its environmental impact, and maintain production feasibility? In response to this question, we create a multi-objective mathematical model that explicitly minimizes energy use, carbon emissions, and setup costs, and accounts for earliness/tardiness penalties to ensure on-time delivery. By incorporating these aims into an optimization framework based on the Benders Decomposition Algorithm (BDA), the suggested solution will contribute to both theoretical developments in sustainable manufacturing and the practice of green supply chain integration.

The concepts of green supply chain should be incorporated into mixed-model assembly lines to improve sustainability factors. Green Supply Chain Management refers to integrating environmental considerations into supply chain management, with the ultimate objective of delivering environmental benefits, such as reductions in waste, energy use, or greenhouse gas emissions. By promoting GSCM practices, the firm is positioned to achieve economic and ecological bottom lines, thereby contributing to the global fight against global warming [1].

The paper proposes a multi-objective assembly line sequencing problem in a sustainability context. Of the key

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dimensions of sustainability that have been covered, there is the environmental impact indicator that is measured in terms of the use of lesser fuel and carbon emission generated by delivery vehicles, and the operational efficiency indicator that is obtained in terms of lower cost of set up, idle times, and penalties for deviation from orders. These are modeled explicitly and incorporated into a new Benders Decomposition Algorithm (BDA) and will allow structured trade-off between economic and environmental performance."

In contrast to other existing works, which usually employ generic terms such as green supply chain integration that are often not tied to particular model variables or components, this paper directly incorporates sustainability objectives into the mathematics and algorithmic implementation. This enables a trade-off between environmental performance and operational effectiveness, which is a global sustainable development goal.

The present study on green mixed-model assembly lines has made the following important contributions to the sustainable manufacturing literature. This research seeks to fill a gap in the extant literature by focusing on improving product delivery patterns and routes to reduce fuel consumption and, therefore, the supply chain's environmental impact. Thus, by applying green supply chain management principles, including optimizing the distance between products and customers, the study will minimize overall energy consumption and greenhouse gas emissions. It also emphasizes satisfying customers' needs and delivering the product on time, all while maintaining cost-effectiveness and being environmentally sensitive. Thus, the contribution of this research lies in presenting fresh data on sustainable manufacturing practices and real-world solutions that can be implemented across different industries. The study not only contributes to improving existing conceptual models but is also useful in practice, helping to change approaches to production for the better. This work is significant and relevant to the academic community and the rest of the world because it addresses the current global challenge of improving the sustainability of industrial processes to fight climate change and protect the environment.

This paper examines sequencing decisions in mixed-model assembly lines (MMAL) from a green supply chain management perspective. The overall aim will be to enhance product sequencing at the level of production lines, while addressing environmental issues, including the carbon emissions from the delivery vehicle and energy use in production. The deterministic demand, constant cycle time, and known processing times in models are consistent with MMAL sequencing problems. All these assumptions enable us not to overcomplicate the model with stochastic or dynamic parameters, while still allowing us to select the trade-off they pursue between operational efficiency and environmental sustainability.

The uniqueness of this work is that a multi-objective mathematical model has been developed to minimize setup costs, work imbalance (idle/overload time), earliness/tardiness penalties based on order priorities, and fuel consumption and CO2 emissions. This type of combined approach has never been discussed in depth in prior research. We further suggest an extended Benders Decomposition Algorithm (BDA) with valid inequalities, optimality cuts, and acceleration methods, which outperforms heuristic methods such as MOPSO in convergence behavior and solution quality, especially on large-scale instances. This framework offers an in-depth, multi-objective ranking based on various performance indicators, making methodological and operational contributions to the study of green manufacturing and sustainable operations.

The remainder of the article is organized as follows: Section two presents the literature review, primarily addressing the sequencing concerns of this study. Specifically, Section 3 elaborates on the problem formulation and the mathematical model. Section 4 is dedicated to describing the Benders decomposition and the multi-objective particle swarm optimization algorithm; we define the features of the algorithms and their mathematical models. Section 5 covers the numerical experiments and computational results. Lastly, Section 6 presents the conclusion, where the authors also offer suggestions for future research on the variables studied.

2. Literature Review

In this paper, the literature review is presented in several comprehensive sections, ensuring a thorough examination of mixed-model assembly-line sequencing problems. The following are the usual objectives in sequencing research of mixed-model assembly lines:

The main goals and problem statements in the context of MMAL sequencing in the literature focus on maximizing production efficiency, minimizing costs, and maximizing flexibility. Cortez and Costa considered the effect of task execution time by the worker and the model on line productivity. By acknowledging differences in the workforce, their study provided a structure that benefits overall productivity by assigning tasks to efficient workers, thereby avoiding waste [2]. Likewise, Jiao et al. stressed the importance of maximizing the use of parts, the utility of activities performed, and setup time, especially in line balancing. This research also highlighted that the best way to manage objectives is to have more than one at a time, ensuring that work runs continuously and is interrupted as little as possible [3].

The studies conducted by Zhang et al. expanded on the mathematical models that balance and sequence the work to minimize idle time and utility work [4]. Akpınar and Bayhan proposed an integrated model to increase the flexibility and productivity of the lines, accounting for interactions between balancing and sequencing activities [5]. Later, Tanhaie et al. developed this approach to identify balancing and sequencing issues that minimize utility work and waiting time, making it a holistic approach to two-sided assembly lines [6]. Further, Li et al. focused on the general issues of task assignment and model sequencing in a robotic assembly line environment, with special emphasis on task allocation and sequencing to improve efficiency. They implemented new approaches to task and robot distribution that are crucial for the efficient operation of robotic lines and for maintaining business continuity [7].

2.1. Sequencing Techniques of Mixed-Model Assembly Lines

There are several methods for addressing various issues related to MMAL sequencing and balancing across different car models. Krenczyk and Dziki proposed a simulated annealing algorithm to achieve full line synchronization. They demonstrated that this algorithm can substantially enhance synchronization and reduce cycle times, which is imperative for the synthesis of effective production lines [8]. Paprocka and Krenczyk present the GWO for sequencing and find that it decreases energy consumption by 50% while increasing productivity [9].

Also, Bock and Boysen discussed optimizing part-feeding processes using real-time launch control. They created a solution concept for real-time launch control, focusing on the operational sequence on the assembly line and the part-feeding consequences. This real-time optimization process proved to be much more effective than classical methods for optimizing inventory safety stocks and overall material flow [10].

There has been a shift from basic research in MMAL sequencing to the development of complex algorithms and the need to solve real-life production issues. Tanhaie proposed a Multi-Objective Particle Swarm optimization algorithm for sequencing and balancing an MMAL problem with setup times between tasks. Based on the outcomes from comparison metrics and experimental problems, the proposed benders decomposition algorithm was the appropriate approach for the problem [11].

Rabbani et al. used the MOPSO and NSGA-II algorithms to address balance and sequencing issues grouped under multi-objective optimization. These algorithms were able to handle more complex sequencing dilemmas, in which it was possible to balance numerous goals, including cycle time and workload variance [12]. The application of multi-objective evolutionary algorithms for MMALs highlighted the potential for further increases in efficiency and flexibility, and ongoing research has incorporated new strategies and improvements in MMAL sequencing methodologies.

2.2. Mixed-Model Assembly Line Sequencing Problems about Customer Satisfaction

Customer satisfaction has emerged as a critical factor in planning and formulating MMAL's production sequences. Current studies have focused more on determining customer-oriented solutions to sequencing and balancing problems. Rabbani, Heidari, and Farrokhi-Asl focused on sequencing issues, ordering items according to customer satisfaction criteria using a bi-objective mathematical model [13]. Tanhaie, Rabbani, and Manavizadeh presented the ATP concept for ordering customer demand to determine the profitability of delivery quantities and delivery schedules [14]. In another study, the same authors used the fuzzy TOPSIS approach to prioritize customer orders in an MTO context; they employed a multi-objective particle swarm optimization (MOPSO) to handle the problem's NP-hard nature [15].

Further, Rabbani, Aliabadi, and Farrokhi-Asl put forward a framework to determine the order of customer orders with the help of the ANP in MM2SAL to minimize the utility work, idle time, and tardiness/earliness costs; the authors find that genetic algorithms are superior to other types of algorithms in large-scale cases [16]. Altogether, these studies stress the need to incorporate customer satisfaction into MMAL sequencing problems and the fact that satisfying customer orders and optimizing sequencing methods can improve both customer satisfaction and organizational performance. The discussed methodologies and algorithms can be viewed as a set of useful ideas and tools for addressing the emerging challenges of MMAL sequencing in customer-oriented environments.

2.3. The Problems of Mixed-Model Assembly Line Sequencing Considering Green Supply Chain

Indeed, in the recent past, the integration of green supply chain principles into MMAL sequencing has been a focus of interest. These studies focus on improving production processes, with an emphasis on factors such as energy use, waste, and resource utilization. Nie et al. proposed an integrated mathematical model for balancing and sequencing problem of MMALs, highlighting the flexibility and environmental issues [17]. Saif et al. were the first to incorporate sustainability into the sequencing algorithm, employing a multi-objective artificial bee colony algorithm to minimize variability in material usage and penalty costs for late deliveries [18].

In addition, Abdul-Rashid et al. paid more attention to reducing setup time and utility work, as well as incorporating green manufacturing elements to address concerns about green operations [19]. Defersha and Mohebalizadehgashti proposed a hybrid genetic algorithm for balancing and sequencing with the objectives of minimizing make-span and environmental cost; the latter involved assigning tasks common to different models to different workstations [20]. Kucukkoc and Zhang put forward an agent-based ant colony optimization method that integrates green supply chain factors and optimizes task allocation with lower environmental impact. Taken together, these studies indicated that, to achieve sustainable manufacturing goals, the green supply chain factor should be incorporated into the proposed MMAL sequencing [21].

Thus, as was seen from the literature review of MMAL sequencing, the focus is on the improvement of efficiency, reduction of costs, and increased flexibility of the line. Nevertheless, the application of the green supply chain in MMAL sequencing remains a relatively new practice. This work is unique in that sustainability objectives are integrated into the heart of MMAL sequencing problems. In this study, an effort is made to formulate a mathematical model of a supply chain that uses the state-of-the-art optimization technique of Benders decomposition while optimizing operational efficiency and environmental objectives. Besides, it effectively solves the problem due to the NP-hard property and offers real-world and efficient solutions that meet worldwide sustainability objectives. The following sections will describe the proposed model and the approaches used to demonstrate its efficacy, thereby informing the theory and practice of sustainable manufacturing.

We have developed a comprehensive comparative table summarizing key studies in MMAL sequencing.

Table 1. Comparative Summary of Key Literature in MMAL Sequencing

Papers	Objectives	Methodology	Key Findings	Green integration	Multi objectives
Cortez & Costa (2015)	Worker-task assignment, productivity	Heuristic-based model	Improved task allocation by worker skill level	✕	✓
Jiao et al. (2024)	Parts usage, utility time, setup time	Line balancing model	Multiple objectives improve system continuity	✕	✓
Zhang et al. (2021)	Minimize idle and utility time	Mathematical model	Effective in two-sided assembly lines	✕	✕
Akpınar & Bayhan (2011)	Flexibility, productivity	Integrated balancing/sequencing	Enhanced interaction between tasks	✕	✓
Tanhaie et al. (2020)	Utility work, waiting time	MOPSO	Holistic approach for two-sided lines	✕	✓
Paprocka & Krenczyk (2023)	Energy consumption, productivity	GWO algorithm	50% energy saving achieved	✓	✕
Rabbani et al. (2019)	Utility work, idle time, tardiness	ANP + GA	Genetic algorithm outperforms others in large-scale	✕	✓
Saif et al. (2014)	Material usage variation, penalty costs	Multi-objective ABC	First to consider sustainability in sequencing	✓	✓
Defersha & Mohebalizadehgashti (2018)	Makespan, environmental cost	Hybrid GA	Task assignment affects environmental impact	✓	✓
Kucukkoc & Zhang (2014)	Environmental impact, task allocation	Agent-based ACO	Green factors can guide optimal task assignment	✓	✓
proposed paper	Setup cost, idle time,earliness/tardiness, CO_2 emissions	BDA + MOPSO	Exact and heuristic solutions for the integrated problem	✓	✓

3. Problem Definition and Mathematical Model

Today, manufacturing is a highly dynamic process, made even more challenging by the need to implement as many improvements on the assembly line as possible to ensure customers are happy with what they get. This research focuses on the problem of scheduling mixed-model assembly lines, which are central to today's manufacturing systems. Essentially, mixed-model assembly lines require producing different models on a single line, making sequencing decisions critical to operations. Another goal of this research is to deliver products on time to customers, with paramount concern for customer satisfaction. Punctuality is crucial, as being late or arriving too early can demoralize the customer. Every order has a delivery time, and any time outside this schedule is chargeable and included in the objective function.

Besides, this research focuses on the extent of customer satisfaction with the specific product and on the environmental concerns associated with the manufacturing process. Hence, the desire to minimize the fuel consumption of vehicles used to deliver goods and services is also an aim now. The novelty of this work lies in how the problem is formulated, with product delivery via various vehicles, each with distinct travel routes associated with carbon emissions. The objective is to enhance plans to reduce setup costs at assembly stations and minimize idle time on production lines. By achieving these goals, this research aims to develop a new model that optimizes the efficiency and sustainability of hybrid model assembly lines.

What we have done is to indicate the assumptions that underlie the model:

- Station Layout: We take as the basic assumption a straight assembly line, in which the stations are arranged in line and workpieces move along the line at a constant velocity.
- Homogeneity of Stations: Stations are assumed to be homogeneous in processing capabilities and worker skill levels.
- Setup Types: The cost/time of the setup depends on the sequence; for example, when the product model is changed, some costs are incurred depending on the pair of models that are interchanged.

Achieving these objectives – fast delivery, low fuel consumption, and low carbon emissions – represents a breakthrough in the literature on mixed-model assembly lines. This new model is not only efficient in operation but also sustainable in its relationship with the environment; thus, it is useful for enhancing production and advancing environmental conservation. Based on the analyzed literature and the innovations introduced earlier, here is the mathematical model to solve the problem under consideration. Clear descriptions of these critical measures and decision factors will be provided in subsequent sections. The sets, parameters, and decision variables used in the mathematical model are defined in Tables 2 and 3.

Table 2. Sets

M	Set of product models $\{1,...,M\}$
N	Set of positions in the sequence $\{1,...,N\}$
V	Set of delivery nodes (customers + depot)
K	Set of delivery vehicles
C	The time taken to complete one cycle of the MPS product range.
M	The different models are collected in one set
T	It is the total working time required to produce one cycle of MPS products.
J	The total number of stations,
I	Total number of products which is indexed according to the positions
L_j	The length of station j.
d_m	The required demand of model m.
$cost_{vw}$	The cost of traveling from node v to node w.
cap	vehicle capacity
$G(V,A)\colon\left\{\begin{matrix} V=\{0,1,\ldots,m\} & 0 = The\ origin\ point \\ A=\{(v,w)\} & v,w\in V and \quad v\neq w \end{matrix}\right\}$	The representation of the traveling graph
S	The collection of vehicles.
p_{mj}	The processing time of model m at station j.
V_c	The rate at which the conveyor moves.
γ	The time interval between products
S_{jmn}	The cost incurred for setting up station j when changing from model type m to model type n.
$C_m^{tar}(C_m^{ear})$	The cost of tardiness or earliness of model m
$D_m(E_m)$	lateness or earliness of model m
$d_m(dd_m)$	Due date of model m

Table 3. Decision variables

$$r_{vw}=\begin{cases}1 & if\ the\ vehicle\ travels\ directly\ from\ v\ to\ w,\\ 0 & \text{otherwise}\end{cases}$$

$$f_{vs}=\begin{cases}1 & if\ node\ v\ is\ visited\ by\ vehicle\ s\\ 0 & \text{otherwise}\end{cases}$$

Q_m: $The\ total\ time\ required\ to\ complete\ the\ production\ of\ model\ m$

T_m: $The\ positional\ index\ of\ model\ m\ in\ the\ production\ sequence$

b_{ji} $continuous\ variable$: $Processing\ time\ of\ the\ model\ assembled\ in\ cycle\ i\ at\ station\ j$

O_{ji} continuous variable: Starting position of regular worker in station j when cycle i begins

$$x_{im}=\begin{cases}1 & \text{if ith product in a sequence is model m}\\ 0 & \text{otherwise}\end{cases}$$

$$y_{ji}=\begin{cases}1 & \text{if a utility worker is required at station j in cycle i}\\ 0 & \text{otherwise}\end{cases}$$

$$Z_{imn}=\begin{cases}1 & \text{if ith and i + 1th products in a sequence are model m and n}\\ 0 & \text{otherwise}\end{cases}$$

3.1. Multi-Objective Modeling

The first objective function will help minimize overload on the assembly line and reduce carbon dioxide emissions by optimizing product delivery to customers. The second objective function, on the other hand, seeks to minimize the total setup costs, the cost of order delay, and the cost of ordering ahead. Constraints 3 and 4 define the degree of tardiness and earliness for model m, respectively. The fifth constraint states that an edge can enter only one node, which is true for all nodes except node m, the depot node.

The following constraint ensures that there is at most one edge at each node, except at the depot node. Constraint 7 ensures that each model's demand is met, and constraint 8 requires that one model be assigned to each position in the sequence. Constraint 9 prevents the algorithm from producing solutions that contain subtours. Constraint 10 ensures that the load is not more than the permissible limit of the capacity Q in any of the vehicles. Constraints 11 and 12 carry forward the product sequence from the current cycle to the next production cycle. Constraint (13) gives the time taken to process the model assembled at station j during cycle i.

Constraints 14 and 15 determine the number of overload situations and the need for a utility worker when time and space are insufficient to complete the current workpiece. Constraint 16 defines the left station's extents. Constraint 17 defines the start configuration of the start positions. Constraint 18 requires that workers immediately start on the next product upon completion of the current one. Constraint 19 calculates the makespan, which is the time at which the first product is complete plus the time needed to satisfy the demand of model m. Constraints 20 and 21 are similar to constraints 17 and 18 and aim to identify the ith product until the demand of model m is met. Last but not least, Constraints 22 to 26 identify decision variables.

$$Min\, z_1 = \sum_{i=1}^{I}\sum_{j=1}^{J} y_{ji} + \sum_{v=0}^{m}\sum_{w=0}^{m} c\,ost_{vw} r_{vw} \tag{1}$$

$$Min\, z_2 = \{\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{n=1}^{M} z_{imn} \times s_{jmn} + \sum_{m=1}^{M}(C_m^{tar} \times D_m + C_m^{ear} \times E_m)\} \tag{2}$$

$$D_m = max\{0, Q_m - dd_m\} \quad m \in M \tag{3}$$

$$E_m = max\{0, dd_m - Q_m\} \; m \in M \tag{4}$$

$$\sum_{v=0}^{m} r_{vw} = \begin{cases} 1 & \text{if } w = 1,2,\dots,m \\ m & \text{if } w = 0 \end{cases} \tag{5}$$

$$\sum_{w=0}^{m} r_{vw} = \begin{cases} 1 & \text{if } v = 1,2,\dots,m \\ m & \text{if } v = 0 \end{cases} \tag{6}$$

$$\sum_{i=1}^{I} x_{im} = d_m \quad \forall m \in M \tag{7}$$

$$\sum_{m=1}^{M} x_{im} = 1 \quad \forall i \in I \tag{8}$$

$$\sum_{v,w \in U} r_{vw} \le |U| - 1 \quad (\forall U \subseteq \{1,2,\dots,m\}, |U| \ge 2) \tag{9}$$

$$\sum_m d_m f_{vs} \le \text{cap} \quad (s = 1,2,\dots,mm) \tag{10}$$

$$\sum_{m=1}^{M} z_{imn} = \sum_{k=1}^{M} z_{i+1nk} \qquad i = 1,2,\dots, I-1, \forall n \in M \tag{11}$$

$$\sum_{m=1}^{M} z_{Imn} = \sum_{k=1}^{M} z_{1nk} \qquad \forall n \in M \tag{12}$$

$$b_{ji} = \sum_{m=1}^{M} p_{jm} * x_{im} \qquad \forall i \in I, j \in J \tag{13}$$

$$o_{ji} + b_{ji} * v_c - l_j * y_{ji} \le l_j \qquad \forall i \in I, j \in J \tag{14}$$

$$o_{ji} + b_{ji} * v_c - l_j * y_{ji} - c * v_c \le o_{ji+1} \qquad \forall i \in I, j \in J \tag{15}$$

$$o_{ji} \ge 0 \qquad \forall i = 2,3,\dots. I, j \in J \tag{16}$$

$$o_{j1} = 0 \qquad \forall j \in J \tag{17}$$

$$o_{jI+1} = 0 \qquad \forall j \in J \tag{18}$$

$$Q_m = \left((T_m - 1) * \gamma + \sum_{j=1}^{J} \frac{l_j}{v_c}\right) \qquad m \in M \tag{19}$$

$$\sum_{i=1}^{T_m} x_{im} = d_m \qquad m \in M \tag{20}$$

$$d_m \le T_m \le I \quad m \in M \tag{21}$$

$$z_{imn} \in \{0,1\} \tag{22}$$

$$r_{vw} \in \{0,1\} \quad v = 0,1,\dots,m; \quad w = 0,1,\dots,m \tag{23}$$

$$f_{vs} \in \{0,1\} \quad V = 0,1,\dots,m; \quad S = 1,2,\dots,mm \tag{24}$$

$$x_{im} \in \{0,1\} \qquad \forall i \in I, m \in M \tag{25}$$

$$y_{ji} \in \{0,1\} \qquad \forall i \in I, j \in J \tag{26}$$

The model addressed the issue of earliness and tardiness broadly. We now formally learn to represent them using piecewise-linear functions and introduce additional constraints to account for deviations from due dates.

Objective Function Contribution:

$$minZ = \sum_{m \in M}(\alpha m \cdot Em + \beta m \cdot Lm) \tag{27}$$

Earliness and Tardiness Constraints:

$$Em \geq \delta m - Cm \qquad \forall m \in M \tag{28}$$

$$Lm \geq Cm - \delta m \qquad \forall m \in M$$
$$Em, Lm \geq 0 \qquad \forall m \in M \tag{29}$$

where *Cm* is the actual completion time of model *m,* and *δm* is the predefined due date for model *m*

Non-linear penalties can be represented linearly in this formulation, which is necessary for computational tractability, and the problem can also be addressed directly with standard solvers such as CPLEX.

4. Problem Solving

The sequencing problem studied in this work is a complex issue in mixed-model assembly lines and is NP-hard. As seen with the newly included innovative constraints and variables in the problem, it becomes even more difficult to solve efficiently. Due to the complexity of the constraints and the binary decision variables involved in this model, solving it by conventional methods is out of the question. Because of the NP-hardness of the problem, we have decided to employ Benders Decomposition – an algorithm that is suitable for large-scale optimization problems with a particular structure. Benders Decomposition applies to problems that can be separated into a master problem and one, two, or more subproblems. This algorithm works by breaking down the original problem into two parts: the master problem and the subproblem, which are well-defined and include all the given constraints. The master problem includes the decision variables in the problem's framework, while the subproblem includes the other variables and constraints.

4.1. Applying the Benders Decomposition Algorithm

In the MOPs, the objectives are always competing, and they are usually expressed in different units. Consequently, there is no single global solution that can simultaneously achieve the best possible value for all [22]. However, Pareto-optimal solutions offer a more general framework for optimization. They can be defined as solutions that outperform others in the search space when all objectives are considered. To solve this multi-objective balancing problem, an integrated Benders Decomposition Algorithm (BDA) based on the LP-metric method is employed.

The integrated BDA is tested using the LP-metric method, and for this purpose, several small-sized test instances are solved with the CPLEX solver in the GAMS environment.

4.2. LP-Metric Method

The method under review, called the LP-metric method, is a technique for solving multi-objective problems that converts conflicting objectives into a single objective [23]. This method involves determining the optimal solutions to the principal and subsidiary objectives by solving the multi-objective problem for each objective. In this paper, we use the following notation for the optimal solutions of the first and second objectives. It then sums the normalized deviations of each objective from its optimal value to obtain a single value of the objective function. The LP-metric.

$$Z_{LP\_metric} = \left((\frac{Z_1 - Z_1^*}{Z_1^*}) \times w_1 + (\frac{Z_2 - Z_2^*}{Z_2^*}) \times w_2 \right)$$
$$w_1 + w_2 = 1, \qquad w_1, w_2 \geq 0 \tag{30}$$

In this model, Z1 and Z2 are the primary and secondary objective functions, and w1 and w2 are the weights assigned to these objectives by the DM. With respect to the above, it is possible to apply AHP to assign weights to the objectives.

4.3. Benders Decomposition Algorithm

Frequently, Benders decomposition has been proven very useful in solving other large-scale optimization problems, particularly those with block-diagonal structure in the constraints [24]. This technique did help, as it broke down one grand problem into several sub-problems, making it easier to determine the best way to solve them. It is particularly useful when the problem to be solved is a mixed-integer programming (MIP) problem, and there are complicating variables that interconnect the various areas of the problem. The model discussed here is well-suited for Benders decomposition due to the following features.

This transforms the initial problem into what is known as the Master Problem (MP), which begins with an exponential number of cuts and is only introduced in the iteration with the Sub-problem (SP). The Benders Decomposition Algorithm (BDA), on the other hand, solves the master problem by iteratively solving a subproblem to obtain the solution. The framework of the proposed BDA is depicted in Figure 1.

To mitigate an issue with the proposed integrated BDA, the problem is decomposed into two distinct parts: the first part (MP) concerns rebalancing, and the second part (SP) concerns reducing the average cycle time of two lines and reducing the variability of station workloads in vertical balancing. It consequently follows that the optimal MP solution provides the minimum ceiling for the objective function. Taking the solution from MP and freezing the integer variables that specify them as inputs in the DSP gives the upper bound of the objective function. Also, solving DSP produces an optimal solution, and Benders cut containing additional continuous variables, which are then added to the master problem MP. Therefore, two problems are solved within a specific iterative process to obtain a desirable solution to the main problem.

- Optimality Gap Threshold: By a set threshold, the algorithm stops when the relative difference between the upper bound (UB) and lower bound (LB) becomes less than a given threshold:

$$Gap = \frac{UB - LB}{UB} \leq \epsilon \tag{31}$$

where ϵ is 0.1%.

- The Maximum number of iterations: The maximum number of iterations was set to 500 to prevent too much time wastage in computation.
- No Improvement Criterion: In case no change in the value of the objective is seen after the 50 iteration period, then the algorithm will be terminated early.

4.4. Bender's Master Problem

The formulation of the Benders MP in terms of the objective function and constraints is as follows.

$$min\left(\left(\frac{\sum_{i=1}^{I}\sum_{j=1}^{J}y_{ji}+\sum_{v=0}^{m}\sum_{w=0}^{m}cost_{vw}r_{vw}-Z_1^*}{Z_1^*}\right)\times w_1+\left(\frac{\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{n=1}^{M}z_{imn}\times s_{jmn}-Z_2^*}{Z_2^*}\right)\times w_2\right) \quad (32)$$

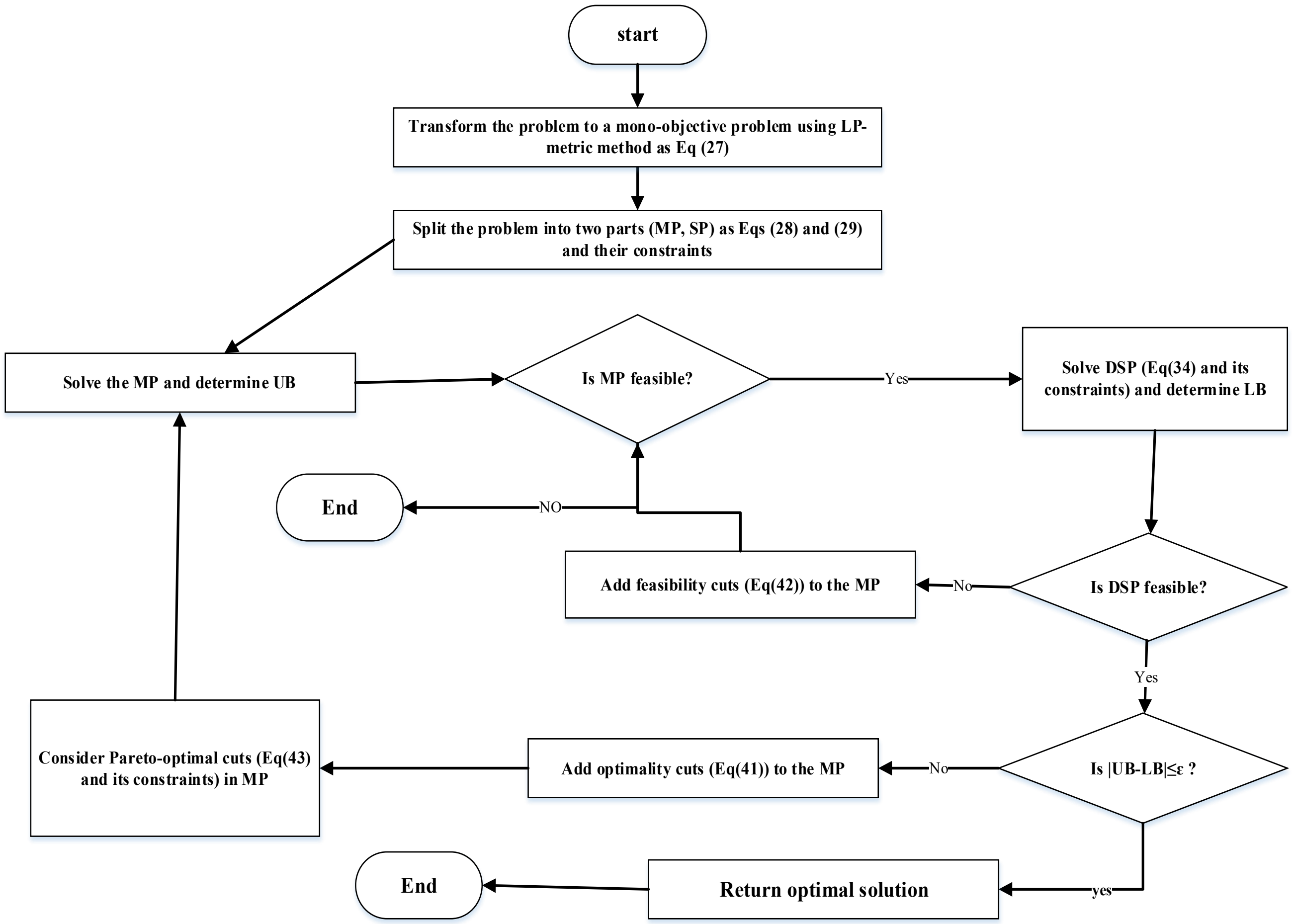


Figure 1. Proposed BDA

St. Constraints 5 to 15 and 20. The subproblem employs the same constraints as the main mathematical model. To avoid redundant presentation of identical mathematical expressions, the constraints are referred to by their corresponding equation numbers.

4.5. Benders Sub-Problem

The following is a list of the objective function and constraints formulated in the Benders SP.

$$Z_{SP}=min\left(\left(\frac{\sum_{m=1}^{M}(C_m^{tar}\times D_m+C_m^{ear}\times E_m)}{Z_2^*}\right)\times w_2\right) \quad (33)$$

St. Constraints 3 and 4, 16 to 19, and 21.

The subproblem employs the same constraints as the main mathematical model. To avoid redundant presentation of identical mathematical expressions, the constraints are referred to by their corresponding equation numbers. The linear transformation of constraint 3 is represented by equations 30 and 31, while the linear transformation of constraint 4 is expressed by Equations 32 and 33.

$$D_m \geq Q_m - dd_m \qquad \forall m \quad (34)$$

$$D_m \geq 0 \qquad \forall m \quad (35)$$

$$E_m \geq dd_m - Q_m \qquad \forall m \quad (36)$$

$$E_m \geq 0 \qquad \forall m \quad (37)$$

Here, integer variables determined in the previous step are fixed by solving the MP. By utilizing the corresponding dual variables, the Benders DSP is derived from the SP as follows:

$$Z_{Dsp} = \max \sum_m \left(dd_m \times W_m^1 + dd_m \times W_m^2 + \left(\frac{\sum L_j}{V_c} - \gamma \right) \times W_m^6 \right) + d_m \times W_m^7 + I \times W_m^8 \tag{38}$$

$$u_{kl}{}^1 - (1/N) \times u_l{}^2 \le 0 \quad \forall k,l \qquad W_m^1 + W_m^2 + W_m^6 \le 0 \quad \forall m \tag{39}$$

$$u_l{}^2 \le 0 \quad \forall k,l \qquad W_{ij}^3 + W_j^5 \le 0 \quad \forall m \tag{40}$$

$$u_l{}^3 - u_l{}^4 \le \left(\frac{order_l \times w_1}{z_1^*} \right) \quad \forall l \qquad W_m^7 + W_m^8 - \gamma W_m^6 \le 0 \quad \forall m \tag{41}$$

$$u_{kl}{}^5 + u_{kl}{}^6 \le \left(\frac{w_1}{z_1^*} \right) \quad \forall k,l \qquad \frac{\sum_m c_m^{tar}}{z_2^*} \times W_m^2 + W_m^2 \ge 0 \quad \forall m \tag{42}$$

$$u_l{}^1, u_l{}^2 \quad free \qquad \frac{\sum_m c_m^{ear}}{z_2^*} \times W_m^2 - W_m^2 \ge 0 \quad \forall m \tag{43}$$

$$u_l{}^3 \ge 0, u_l{}^4 \ge 0, u_l{}^5 \ge 0, u_l{}^6 \ge 0 \qquad W_m^1 \le 0, W_m^2 \ge 0, W_{ij}^3 \ge 0, W_j^5 : free, W_m^6 : free, W_m^7 \ge 0, W_m^8 \ge 0 \tag{44}$$

Firstly, the MP is solved to obtain a solution for the DSP, which will then be used as input to solve the optimization problem. Each time, it is sufficient to derive only one type of cut out of an optimality cut as well as a feasibility cut. The Benders cuts are formulated as follows: The Benders cuts are formulated as follows:

optimality cut:

$$Z_{MP} \ge \left(\left(\left(\frac{\sum_{m=1}^{M} (C_m^{tar} \times \mathrm{D}_m + C_m^{ear} \times \mathrm{E}_m)}{Z_2^*} \right) \times w_2 \right) \right)_2 + \sum_m \left(dd_m \times W^1_{m(\text{O\_iter})} + dd_m \times W^2_{m(\text{O\_iter})} + \left(\frac{\sum L_j}{V_c} - \gamma \right) \times W^6_{m(\text{O\_iter})} \right) + d_m \times W^7_{m(\text{O\_iter})} + I \times W^8_{m(\text{O\_iter})} \tag{45}$$

feasibility cut:

$$\sum_m \left(dd_m \times W^1_{m(\text{F\_iter})} + dd_m \times W^2_{m(\text{F\_iter})} + \left(\frac{\sum L_j}{V_c} - \gamma \right) \times W^6_{m(\text{F\_iter})} \right) + d_m \times W^7_{m(\text{F\_iter})} + I \times W^8_{m(\text{F\_iter})} \le 0 \tag{46}$$

$$(\text{O\_iter}) + (\text{F\_iter}) = g$$

Constraint 29 is used when the DSP is feasible, while constraint 30 is used when the DSP is unbounded, to generate optimality and feasibility cuts for the MP, respectively. The z variable serves as the constraint in the application of the MP, which serves as a supplementary objective function in this circumstance. The first is n, with A as the number of optimality cuts and B as the number of feasibility cuts in the g-th iteration.

4.6. Acceleration Techniques for Improved BDA

To further improve our modeling and results, we use acceleration techniques and solve the model in two ways, both of which are very time-consuming. The classical BDA, when used to solve an MILP without any acceleration, is relatively slow. Thus, this process is accompanied by several improved acceleration techniques. In this paper, the quality of these cuts is enhanced to boost the convergence rate and thus guarantee fewer total cuts, resulting in fewer total iterations.

4.7. Pareto-optimal Cuts (POC)

Recognizing this, the methods of Magnanti & Wong are used in this paper to improve the optimality cut (Magnanti & Wong, 1981). The DSP has multiple dual optimal solutions; hence, selecting different dual solutions for the continuous variables may affect the strength of the optimality cut added to the MP in any iteration. By selecting the most effective dual optimal solutions, we can construct an optimality cut that outperforms others, such as the Pareto-optimal cut (POC). According to Magnanti and Wong, a Pareto-optimal cut can be derived from DSP by solving the following problem:

$$\max \sum_m \left(dd_m \times W_m^1 + dd_m \times W_m^2 + \left(\frac{\sum L_j}{V_c} - \gamma \right) \times W_m^6 \right) + d_m \times W_m^7 + I^{corepo\,\mathrm{int}} \times W_m^8 \tag{47}$$

St. Constraints 35 to 40 and

$$\sum_m \left(dd_m \times W_m^1 + dd_m \times W_m^2 + \left(\frac{\sum L_j}{V_c} - \gamma \right) \times W_m^6 \right) + d_m \times W_m^7 + (I\,fix) \times W_m^8 = Z_{DSP}^* \tag{48}$$

In this context, Z_{DSP}^* represents the optimal objective function of the DSP.

4.8. Applying Multi-objective Particle Swarm Optimization (MOPSO)

In addition to Bender's decomposition, this research employs Multi-Objective Particle Swarm Optimization

(MOPSO) as a computational technique. It can be seen that MOPSO has performed well in converging to optimal or near-optimal solutions for a large number of multi-objective problems. Due to its ability to handle continuous and discrete variables, the model is useful for solving sequencing issues in sequence flow on mixed-product assembly lines. MOPSO has also shown strength in multidimensional problem-solving, where multiple conflicting objectives, such as environmental pollution levels and budget constraints, can be addressed simultaneously.

Applying MOPSO in our research enables us to compare the results obtained with the application of the Benders decomposition algorithm. When applying the same solution with both methods, it is easier to compare the pros and cons of the approaches and, most importantly, the implications for computational time and the quality of the results.

4.9. MOPSO Algorithm

One of the well-known methods utilized for solving multi-objective optimization problems is the Evolutionary Multi-Objective Particle Swarm Optimization (MOPSO). It starts a population of particles with random velocities and positions, and they search for the optimal solution as a swarm [25]. The swarm changes generations to reach the best solutions. It shows that particle velocities have both local and social factors: In MOPSO, Pareto dominance is employed to assess the most beneficial solutions as leaders. They are stored in an external archive of limited size that contains the best non-dominated solutions generated up to a given iteration. The velocities and positions of particles are calculated based on specific equations: The velocities and positions of particles are calculated based on specific Equations:

In this context, $\vec{y}(t)$ and $\vec{v}(t)$ are the coordinates and $\vec{v}(t)$ is the velocity at the present step of the particle i. The symbols r1 and r2 are used as random uniform in the interval [0, 1] to define the best position of the particle in each iteration. C1 and C2 represent the coefficients of personal learning and global learning, respectively, and 'w' is the inertia weight, which determines the degree of the old velocity that will be incorporated in the new velocity vector. $\vec{p}_{best}(t)$ denotes the best experience of particle i, and $\underline{rep^{\rightarrow}{}_{h}(t)}$ is the best particle selected from the various repositories offered by the global neighborhood. Figure 2 illustrates the configuration and the core process of MOPSO.

This article also employed the following parameters because of their relation to MMAL sequencing, as postulated by Clerc & Kennedy [26].

4.10. Parameters Setting

Here, the authors emphasized that accurate parameter tuning of the multi-objective particle swarm optimization (MOPSO) algorithm is crucial for achieving the best results. It helps to have an efficient, fast search and to avoid the appearance of the so-called 'premature convergence'. We used an orthogonal array for four algorithmic parameters at three levels each (Table 4):

Table 4. Parameters Setting

Parameters	Level 1	Level 2	Level 3
Inertia Weight (w)	0.4	0.6	0.8
Cognitive Coefficient (c1)	1.5	2.0	2.5
Social Coefficient (c2)	1.5	2.0	2.5
Swarm Size	30	50	70

Each combination of levels was evaluated in 10 independent runs, and the average objective value was recorded.

As applied in this work, the parameters of all MOA methods were previously adjusted based on the experimental outcomes. The MOPSO algorithm developed in this study employed the Taguchi method to identify the optimal parameters. At four levels representing each parameter, as shown, the experiments were designed and analyzed using the Taguchi methodology, which helps determine the minimum number of experiments. The use of orthogonal arrays (OAs) and signal-to-noise ratio (S/N) allows for multi-objective optimization. The OA matrix has the numerical data arranged in rows and columns, and the S/N ratio gauges the issue of parameters following every complete run of the experiment, as Equation 49 demonstrates [27].

$$(\frac{S}{N})\,ratio = 10 \times log(\frac{\bar{y}^2}{s^2}) \tag{49}$$

$\bar{y}$ represents the average response, while s^2 signifies the variance of the sample responses. The data were then analyzed using Taguchi's graphical method, as shown in Figure. The S/N ratio is assigned in direct proportion to the value of the objective function: the least value of which indicates the best solution. As such, the optimal factor level is the level that corresponds with the highest value of S/N. Average S/N ratios of each level are shown in Figure 3.

Referring to Figure 3 above, it is evident that by segregating the average S/N ratios, the optimal parameter values for improving results can be determined. The best levels for the factors are determined as follows: Swarms at level one; Cognitive coefficient, c1, at level three; Social coefficient, c2, at level two; and Inertia weight, w, at level four.

5. Computational Results

Thus, to assess the effectiveness of the specified Benders decomposition algorithm, we first apply it to solve a series of small problems. These problems, as listed in Table 5, involve one or more of the following input data: processing time, dependencies among tasks, and delivery times for models. The efficiency analysis of our algorithm is made with the aid of the LP-metric method, which is found in the CPLEX solver of the GAMS environment. This first step establishes the credibility and validity of the Benders decomposition method for smaller-scale problems, in addition to other kinds of problems.

After demonstrating its viability on these simpler problems, the next step is to apply the Benders decomposition algorithm to larger problems. These larger problems will be solved, alongside the Benders decomposition algorithm, by the Multi-Objective Particle Swarm Optimization (MOPSO) algorithm. The comparison of the two methods allows for the comparison of the outcomes with the operational requirements for the analysis of the meeting of the environmental sustainability goals. Apart from the condition that this comprehensive analysis guarantees that our strategy can be scaled, it also makes practical and efficient contributions to the creation of initial mixed-model assembly-line models, in order to first enhance efficiency to meet international sustainable development strategies

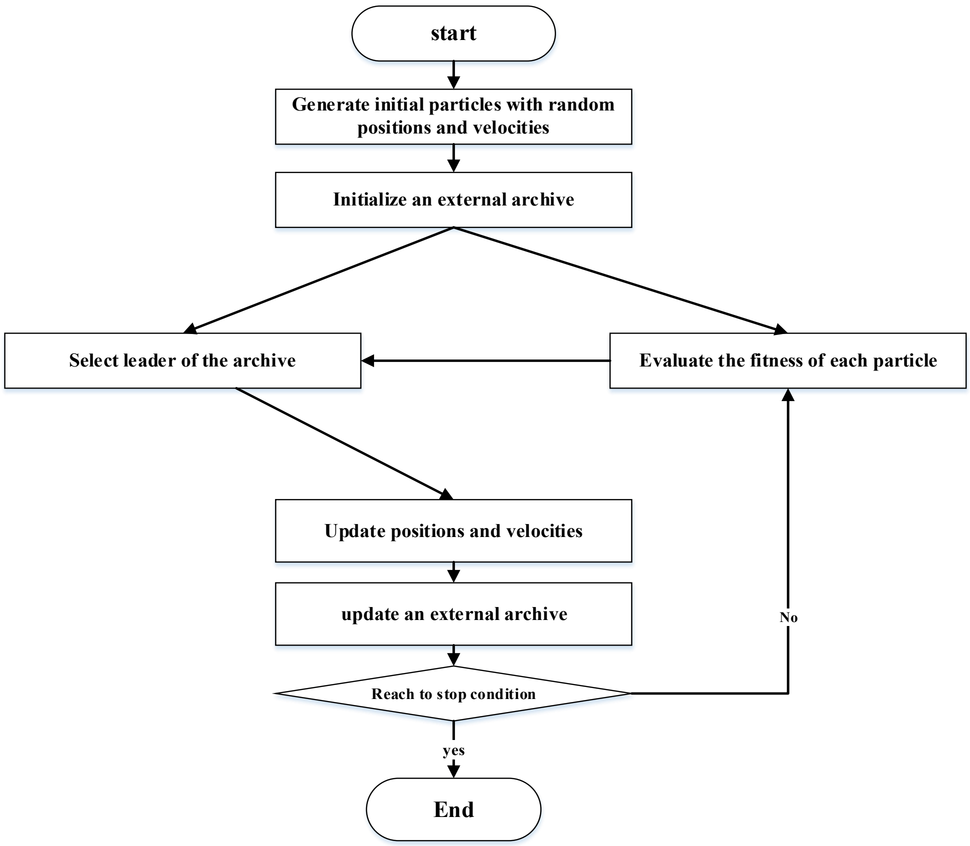


Figure 2. MOPSO flowchart

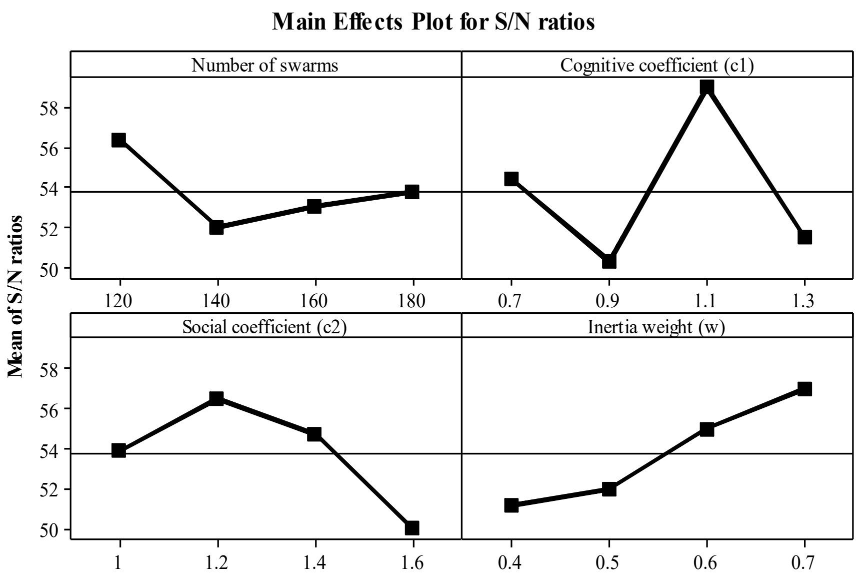


Figure 3. The mean S/N ratio plotting for each level of the factors

Table 5. Input data for test problems (s)

Test Problem 1						Test Problem 2						
	1	6	3		-		1	8	6	5	-	-

	2	3	-	-	-		2	5	-	-	-	-
	3	-	4	-	1,2		3	-	-	4	-	2
	4	4	-	-	3		4	6	7	7	-	3
	5	5	3	-	3,4		5	-	5	6	-	3,4
	6	7	7	-	5		6	4	8	-	-	3,5
	7	12	9	-	2,4		7	8	6	5	-	6
	8	3	4	-	6		8	7	5	-	-	7
	-	-	-	-	-		9	12	7	4	-	6
	-	-	-	-	-		10	12	5	-	-	3,7
	-	-	-	-	-		11	12	5	7	-	8
Delivery Time		93	75	-	-	**Delivery Time**		45	37	52	-	-
Test Problem 3	1	9	7	2	-	**Test Problem 4**	1	8	6	5	-	5
	2	6	-	4	-		2	6	7	9	4	9
	3	7	10	7	2		3	5	-	6	6	6
	4	9	4	-	3		4	-	5	7	8	1,3
	5	4	-	6	3,4		5	-	9	10	-	4
	6	-	9	8	-		6	7	10	7	7	-
	7	9	6	3	3,5		7	4	5	-	9	4,5
	8	10	7	-	7		8	9	6	-	3	-
	9	-	-	-	8		9	-	-	-	-	7,8
	10	11	5	7	9		10	11	-	7	2	5
	11	6	8	4	10		11	-	6	10	-	7,10
	12	10	12	9	8,9		12	7	8	9	-	8,11
	13	6	8	8	11		13	11	3	9	6	12
	-	-	-	-	-		14	9	10	7	8	9,10
	-	-	-	-	-		15	12	6	6	4	13
Delivery Time		72	53	64	-	**Delivery Time**		92	26	50	71	-

To provide evidence of the developed optimization model's favorability, the results of the CPLEX solver, BDA, and the enhanced BDA model are summarized in Table 6. This again proved the strength of the argument in the comparative analysis in favor of the proposed BDA and outlined its reliability and accuracy.

Table 6. Comparison of CPLEX solver and proposed BDA (small-sized problems)

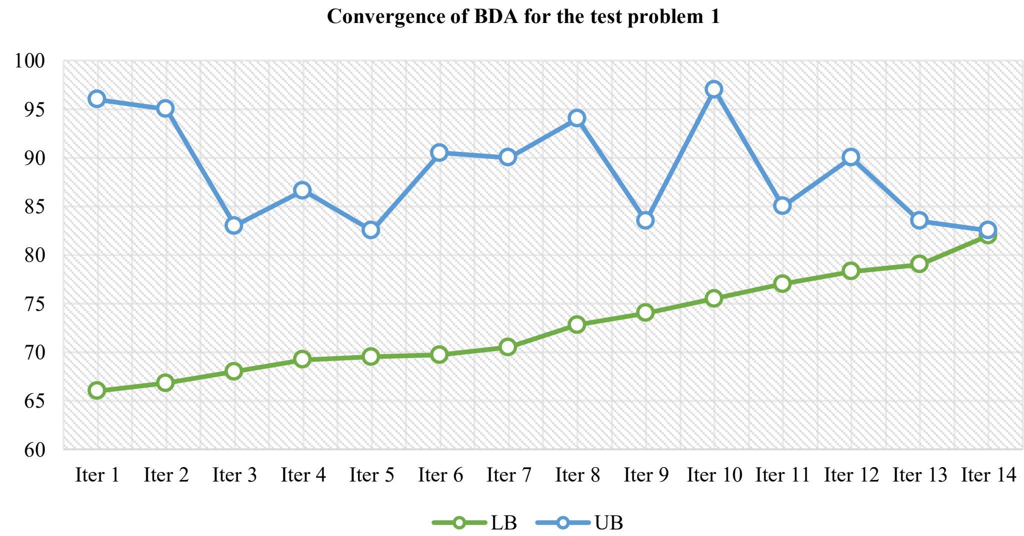
Number	Models	Tasks	Stations	Sequencing solution (BDA)	Approach	Z1	Z2	Run time (s)
					BDA	169	221	8.758
1	2	8	4	{1,3/2,4/5,8/6,7}	Improved BDA	168	217	7.453
					CPLEX	168	217	4.673
					BDA	197	243	47.979
2	3	11	6	{1,2,4/3,6/5/7,9/8,10/11}	Improved BDA	193	239	41.987
					CPLEX	193	239	30.278
					BDA	234	260	68.498
3	3	13	7	{1,3/2,3,6/4/7,10/8,9/12/11,13}	Improved BDA	234	253	60.908
					CPLEX	234	253	66.346
					BDA	26	310	157.949
4	4	15	9	{1/2,4/3,4,7/5/8,11/9,10/13/11,14/12,15}	Improved BDA	259	302	122.567
					CPLEX	259	302	223.075

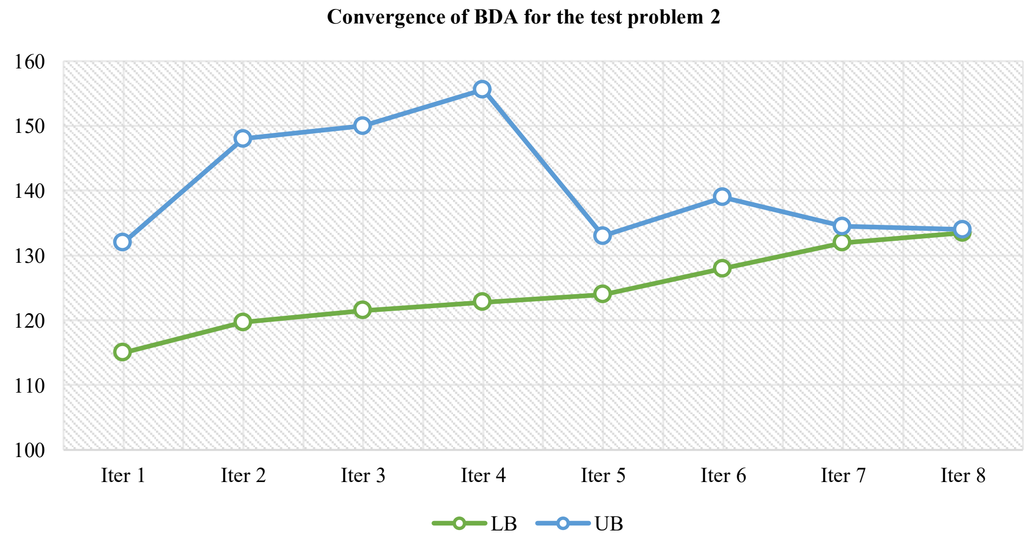
This is shown in Figure 4, where the upper bound, including the Benders cut (UB), and the lower bound (LB are obtained after one has solved the problem using the improved Benders Decomposition Algorithm (BDA). Such a coincidence proves that the algorithm helps minimize the difference between the bounds, thereby finding the optimal solution. Therefore, the convergence of UB and LB, as observed through iterations, demonstrates the success and competence of the algorithm in the optimization problem.

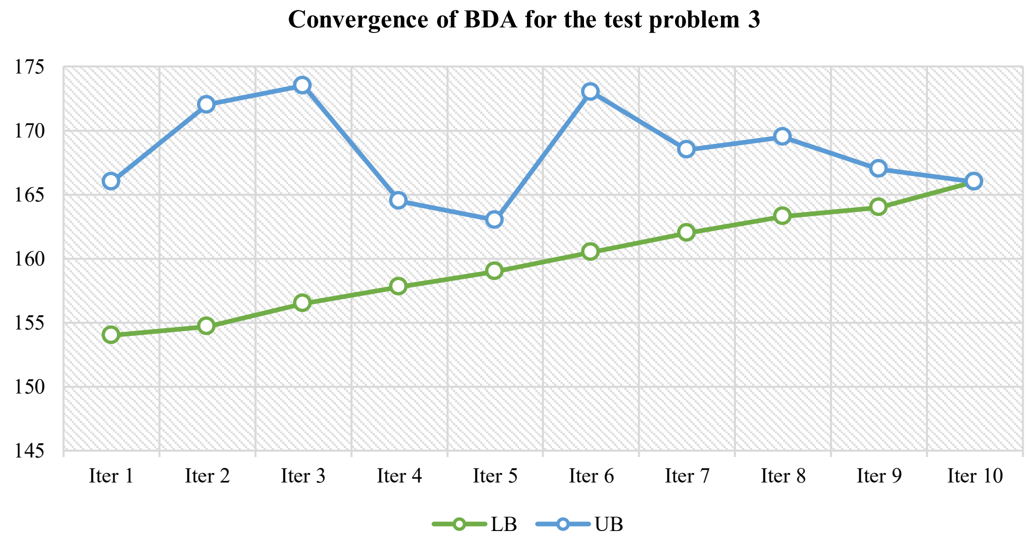
5.1. Large Sized Test Problems

Given the efficiency of the improved BDA on small problems, the next target is to solve larger instances. In the next phase of the present work, we plan to compare our proposed BDA with the Multi-Objective Particle Swarm Optimization (MOPSO) algorithm described in Table 7. This comparison will also extend to analyzing the effectiveness of tuned parameters on larger, more scalable datasets for our proposed method.

The summary of results from the three methods is shown in Table 7 below, where the optimality gap and computational time are compared. The main drawback of the LP-metric method is that it is computationally intensive and therefore not well-suited to large problems in a reasonable amount of time. Scalability analysis using the standard deviation of the Average number of Iterations shows that the enhanced Benders Decomposition Algorithm (BDA) reaches a 0% Optimal Gap. In contrast, the Gap ranges from 0 to 3.9% using the Multi-Objective Particle Swarm Optimization (MOPSO) Algorithm. In terms of computational time, MOPSA has a relatively smaller value than that of the improved BDA. However, the computational times for the improved BDA remain relatively low, given that BDA solves problems exactly and the sizes of the problems under consideration.







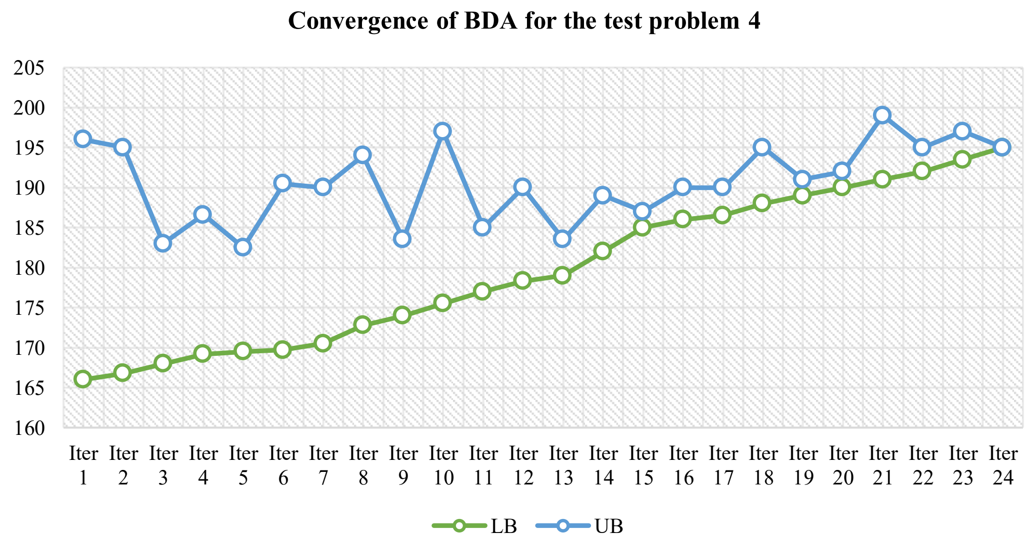


Figure 4. **Convergence of improved BDA based on the objective function**

Table 7. **Computational results**

		LP metric	Improved BD		MOPSO	
problems	**task**	**Run time (s)**	**Run time (s)**	**% from optimal**	**Run time (s)**	**% from optimal**
1	8	4.673	5.453	0	5.206	0
2	11	30.278	41.987	0	40.398	3
3	13	66.346	60.908	0	51.095	2.7
4	15	223.075	122.567	0	101.389	3.9
5	19	>3400s	438.879	-	378.498	-
6	21	>3400s	889.230	-	749.408	-
7	26	>7000	1278.097	-	1108.6898	-
8	29	>7000s	1630.768	-	1428.389	-
9	34	>10500s	2104.790	-	1903.489	-
10	40	>10500s	3739.397	-	2849.678	-

5.2. The use of Comparison Metrics of improved BDA and MOPSO

For comparison with multi-objective algorithms, different metrics are used than in the case of single-objective algorithms. In this context, familiar comparison measures such as Spacing, Number of Pareto solutions, and Diversity have been used to measure the quality of solutions for the improved BDA and MOPSO. These metrics assist in meaningful comparisons and assessments of the performance and efficiency of these algorithms.

Spacing metric: This metric's value is defined by Equation 50.

$$Spacing = \left[\frac{1}{n-1} \times \sum_{i=1}^{n} \left(\bar{d} - d_i\right)^2\right]^{\frac{1}{2}} \qquad (50)$$

Here, $d_i$is is the Euclidean distance that solution i has from the nearest solution obtained from the computations on the Pareto-optimal frontier. The parameter ‘n’ is the total enumeration of the identified non-dominated solutions and $\bar{d}$ is the average of all d_i. Spacing measures the uniform distribution of points on S in terms of evenness or uniformity. It means that if the value of this metric is low, the algorithm's performance is better.

Diversification metric: It is calculated using the formula given below. This is done by computing the directed cosines from node 58 to each of its first neighbors to find the proportions of the solution set, as described by McMullen [28].

$$D = \sqrt{\sum_{i=1}^{n} max(\|x_i - y_i\|)} \qquad (51)$$

Here, $max(\|x_i - y_i\|)$ is the Euclidean distance between non-dominated solutions y_i and x_i , and n is the number of Pareto-optimal solutions that have been detected. The power of this measure increases with the increase, and higher values imply that the algorithm produces higher-quality solutions that are more diverse in the solution space. In Table 8 and Figure 5, the results for the spacing metric, the number of Pareto solutions, and the diversification metric are provided.

Based on the spacing metric values obtained from the computation above, it is clear that the MOPSO algorithm is better due to its smaller spacing metric value. This means that the solutions produced by the MOPSO algorithm, devoid of dominance, are more dispersed than those developed by the improved BD algorithm. The average values of the diversification metric presented in Table 7 indicate that the improved BD algorithm is superior to MOPSO in terms of the size of the solution space it explores.

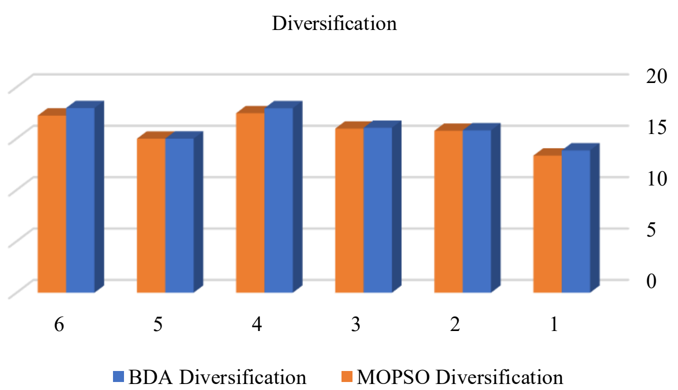
Table 8. Comparison between the algorithms

		BDA	MOPSO	BDA	MOPSO	BDA	MOPSO
TP	**NT**	**Spacing**	**Spacing**	**Number of Pareto solutions**	**Number of Pareto solutions**	**Diversification**	**Diversification**
5	19	5.95	5.15	42	45	13.85	13.35
6	21	6.84	6.20	38	40.0	15.80	15.76
7	26	7.57	7.15	39	38	16.05	15.97
8	29	7.04	6.85	47	45	17.95	17.47
9	34	7.19	6.13	53	50	15	14.99
10	40	7.52	7.37	37	42	17.97	17.24

5.3. Run Time Comparison

The outcomes clearly show that the MOPSO computational time is slightly shorter than that of the improved BD algorithm. This is well illustrated in the following chart (Figure 6), which shows the computation time of MOPSO. Thus, to make the comparison of the analyzed algorithms more precise, inclusive statistical analyses were conducted, and the comparison of the algorithms for some indicator was made at the 90% confidence level in Table 7. Table 3 below presents ANOVA and t-test results to assess the null hypothesis regarding the duration of the intervention.

$$\begin{cases} H_0: BDA_{(criteria)} = MOPSO_{(criteria)} \\ H_1: BDA_{(criteria)} \neq MOPSO_{(criteria)} \end{cases} \qquad (52)$$



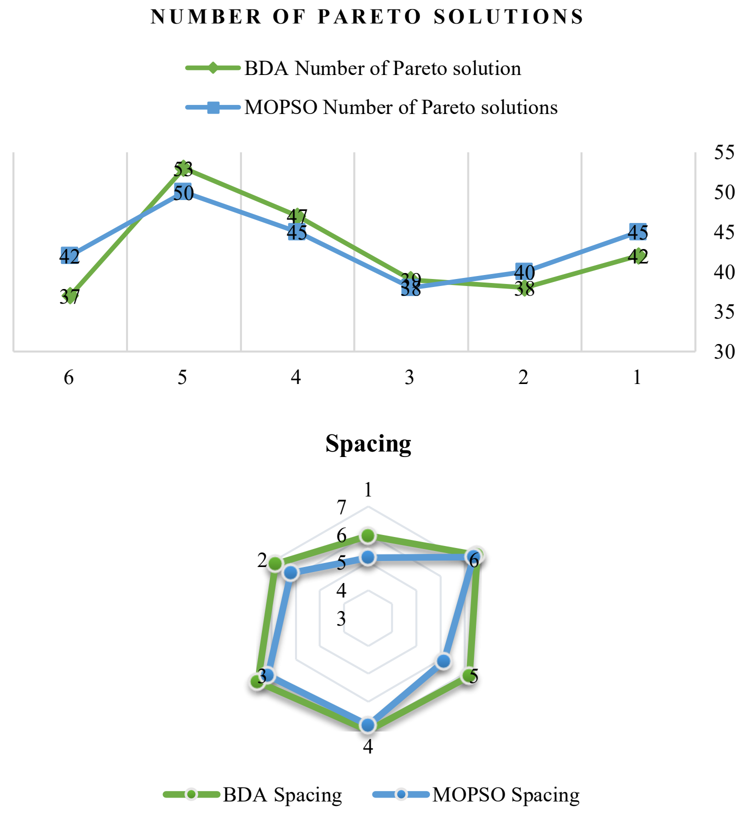


Figure 5. Comparison between the algorithms

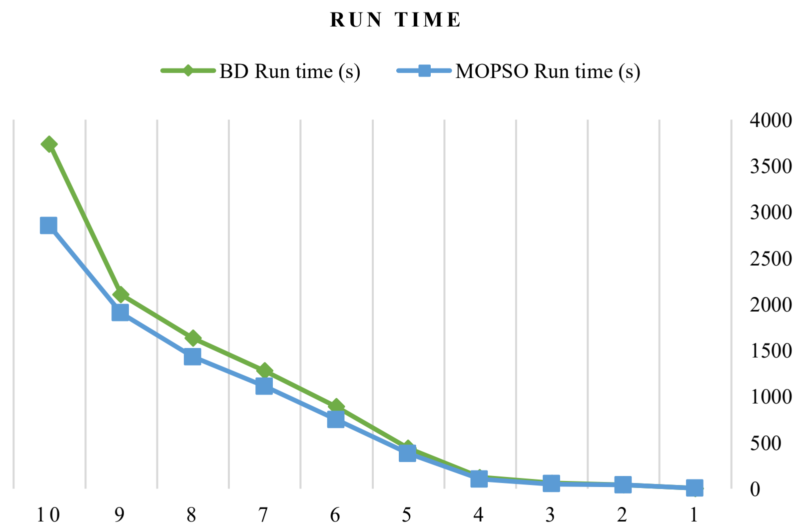


Figure 6. Compared the computation time of the algorithm BDA

Table 9. Computational of variance and t-test

indicator	P-Value	Test result	Better performance
Model solving time	0.669	Failure to turn down the null hypothesis	The differentiation between the two algorithms is not much
Objective function	0.076	Reject the null hypothesis	Algorithms of the improved Benders decomposition also prove to be quite efficient
Spacing metric	0.406	Failure to turn down the null hypothesis	The differentiation between the two algorithms is not much
Diversification metric	0.093	Reject the null hypothesis	Algorithms of the improved Benders decomposition also prove to be quite efficient

The proposed Benders decomposition algorithm achieves higher accuracy in the objective function and Diversification metric than the MOPSO algorithm. However, it can be seen that there is almost no nearly significant difference in the performance of the two algorithms in terms of the time indicator and Spacing metric. Hence, based on the findings from the test problems and the evaluation criteria, the improved BD algorithm has been identified as the most appropriate solution.

5.4. Sensitivity Analysis

To study the randomness in overtime, idle time, and the objective function, a value sensitivity analysis was again conducted on a parameter that influences the objective function, as shown in Table 8. To determine how sensitive the problem is to the selected parameter, the parameter is varied over the range ±20%. Here, the parameter of interest is the station length, as the amount of overtime and idle time for operators at each station during product sequencing causes long waiting times at the station.

Table 10. Alterations of the station length parameter

Number	Change parameter	Over time	Idle time	Z
1	0.8	83	42	3365
2	0.9	68	59	3214
3	1	54	73	3128
4	1.1	47	81	3014
5	1.2	83	42	3365

The increase in station length has two main consequences:

Firstly, as station length increases, operator idleness is also higher at some stations, as depicted in the graph. This increases idling because, by the time the cycle time is reached, all the station's activities have already been completed.

Secondly, this expansion in station length results in time savings for operators, meaning that overtime is decreased at some stations, as shown in Figure 6. This diminution in overtime is a result of increased cycle time as a result of prolonged station length, through which all requisite tasks may be accomplished despite high workloads. As a result, the requirement for overtime shrinks.

Indeed, the net effect of this reduction in overtime and increase in idleness is positive, resulting in a decrease in the objective function. However, the extent of this decrease is very small; as evident in Figure 7, the range of reduction is minimal in the region of 1.1 to 1.2.

Thus, with a marginal increase in workstation length per line to accommodate more stations, the overall model cost decreases. But to the same extent, one has to seriously contemplate the correlation between these savings and the possible rise in costs due to the increased station length. One should also note that an excessively large increase or decrease in the station length can pin down the feasible area between the constraints, rendering the model infeasible. For all the problems, the critical length threshold is different.

Short Managerial Insights: The study introduces practical implications for production and logistics managers seeking to enhance the efficiency and environmental outcomes of the mixed-production-line assembly system. By incorporating green supply chain principles into the process of making sequencing decisions, it will be possible to minimize carbon emissions and fuel consumption and achieve a similarly high level of production performance. Sensitivity analysis shows that more can be done to make the system more responsive and economical in response to changes in order priorities and delivery schedules. The findings indicate that implementing the Improved Benders Decomposition Algorithm would help make more informed decisions, as these decisions will be data-driven, especially in large-scale manufacturing settings. Strategies involving optimization can enable other firms to contextualize sustainability agendas on the global front, as well as be competitive in dynamic markets.

6. Conclusion

This paper reports a literature review and a methodology for addressing part sequencing issues in green MMALs, with a focus on productivity and the environment. Comparable to the results achieved above, the integration of the GSCM concept to sequencing results in the achievement of the two goals of reducing the environmental footprint and increasing the production cycles. Based on the described approach of applying Benders decomposition together with acceleration techniques and Pareto-optimal cuts, the improvements in both the convergence rate and solution quality are remarkable. Based on the outcomes derived from the improved Benders decomposition algorithm, the current study predicts a better optimization rate than the adopted Multi-objective Particle Swarm Optimization (MOPSO) algorithm, even in large-scale problems, despite reasonable computational time. Comparing the two methodologies, where some activities were replicated, made it easier to understand how the improved Benders decomposition can produce better, more efficient solutions that satisfactorily meet the stringencies of modern sustainable manufacturing systems. Moreover, the incorporation of a real-world case and the sensitivity analysis gives practical implications of the proposed model in a real-world environment, making the model more realistic and suitable for real-population conception.

The study makes a valuable contribution to the literature by presenting a concept and a framework for a practical, easily implementable approach that, at the same time, fits within the boundaries of sustainable development goals at an international level. Environmentally sustainable manufacturing practices are currently a great concern. There are possibilities for future studies by expanding other environmental objectives and developing improvement methods to increase the flexibility of acceleration methods for more general manufacturing processes. This work thereby provides a solid background for the progressive sustainability of green manufacturing, aiming to improve operational efficiency while also considering sustainability.

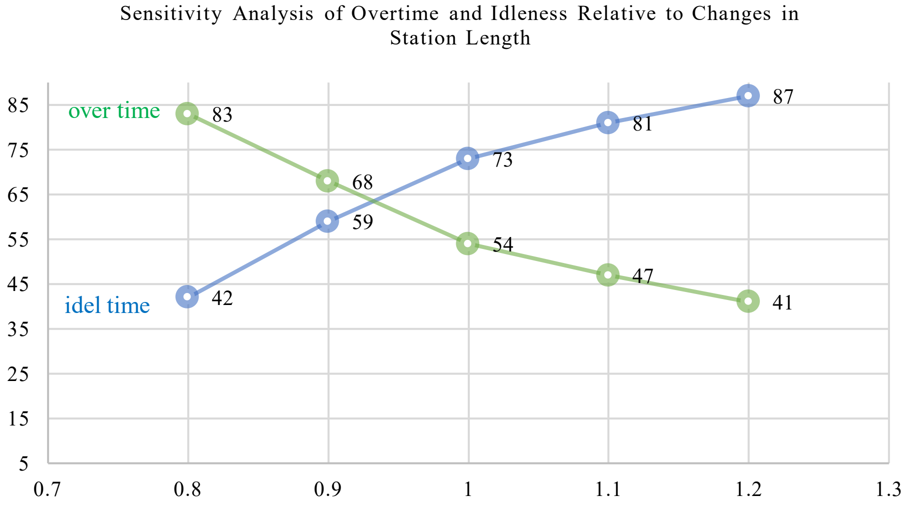


Figure 7. Changes in Overtime and Idleness Relative to Changes in Station Length

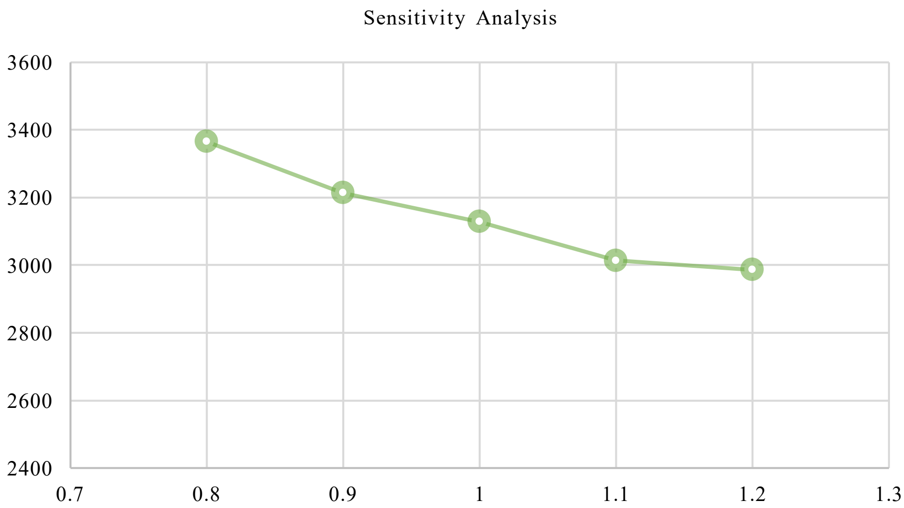


Figure 8. The parameter of the main objective function in Station Length changes compared to in Station Length changes

7. Statements & Declarations

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7.3. Author Contributions

Fahimeh tanhaie is the sole author of this manuscript and was responsible for the conception, design, data analysis, and writing of the manuscript.

7.4. Data Availability

The authors confirm that the data supporting the findings of this study are available within the article.

7.5. Competing Interests

The authors have no competing interests to declare that are relevant to the content of this article

7.6. Ethics Approval and Consent to Participate

This study does not involve human or animal subjects. Therefore, ethical approval and consent to participate are not applicable.

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