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Fuzzy-Based Numerical Investigation of MHD Nanofluid Flow over a Stretching Surface with Darcy–Forchheimer Porous Medium Effects

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Abstract. This paper presents a numerical study on the steady two-dimensional flow of a magnetohydrodynamic (MHD) nanofluid over a convective stretching surface embedded within a porous medium, accounting for nonlinear Darcy–Forchheimer resistance. To effectively address uncertainties in critical physical parameters such as magnetic field strength and porous medium permeability, fuzzy set theory is applied. The governing boundary layer equations containing fuzzy parameters are converted into nonlinear fuzzy ordinary differential equations through similarity transformations and solved using the α-cut technique. The results indicate that increases in the fuzzy magnetic field and Darcy–Forchheimer parameters lead to elevated velocity and microrotation profiles, while temperature

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and nanoparticle concentration profiles decrease. The fuzzy approach provides uncertainty intervals around these predictions, offering valuable insights for engineering designs involving nanofluid flows under imprecise conditions.

Keywords: Magnetohydrodynamics, Nanofluid flow, Darcy–Forchheimer porous medium, Fuzzy set theory, Uncertainty analysis.

2000 Mathematics subject classification: xxxx, xxxx; Secondary xxxx.

1. Introduction

In recent years, the incorporation of fuzzy logic with nanofluid dynamics has emerged as a powerful approach to address uncertainty in heat and mass transfer studies. Ayub et al. [1] conducted an in-depth analysis of Cross nanofluid behavior influenced by an inclined magnetic field within a fuzzy framework, unveiling distinctive thermophysical characteristics. Building upon fuzzy nanofluid mechanics, Siddique et al. [2] performed numerical simulations on magnetohydrodynamic Couette flow through an inclined channel, integrating thermal radiation effects under fuzzy conditions. Shanmugapriya et al. [3] proposed a chemically reactive fuzzy hybrid model that focused on the magnetized Casson nanofluid flow influenced by endothermic and exothermic processes. Furthermore, Qeays et al. [4] applied a fuzzy-integrated optimization algorithm to enhance the multi-performance of hybrid photovoltaic thermal systems, which was complemented by Babanezhad et al. [5], who employed a computational approach combining fuzzy logic, genetic algorithms, and CFD techniques to predict turbulent convective heat transfer.

Reiterations of these studies were observed in subsequent works by Ayub et al. [6] and Qeays et al. [7], reinforcing the applicability of fuzzy modeling in thermal systems. Meanwhile, Ramya and Deivanayaki [8] numerically modeled the Casson micropolar fluid flow over an inclined porous surface, highlighting flow and heat transfer complexities. In follow-up studies, they examined the effect of radiative heat and mass diffusion on Casson nanofluid flow over stretching surfaces [9], and later incorporated the Soret and Dufour mechanisms under magnetic influence [10]. Additionally, Ramya et al. [11] explored the combined role of Brownian motion, thermophoresis, and microbial interactions in Casson-based ternary hybrid nanofluids over a horizontal plate.

Further advancement in fuzzy nanofluid theory was presented in Ramya et al. [12], where the influence of homogeneous–heterogeneous chemical reactions was analyzed using the Cattaneo–Christov heat flux model. In another significant extension, the authors explored the Carreau nanofluid flow in Darcy–Forchheimer porous media under magnetohydrodynamic conditions [13], emphasizing flow resistance and nonlinear characteristics.

Parallel to fluid studies, graph-theoretic contributions in fuzzy environments have grown substantially. Lakdashti et al. [14] addressed structural aspects of edge irregular product vague graphs, offering mathematical insight into vagueness in complex networks. Chen et al. [15] focused on elementary abelian covers of notable graphs such as the Wreath graph $W(3,2)$ and Foster graph $F26A$, enriching the domain of algebraic and topological graph theory. Talebi et al. [16] introduced interval-valued intuitionistic fuzzy soft graphs by blending soft set theory and intuitionistic fuzzy logic to handle uncertainty in graph modeling. This effort expanded earlier work by Talebi et al. [17], who developed new regularity principles for interval-valued fuzzy graphs.

Rashmanlou and Borzooei [18] contributed foundational properties and practical applications of vague graphs, particularly under imprecise data conditions. Kosari et al. [19] analyzed a specialized domination metric known as the restrained Roman reinforcement number, pertinent to securing networks. In the field of colored graph theory, Kosari et al. [20] investigated the independent k-rainbow bondage number, a crucial parameter for combinatorial optimization.

Further computational and complexity aspects were tackled by Kosari et al. [21], who examined the NP-hardness of the signed total Roman domination problem. Rashmanlou et al. [22] introduced bipolar fuzzy graphs, allowing for simultaneous representation of positive and negative information — a structure valuable in opinion dynamics and social networks. Kosari [23] addressed spectral characteristics such as spectral radius and Zagreb Estrada index, tools with relevance in molecular graph theory.

Expanding on categorical theory, Rashmanlou et al. [24] formulated bipolar fuzzy graphs embedded with categorical properties. Borzooei and Rashmanlou [25] introduced vague domination parameters to control and monitor uncertain networks. Additionally, Rashmanlou and Jun [26] examined complete interval-valued fuzzy graphs, emphasizing network completeness under uncertainty. In a study on regularity, Borzooei et al. [27] extended the classical idea of regular graphs to vague graph environments. Lastly, Rashmanlou and Pal [28] proposed balanced interval-valued fuzzy graphs, which aim to represent equilibrium states in fuzzy systems.

2. Mathematical Analysis in Fuzzy Environment

The two-dimensional flow of magnetohydrodynamic (MHD) Casson fluid over a convective stretching sheet is studied under uncertain conditions modeled via fuzzy sets. Uncertain physical parameters such as the Casson parameter, magnetic field intensity, and diffusion coefficients are treated as fuzzy numbers to capture inherent imprecision. Unlike classical models, the magnetic Reynolds number assumption is relaxed, and cross-diffusion effects are incorporated.

The stretching velocity of the sheet is given by

$$u_m(x) = qx,$$

where q is a fuzzy stretching rate with membership function $\mu_{\tilde{q}}(q)$. A steady fuzzy magnetic field $\tilde{B}_0$ is applied in the y-direction. The flow is governed by fuzzy-valued PDEs involving fuzzy parameters $\tilde{\beta}, \tilde{N}_b, \tilde{N}_t, \tilde{M}$, etc.

The continuity equation remains deterministic:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

The fuzzy momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(1 + \frac{1}{\tilde{\beta}}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\tilde{k}_1}{\rho}\frac{\partial N}{\partial y} + g\left[\tilde{\beta}_T(T - T_\infty) + \tilde{\beta}_C(C - C_\infty)\right]\cos\alpha - \frac{\tilde{\sigma}\tilde{B}_0^2 u}{\rho} - \frac{\nu u}{\tilde{k}} - \frac{\tilde{C}_b}{\sqrt{\tilde{k}}}u^2.$$

The microrotation equation:

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\tilde{\gamma}}{\rho}\frac{\partial^2 N}{\partial y^2} - \frac{\tilde{k}_1}{\rho}\left(2N + \frac{\partial u}{\partial y}\right).$$

The energy equation:

$$\rho C_p\Big[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \tilde{\lambda}_1\mathcal{F}(u, v, T)\Big] = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \tilde{q}''' + \nu\left(1 + \frac{1}{\tilde{\beta}}\right)\rho\left(\frac{\partial u}{\partial y}\right)^2 + \tau\rho C_p\left[\tilde{D}_m\frac{\partial T}{\partial y}\frac{\partial C}{\partial y} + \frac{\tilde{D}_T}{D_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] + \tilde{\sigma}\tilde{B}_0^2 u^2 + \frac{\rho\tilde{D}_m\tilde{k}_T}{C_s}\frac{\partial^2 C}{\partial y^2}.$$

The concentration equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+\tilde{\lambda}_2\mathcal{G}(u,v,C)=\tilde{D}_m\frac{\partial^2 C}{\partial y^2}-\tilde{k}^*(C-C_\infty)$$
$$+\frac{\tilde{D}_m\tilde{k}_T}{T_m}\frac{\partial^2 T}{\partial y^2}+\frac{\tilde{D}_T}{D_\infty}\frac{\partial^2 T}{\partial y^2}.$$

Fuzzy heat generation:

$$\tilde{q}'''=\frac{\tilde{k}u_w}{x\nu}\left[\tilde{C}^*(T_w-T_\infty)f'+\tilde{D}^*(T-T_\infty)\right].$$

Radiative heat flux:

$$q_r=-\frac{16}{3}\frac{\tilde{\sigma}^*}{\tilde{k}^*}T^3\frac{\partial T}{\partial y}.$$

Fuzzy boundary conditions at $y=0$:

$$u=u_w+S\frac{\partial u}{\partial y},\quad -k\frac{\partial T}{\partial y}=h_1(T_\infty-T),$$
$$-N=-m\frac{\partial u}{\partial y},\quad D_m\frac{\partial C}{\partial y}=h_2(C_w-C).$$

As $y\to\infty$:

$$u\to 0,\quad T\to T_\infty,\quad C\to C_\infty.$$

Similarity variables:

$$\phi=\sqrt{\tilde{q}\nu}xf(\eta),\quad u=qxf'(\eta),\quad N=qx\sqrt{\frac{q}{\nu}}h(\eta),\quad v=-\sqrt{\tilde{q}\nu}f(\eta),$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}.$$

Fuzzy ODEs (-cut form):

$$\left[1+\frac{1}{\tilde{\beta}_\alpha}\right]f'''+ff''-(f')^2+\tilde{K}_\alpha h'+(\tilde{G}r_{T,\alpha}\theta+\tilde{G}r_{C,\alpha}\varphi)\cos\alpha$$
$$-(\tilde{M}_\alpha+\tilde{k}_{p,\alpha})f'-\tilde{F}_{r,\alpha}(f'')^2=0,$$

$$\left[1+\frac{\tilde{K}_\alpha}{2}\right]h''+fh'-f'h+\tilde{K}_\alpha(2h+f'')=0,$$

$$\Big(1+\tilde{N}_{r,\alpha}\big[1+3(\theta_m-1)+3(\theta_m-1)^2\theta^2+(\theta_m-1)^3\theta^3\big]\Big)\theta''$$
$$+\tilde{N}_{r,\alpha}\left[(\theta_m-1)(\theta')^2+6(\theta_m-1)^2\theta(\theta')^2+3(\theta_m-1)^3\theta^2(\theta')^2\right]$$
$$+\tilde{P}_{r,\alpha}f\theta'-\tilde{P}_{r,\alpha}f'\theta+\tilde{A}^*_\alpha f'+\tilde{B}^*_\alpha\theta-\alpha_t(f^2\theta''+ff'\theta')+\tilde{P}_{r,\alpha}\tilde{D}_u\varphi''$$
$$+\tilde{P}_{r,\alpha}\tilde{E}_c\left(1+\frac{1}{\tilde{\beta}_\alpha}\right)(f'')^2+\tilde{P}_{r,\alpha}\tilde{E}_c\tilde{M}_\alpha(f')^2+\Big(\tilde{N}_{b,\alpha}\theta'\varphi'+\tilde{N}_{t,\alpha}(\theta')^2\Big)\tilde{P}_{r,\alpha}=0,$$

$$\varphi''+\tilde{S}_{c,\alpha}f\varphi'-\tilde{k}_{r,\alpha}\tilde{S}_{c,\alpha}\varphi-\alpha_C\tilde{S}_{C,\alpha}(f^2\theta''+ff'\theta')+\tilde{S}_{c,\alpha}\tilde{S}_{r,\alpha}\theta''+\frac{\tilde{N}_{t,\alpha}}{\tilde{N}_{b,\alpha}}\theta''=0.$$

Boundary conditions at $\eta = 0$, $\eta \to \infty$:

$$f'(0) = 1 + \gamma f''(0), \quad f(0) = 0,$$
$$\theta'(0) = Bi_{T,\alpha}(1-\theta(0)), \quad \varphi'(0) = Bi_{C,\alpha}(1-\varphi(0)),$$
$$f'(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \varphi(\infty) \to 0.$$

Fuzzy dimensionless numbers:

$$\tilde{Gr}_T = \frac{g\tilde{\beta}_T(T_w - T_\infty)}{\tilde{q}^2 x}, \quad \tilde{Gr}_C = \frac{g\tilde{\beta}_C(C_w - C_\infty)}{\tilde{q}^2 x}, \quad \tilde{N}_r = \frac{16\tilde{\sigma}^* T_\infty^3}{3\tilde{k}\tilde{k}^*}, \quad \dots$$

Figure 1. Velocity numerous values of M

Figure 2. Temperature numerous values of M

Figure 3. Concentration numerous values of M

Figure 4. Angular numerous values of M

Fuzzy surface characteristics:

$$\tilde{C}_{f_x} Re_x^{1/2} = \left(1 + \frac{1}{\tilde{\beta}}\right) f''(0), \quad \tilde{Nu} Re_x^{1/2} = -\left(1 + \tilde{N}_r(\theta_w)^3\right)\theta'(0), \quad \tilde{Sh} Re_x^{-1/2} = -\varphi'(0).$$

Definition 2.1 (Fuzzy Set)**.** A *fuzzy set* $\tilde{A}$ in a universe of discourse X is characterized by a membership function

$$\mu_{\tilde{A}} : X \to [0,1],$$

where $\mu_{\tilde{A}}(x)$ represents the degree of membership of element $x \in X$ in the fuzzy set $\tilde{A}$.

Definition 2.2 (Fuzzy Number)**.** A *fuzzy number* $\tilde{N}$ is a fuzzy set on the real line $\mathbb{R}$ that is

- Normal, i.e., $\exists x_0 \in \mathbb{R}$ such that $\mu_{\tilde{N}}(x_0) = 1$,
- Convex, i.e., $\mu_{\tilde{N}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{N}}(x_1), \mu_{\tilde{N}}(x_2)\}$ for all $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0,1]$,

FIGURE 5. Velocity numerous values of F_r

FIGURE 6. Temperature numerous values of F_r

FIGURE 7. Concentration numerous values of F_r

FIGURE 8. Angular numerous values of F_r

- Upper semi-continuous,
- With compact support.

Definition 2.3 (α-cut)**.** For a fuzzy set $\tilde{A}$ and $\alpha \in (0, 1]$, the α-cut (or α-level set) is defined as

$$\tilde{A}_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\}.$$

The α-cut is a crisp set representing all elements whose membership degree is at least α.

Definition 2.4 (Fuzzy Arithmetic)**.** Given two fuzzy numbers $\tilde{A}$ and $\tilde{B}$, their sum $\tilde{C} = \tilde{A} + \tilde{B}$ is defined by the extension principle as

$$\mu_{\tilde{C}}(z) = \sup_{x+y=z} \min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)\right).$$

Arithmetic operations (addition, subtraction, multiplication, division) between fuzzy numbers can be computed using α-cuts:

$$\tilde{C}_\alpha = \tilde{A}_\alpha + \tilde{B}_\alpha = [a^L_\alpha + b^L_\alpha,\ a^U_\alpha + b^U_\alpha],$$

where $\tilde{A}_\alpha = [a^L_\alpha, a^U_\alpha]$ and $\tilde{B}_\alpha = [b^L_\alpha, b^U_\alpha]$.

Definition 2.5 (Fuzzy Parameter)**.** A *fuzzy parameter* $\tilde{p}$ in a mathematical model is a fuzzy number representing an uncertain or imprecise quantity whose exact value is unknown but bounded by a membership function $\mu_{\tilde{p}}(p)$.

3. Results and Discussion

In this study, uncertain physical parameters such as the magnetic field intensity $\tilde{M}$ and Darcy–Forchheimer parameter $\tilde{F}_r$ are modeled as fuzzy numbers to capture the inherent imprecision and variability that naturally arise in real-world experimental and environmental conditions. The fuzzy approach allows us to quantify the uncertainty in model inputs and observe its effects on the flow and thermal characteristics of the nanofluid.

Effect of Fuzzy Magnetic Field $\tilde{M}$. The magnetic field applied normal to the flow induces a Lorentz force opposing the fluid motion, thus affecting the velocity and angular velocity of the nanofluid. The fuzzy parameter $\tilde{M}$ represents a range of possible magnetic intensities with associated membership functions, allowing us to evaluate how variations within this range influence the flow.

- **Velocity Profile:** As the fuzzy magnetic field parameter increases (moving toward the upper α-cut values), the velocity profile shows a marked increase in fluid velocity near the stretching sheet. This occurs because the stronger magnetic field enhances electromagnetic forces that can accelerate charged nanoparticles suspended in the fluid, boosting the effective flow momentum. The fuzzy modeling provides bounds on velocity increase, indicating the possible variation due to uncertainty in M.
- **Angular Velocity (Microrotation):** Similarly, the angular velocity of the micropolar fluid particles also increases with the fuzzy magnetic field. This reflects enhanced micro-rotation induced by magnetic torque effects, which is consistent with the nanofluid's microstructure interaction under magnetic influence.
- **Temperature and Concentration Profiles:** Conversely, the temperature and concentration distributions show a decreasing trend with increasing $\tilde{M}$. The intensified magnetic field augments the convective heat and mass transfer rates away from the boundary layer, resulting in thinner thermal and concentration boundary layers. Fuzzy analysis captures the variability in these decreases, providing confidence intervals that reflect the uncertainty in thermal response due to magnetic intensity fluctuations.

Effect of Darcy–Forchheimer Parameter $\tilde{F}_r$. The Darcy–Forchheimer parameter represents nonlinear drag forces in porous media. Modeling F_r as a fuzzy parameter reflects uncertainty in the porous structure and flow resistance characteristics.

- **Velocity and Angular Velocity:** Increasing the fuzzy Darcy–Forchheimer parameter results in enhanced velocity and angular velocity profiles. Physically, higher F_r corresponds to stronger inertial drag in the porous medium, which tends to accelerate the fluid near the surface due to pressure gradients established in the porous matrix. The fuzzy interval analysis shows how these effects vary under uncertain porous media properties.

- **Temperature and Concentration:** An increase in fuzzy $\tilde{F}_r$ leads to a reduction in temperature and concentration profiles, similar to the effect of the magnetic field. This is attributed to the increased drag and flow velocity promoting enhanced convective transport, thus thinning the thermal and concentration boundary layers.

Figures 1 to 8 illustrate the influence of varying magnetic field parameter M and Darcy–Forchheimer parameter F_r on the velocity, temperature, concentration, and angular velocity profiles of the nanofluid flow.

- **Figure 1:** This graph depicts how the dimensionless velocity profile increases with higher values of the magnetic field parameter M. The Lorentz force generated by the magnetic field acts as a resistive force, which in this context enhances the fluid velocity near the stretching sheet due to the coupling effects in the flow.
- **Figure 6:** The temperature distribution shows a decreasing trend as M increases. This reduction occurs because the stronger magnetic field intensifies heat transfer away from the fluid, thereby cooling the boundary layer.
- **Figure 7:** The concentration profile similarly declines with rising magnetic field intensity, indicating that nanoparticle dispersion reduces as the magnetic effects strengthen, limiting mass diffusion near the surface.
- **Figure 8:** Angular velocity increases with M, demonstrating that the micropolar rotation of fluid particles is amplified by the magnetic field, which influences the microstructure dynamics within the flow.
- **Figure 5:** Increasing the Darcy–Forchheimer parameter F_r, representing the porous medium resistance and inertial drag, results in an elevated velocity profile. This behavior suggests that flow acceleration near the sheet is promoted despite the porous resistance due to inertial effects.
- **Figure 6:** The temperature profile decreases as F_r increases, indicating that higher porous drag facilitates thermal energy dissipation from the fluid.
- **Figure 7:** Concentration decreases with increasing F_r, showing reduced nanoparticle transport caused by intensified flow resistance within the porous structure.
- **Figure 8:** Angular velocity grows with the Darcy–Forchheimer parameter, highlighting that micro-rotational effects in the fluid are strengthened by enhanced porous medium inertia.

The fuzzy modeling of magnetic field and Darcy–Forchheimer parameter provides a comprehensive framework for understanding the impact of uncertain physical factors on nanofluid flow. The fuzzy parameters introduce intervals of possible values rather than crisp single values, allowing the capture of uncertainty and variability inherent in practical applications. Results confirm that both increased magnetic intensity and Darcy–Forchheimer parameter amplify velocity and angular velocity while reducing temperature and concentration, with fuzzy sets quantifying confidence in these trends.

4. Conclusion

This study investigated the impact of uncertain physical parameters on the magnetohydrodynamic (MHD) flow of a nanofluid over a convective stretching sheet embedded in a Darcy–Forchheimer porous medium using fuzzy set theory. The fuzzy approach effectively captured the inherent uncertainty in parameters such as the magnetic field strength and porous medium resistance. Numerical results demonstrated that increasing the magnetic field intensity and Darcy–Forchheimer parameter enhances velocity and angular velocity profiles, while temperature and concentration profiles decrease correspondingly. These findings provide deeper insights into controlling nanofluid behavior in porous media under uncertain operating conditions, which is essential for optimizing industrial and engineering applications.

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