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Improved time-censored group sampling schemes based on generalized half-normal distribution

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Abstract. The intent of this article is to propose an attribute group acceptance sampling plan under time-censoring when the lifetime of a product follows the generalized half-normal (*GHN*) distribution. The test plans are formulated by using a traditional two-point method as well as by minimizing and limiting a linear combination of conventional producer and consumer risks. The optimal number of groups and the acceptance number are obtained using integer nonlinear programming. The suggested optimal group plans outperform the traditional optimal two-point plan in terms of sample size when the producer and consumer risks are less than a maximum risk tolerated by the analyst. The proposed plans are applied to a real data set for illustrative purposes.

Keywords: Group acceptance sampling plan, Generalized half-normal distribution, balancing producer and consumer risks, integer nonlinear programing, Operating characteristics.

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1. Introduction

Quality control methods for manufactured products of companies include systematic methods to ensure products comply with quality standards. Acceptance sampling constitutes a critical instrument in the domain of industrial quality control, enabling practitioners to ascertain the acceptability of products predicted on the data derived from a randomly selected sample of the lot. In these schemes, the product lots are subjected to inspection protocols to determine their acceptance or rejection based on randomly selected samples extracted from the lots. A multitude of lot inspection frameworks are present in the academic literature, each presenting distinct viewpoints. Notably, one can reference the work of [13], [22], [7] and, more recently, [15], [19] and [11].

Single acceptance sampling plans (*SASPs*) by attributes are the simplest plans in industrial quality, where a lot of products is accepted if no more than c failures occur during the experiment time. For more detailes, see [18]. The advancement of *SASPs* has culminated in the development of group acceptance sampling plans (*GASPs*), wherein testers are able to assess multiple items simultaneously, thereby resulting in significant savings in both time and cost. This type of acceptance sampling plans has been considered by many researchers based on several probability distributions. Papers by [2], [13] and [17] were focused on the Weibull distribution, while [24], [5] and [6] discussed the generalized exponential distribution. Gamma distribution was analyzed in [4]. The design of *GASPs* under other models such as the inverse Rayleigh, inverse log-logistic, generalized transmuted-exponential, gamma Lindley, transmuted Weibull, odd-Perks-Lomax and type-I heavy-tailed Rayleigh distributions was proposed by [3], [27], [10], [1], [26], [16] and [21].

[25] introduced the generalized gamma (*GG*) distribution with two shape parameters and a scale parameter. Many distributions commonly used for parametric models in reliability analysis such as the exponential, Weibull, gamma, log-normal and half-normal distributions are special cases of the *GG* distribution. The hazard function of the *GG* distribution is bathtub-shaped and hump-shaped for some values of shape parameters. The *GHN* distribution is proposed by [9] to describe the lifetime process under fatigue and is the only member in the family of *GG* distribution which has a bathtub-shaped hazard function with rich statistical properties and mathematically tractable two parameter lifetime density, see [28] and [20] for more detailes.

All works presented in the literature on *GASP* based on lifetime data have focused on finding the optimal number of groups and the acceptance number using the traditional single-point method or two-point method (simultaneous control of producer and consumer risk). Our motivation is to design a *GASP* where the optimal parameters of the plan are determined by limiting and minimizing a weighted-average of the conventional producer and consumer risks.

The remainder of the article is structured as follows. The methodology of *GASP* is proposed in Section 2. The *GASP*s with minimal weighted-average risks are presented in Section 3. The *GASP*s are developed to find the best plans with limited weighted-average risks in Section 4. A real data example is provided in Section 5. Finally, concluding remarks are offered in Section 6.

2. Time censored *GSPSs* for lot sentencing

According to [2], the procedure of the *GASP* under time-censoring is as follows:

(1) Select the number of groups g and allocate k units to each group so that the sample size is $n = gk$.
(2) Select the acceptance number c $(< k)$ for a group and the termination time t_0.
(3) Perform the experiment for the g groups altogether and record the number of failures for each group.
(4) Retain the lot if no more than c failures are observed across each of the groups; otherwise, terminate the experiment and reject the lot.

The above procedure is known as time censored *GASPs*, where the experiment time is censored. For more information, see papers by [14] and [8].

The probability distribution of the quality characteristic aids to design an efficient acceptance sampling plan. [9] proposed the *GHN* distribution for the material specimen failure time t with the probability density function (pdf)

$$f(t;\delta,\lambda) = \sqrt{\frac{2}{\pi}}\left(\frac{\delta}{t}\right)\left(\frac{t}{\lambda}\right)^{\delta} e^{-\frac{1}{2}(\frac{t}{\lambda})^{2\delta}},\ t > 0, \tag{2.1}$$

and the associated cumulative density function (cdf)

$$F(t;\delta,\lambda) = 2\Phi\left[\left(\frac{t}{\lambda}\right)^{\delta}\right] - 1,\ t > 0, \tag{2.2}$$

where $\delta > 0$ and $\lambda > 0$ are the shape and scale parameters, respectively, and $\Phi(.)$ denotes the cdf of the standard normal distribution. We denote a random variable with pdf (2.1) by $T \sim GHN(\delta,\lambda)$.

The mean of the random variable T of the GHN distribution can be computed as

$$\mu = \sqrt{\frac{2^{\frac{1}{\delta}}}{\pi}}\, \Gamma\left(\frac{1+\delta}{2\delta}\right) \lambda, \tag{2.3}$$

which is proportional to the scale parameter λ when the parameter δ is fixed. For simplicity, the mean lifetime is considered the quality characteristic of interest. However, note that in our context, it is equivalent to use the mean or median lifetimes, or even a given percentile.

Assume that the lifespan of products follows the $GHN(\delta, \lambda)$ with pdf given in (2.1). Let μ be the true mean lifetime of products and μ_0 be the specified mean life. The quality level of a product can be expressed in terms of the ratio $r = \mu/\mu_0$. Test termination time t_0 can be determined as a multiple of μ_0, that is $t_0 = f\mu_0$, where f is a positive constant.

The operating characteristic (OC) curve depicts the relationship between the probability of accepting a lot and the true proportion of defective items p. The OC function is defined by $\mathcal{L}(p) \equiv \mathcal{L}(p; g, c)$ and is given by

$$\mathcal{L}(p) = \left[\sum_{i=0}^{c} \binom{k}{i} p^i (1-p)^{k-i}\right]^g, \tag{2.4}$$

where $p \equiv p(f, \delta, r)$ denotes the probability that a product in a group fails before the time t_0 and is given by

$$p(r) = 2\Phi\left[\left(\sqrt{\frac{2^{\frac{1}{\delta}}}{\pi}}\, \frac{f}{r} \Gamma\left(\frac{1+\delta}{2\delta}\right)\right)^{\delta}\right] - 1. \tag{2.5}$$

Assume that the producer and consumer characterize the acceptable and rejectable defective rates, defined as $p_0 \equiv p(f, \delta, r_0)$ and $p_1 \equiv p(f, \delta, r_1)$, respectively, where r_0 is the mean ratio at the producer's risk and r_1 is the mean ratio at the consumer's risk. The conventional producer risk (PR) and the consumer risk (CR) are defined respectively as $\sup_{p \le p_0}\{1 - \mathcal{L}(p)\}$ and $\sup_{p \ge p_1}\{\mathcal{L}(p)\}$. Since $\mathcal{L}(p)$ is a decreasing function of p, then the PR and CR are given by $PR(g, c, p_0) = 1 - \mathcal{L}(p_0)$ and $CR(g, c, p_1) = \mathcal{L}(p_1)$, which can be expressed as

$$PR(g, c, p_0) = 1 - \left[\sum_{i=0}^{c} \binom{k}{i} p_0^i (1-p_0)^{k-i}\right]^g, \tag{2.6}$$

and

$$CR(g, c, p_1) = \left[\sum_{i=0}^{c} \binom{k}{i} p_1^i (1-p_1)^{k-i}\right]^g. \tag{2.7}$$

Table 1. Minimum-WR group number, g_c, and the corresponding risks (%) for selected values of c when $\delta = 1, r_0 = 2,\ r_1 = 1,\ k = 5,\ f = 0.5, 1.0$.

		$(w_0, w_1) = (0.2, 0.8)$				$(w_0, w_1) = (0.5, 0.5)$				$(w_0, w_1) = (0.8, 0.2)$			
f	c	g_c	WR	PR	CR	g_c	WR	PR	CR	g_c	WR	PR	CR
0.5	0	2	18.38	82.11	2.44	1	36.67	57.71	15.63	1	49.29	57.71	15.63
	1	5	15.28	62.91	3.37	3	28.96	44.84	13.08	1	24.54	17.99	50.76
	2	20	10.93	46.45	2.05	11	20.43	29.07	11.79	3	18.32	8.94	55.82
	3	121	6.74	28.16	1.38	78	12.77	19.20	6.34	36	13.09	9.37	27.99
	4	1716	3.70	15.59	0.73	1216	7.19	11.32	3.05	715	8.03	6.82	12.85
1.0	0	1	17.98	84.37	1.39	1	42.88	84.37	1.39	1	67.77	84.37	1.39
	1	2	15.77	74.23	1.16	1	30.00	49.24	10.76	1	41.54	49.24	10.76
	2	4	12.17	54.04	1.71	2	22.63	32.21	13.06	1	21.36	17.66	36.14
	3	12	8.12	34.59	1.50	7	15.28	21.93	8.64	3	15.05	10.07	35.00
	4	73	4.48	18.90	0.87	50	8.63	13.37	3.89	28	9.42	7.72	16.22

3. Optimal *GASP* with minimal weighted-average of risks

[23] analyzed simple and composite hypotheses using minimization of a weighted sum of Type I and II errors. [12] and [20] developed this idea to design optimal acceptance test plans by minimizing and limiting a weighted-average of producer and consumer risks. Our aim is to expand this method to design an *GASP* under time censoring for *GHN* distribution.

Consider the weighted-average of PR and CR risks (WR) as

$$WR(g, c, p_0, p_1) = w_0 PR(g, c, p_0) + w_1 CR(g, c, p_1), \tag{3.1}$$

where the positive constants w_0 and w_1 are the producer and consumer weights, respectively and $w_0 + w_1 = 1$. The optimal *GASP* with a fixed acceptance number c can be obtained by minimizing (3.1). Therefore, the optimization problem can be written as

$$\min\{WR(g, c, p_0, p_1) : \ (g, c) \in \Omega\},$$

where $\Omega = \{(g, c) : \ n \in \mathbb{N}, c \in \mathbb{N}_0, c < k\}$ is the feasible region, $\mathbb{N} = \{1, 2, 3, ...\}$ is the set of positive integers and $\mathbb{N}_0 = \mathbb{N} \bigcup \{0\}$.

The number of groups with minimum WR, g_c, and their associated risks (WR, PR and CR) are presented in Table 1 and 2 for $r_0 = 2,\ r_1 = 1,\ k = 5,\ f = 0.5, 1,\ c = 0(1)4$ and $w_0 = 0.2, 0.5, 0.8$ when $\delta = 1$ and $\delta = 2$, respectively. It can be observed that the minimum-WR group number g_c grows when acceptance number c increases, while the WR and PR decrease. For instance, if $f = 0.5$, $w_0 = 0.2$ and $\delta = 1$, we obtain from Table 1 that $g_0 = 2(n_0 = 2 \times 5 = 10)$, $WR = 18.38\%$, $PR = 82.11\%$ and $CR = 2.44\%$, whereas $g_1 = 5(n_1 = 25)$, $WR = 15.28\%$, $PR = 62.19\%$ and $CR = 3.37\%$.

Moreover, when w_0 increases, g_c and the PR decrease, whereas the CR grows. For instance, if $f = 0.5,\ c = 1$ and $\delta = 1$, we obtain $g_c = 5$,

TABLE 2. Minimum-WR group number, g_c, and the corresponding risks (%) for selected values of c when $\delta = 2, r_0 = 2,\ r_1 = 1,\ k = 5,\ f = 0.5, 1.0$.

		$(w_0, w_1) = (0.2, 0.8)$				$(w_0, w_1) = (0.5, 0.5)$				$(w_0, w_1) = (0.8, 0.2)$			
f	c	g_c	WR	PR	CR	g_c	WR	PR	CR	g_c	WR	PR	CR
0.5	0	5	13.69	57.56	2.73	3	25.86	40.20	11.52	1	22.33	15.75	48.65
	1	29	6.46	26.61	1.42	19	12.25	18.35	6.16	9	12.66	9.15	26.71
	2	277	2.25	9.58	0.42	206	4.46	7.22	1.71	135	5.22	4.79	6.94
	3	4734	0.67	2.93	0.11	3773	1.38	2.34	0.42	2812	1.74	1.75	1.70
	4	-	-	-	-	-	-	-		-	-	-	-
1.0	0	1	12.75	51.35	3.10	1	27.22	51.35	3.10	1	41.70	51.35	3.10
	1	3	7.64	35.60	0.65	2	14.45	25.43	3.47	1	14.64	13.64	18.63
	2	7	3.19	12.92	0.76	5	6.24	9.41	3.07	3	7.08	5.76	12.37
	3	31	1.00	4.39	0.15	24	2.04	3.42	0.66	17	2.52	2.43	2.86
	4	250	0.24	1.08	0.03	206	0.51	0.89	0.14	163	0.67	0.71	0.54

Dashs (-) are presented in the required cells for a large number of groups.

$WR = 15.28\%$, $PR = 62.91\%$ and $CR = 3.27\%$ when $w_0 = 0.2$, whereas $g_c = 3$, $WR = 28.96\%$, $PR = 44.84\%$ and $CR = 13.08\%$ when $w_0 = 0.5$.

For better presentation of the results, the WR percentage is plotted in Figure 1 versus the number of groups g when $c = 1,\ \delta = 2, r_0 = 2,\ r_1 = 1,\ k = 5,\ f = 0.5, 1.0$ and $w_0 = 0.5$. It can be seen that the optimal number of groups are $g = 19$ and $g = 2$, when $f = 0.5$ and $f = 1.0$, respectively. Figure 2 shows the OC functions for the best $GASP$ for $c = 1,\ \delta = 2, r_0 = 2,\ r_1 = 1,\ k = 5$ and the producer weight $w_0 = 0.2, 0.5, 0.8$ when $f = 0.5, 1.0$. It is clear that the lot acceptance probability is higher for the $GASP$ $(29, 1)$ and $(3, 1)$ when $f = 0.5$ and $f = 1.0$, respectively. The WR, PR and CR percentages versus w_0 are depicted in Figure 3 for $c = 1,\ \delta = 2, r_0 = 2,\ r_1 = 1,\ k = 5$ when $f = 0.5, 1.0$. It is evident that the PR reduces and the CR increases, when w_0 increases while the WR first increases and then decreases.

4. Optimal *GASP* with limited weighted-average of risks

The conventional two-point method to determine the optimal $GASP$ controls consumer and producer risks concurrently. The consumer requires that the probability of lot acceptance should be smaller than the specified consumer's risk β at a lower quality level (usually at ratio 1), whereas the producer demands that the lot rejection probability should be smaller than the specified producer's risk α at a higher quality level. The producer wants $PR(g, c, p_0) \leq \alpha$, whereas the consumer wants $CR(g, c, p_1) \leq \beta$. Optimal $(\alpha, \beta)-$(PR,CR) plans, (g_t, c_t), can be

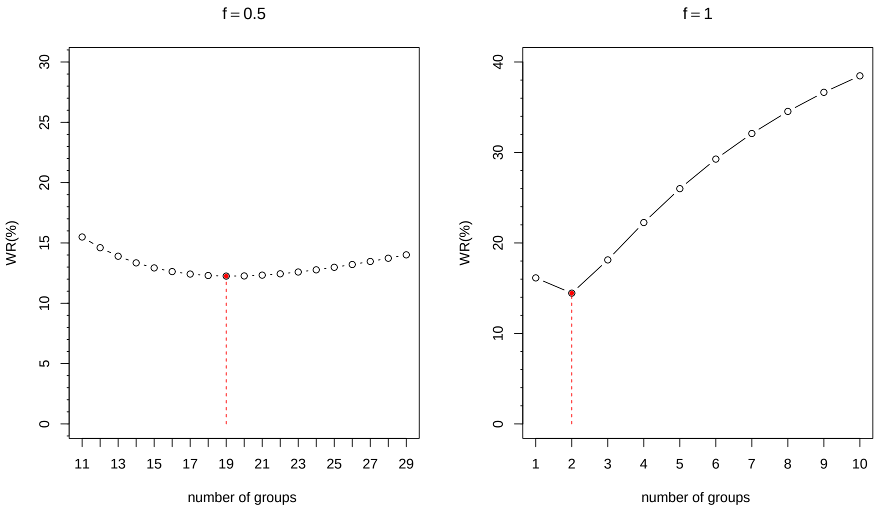

FIGURE 1. WR percentage versus the number of groups, g, when $c = 1,\ \delta = 2, r_0 = 2,\ r_1 = 1,\ k = 5,\ f = 0.5, 1.0$ and $w_0 = 0.5$.

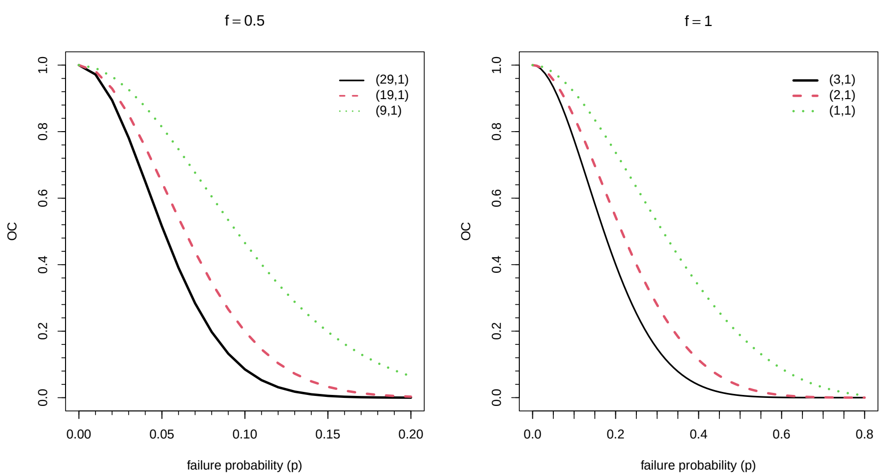

FIGURE 2. OC functions for the best *GASP* when $c = 1,\ \delta = 2, r_0 = 2,\ r_1 = 1,\ k = 5$ and the producer weight w_0 is 0.2, 0.5 and 0.8, which are $(29, 1),\ (19, 1),\ (9, 1)$ for $f = 0.5$ and $(3, 1),\ (2, 1),\ (1, 1)$ for $f = 1.0$.

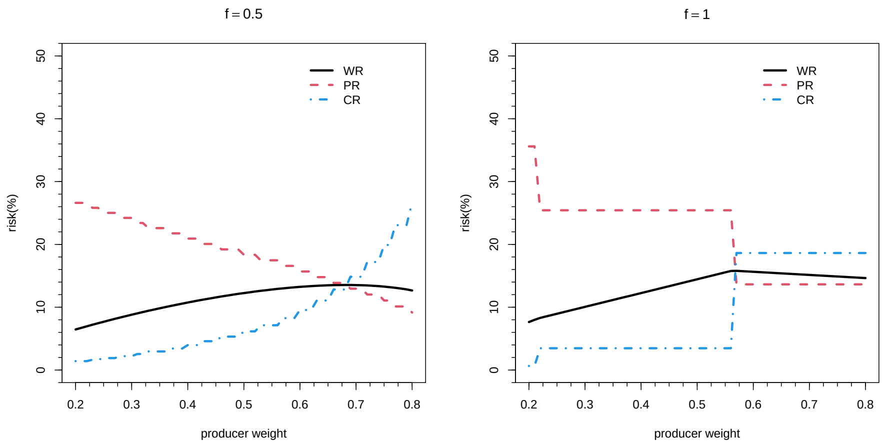

FIGURE 3. WR, PR and CR percentages versus w_0 when $c = 1,\ \delta = 2, r_0 = 2,\ r_1 = 1,\ k = 5,\ f = 0.5, 1.0$

determined by solving the constrained optimization problem

$$\begin{array}{ll} \text{Minimize} & g \\ \text{Subject to} & PR(g, c, p_0) \leq \alpha, \\ & CR(g, c, p_1) \leq \beta, \\ & g \in \mathbb{N},\ c \in \mathbb{N}_0, \\ & c < k. \end{array} \tag{4.1}$$

Suppose that the analyst wants to control the risk incurred by the selected *GASP* by considering $\gamma \in (0, 1)$ as the maximum risk tolerated, where $\gamma \leq \min\{w_0, w_1\}$. Our aim is to determine the optimal number of groups and the acceptance number that satisfy the inequality $WR(g, c, p_0, p_1) \leq \gamma$.

The constrained optimization problem to obtain the optimal group number, $g^\star$, and the optimal acceptance number, $c^\star$, is an integer nonlinear programming problem, which can be stated as follows:

$$\begin{array}{ll} \text{Minimize} & g \\ \text{Subject to} & WR(g, c, p_0, p_1) \leq \gamma, \\ & g \in \mathbb{N},\ c \in \mathbb{N}_0, \\ & c < k. \end{array} \tag{4.2}$$

A step-by-step iterative method to determine the optimal plan parameters, $(g^\star, c^\star)$, can be summarized as follows:

- Step 1. Set the value of parameter δ, the termination ratio f, the group size k, the maximum risk tolerated by the analyst γ, and the producer and consumer weights, w_0 and $w_1 = 1 - w_0$.

- Step 2. Set the initial values of plan parameters, the number of groups $g = 1$ and the acceptance number $c = 0$.
- Step 3. Calculate the PR, CR and WR given in (2.6), (2.7) and (3.1), respectively..
- Step 4. Determine the different feasible plans $S = (g, c)$ verifying the nonlinear inequality constraint $WR(g, c, p_0, p_1) \leq \gamma$.
- Step 5. Find the best plans with minimal g and c from the feasible solutions obtained in the previous step.

Optimal $\gamma-$WR plan, $(g^\star, c^\star)$, and the associated risks(%) are summarized in Tables 3 and 4 for different values of $\gamma = 0.02, 0.05,\ r_0 = 2(2)10,\ r_1 = 1,\ k = 5,\ f = 0.5, 1.0$ and $w_0 = 0.2, 0.5, 0.8$ when $\delta = 1$ and $\delta = 2$, respectively. In view of Tables 3 and 4, it is clear that the optimal group numbers tend to decrease as γ increases. For instance, if $r_0 = 4,\ r_1 = 1,\ k = 5,\ f = 0.5,\ \delta = 2$ and $w_0 = 0.2$, the optimal 0.02$-$WR and 0.05$-$WR plans are $(27, 1)$ and $(20, 1)$, respectively. Moreover, optimal group numbers decrease when r_0 increases. For example, if $w_0 = 0.2,\ r_1 = 1,\ f = 0.5,\ \delta = 2$, then the optimal 0.05$-$WR group numbers are 20 and 5 when $r_0 = 4$ and $r_0 = 6$, respectively.

For a graphical comparison of traditional optimal two-point $\gamma-$(PR,CR) and $\gamma-$WR plans, Figure 4 shows the optimal 0.05$-$(PR,CR) and 0.05$-$WR group number versus w_0 for $r_1 = 1,\ f = 0.5$ and $r_0 = 4$ when $\delta = 1$ and $\delta = 2$. It can be seen that the optimal 0.05$-$(PR,CR) group numbers are 85 and 21 when $\delta = 1$ and $\delta = 2$, respectively, whereas the values of optimal 0.05$-$WR group numbers are less than 85 and 21. This implies that the optimal $\gamma - WR$ plan outperforms the traditional optimal $\gamma - (PR, CR)$ plan in terms of sample size $n = gk$, when the PR and CR are at most γ.

5. Real Data Application

A practical example is presented in this section to illustrate the proposed *GASP*. The data set given in Table 5 reported in [9] represents the stress-rupture life of kevlar 49/epoxy strands, subjected to the constant sustained pressure at the 70% stress level until all had failed.

We fit *GHN* distribution as well as Gamma, Log-normal, Weibull and Birnbaum-Saunders distributions to the data. The criteria of fitness are the Akaike information criterion (AIC), Bayesian information criterion (BIC), the log-likelihood (LL) function, Kolmogorov-Smirnov (K-S) statistic and p-value. Table 6 displays the model fitting summary of the chosen data set. From Table 6, it is clear that the *GHN* distribution has a good fit to considered data set as compared to Gamma, Log-normal, Weibull and Birnbaum-Saunders distributions.

TABLE 3. Optimal $\gamma-$WR plan, $(g^\star, c^\star)$, and the associated risks(%) for selected values of γ, r_0, w_0, f, $r_1 = 1$, $k = 5$ when $\delta = 1$.

			$(w_0,w_1)=(0.2,0.8)$					$(w_0,w_1)=(0.5,0.5)$					$(w_0,w_1)=(0.8,0.2)$				
f	γ	r_0	$g^\star$	$c^\star$	WR	PR	CR	$g^\star$	$c^\star$	WR	PR	CR	$g^\star$	$c^\star$	WR	PR	CR
0.5	0.02	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	111	3	1.99	2.05	1.97	112	3	1.99	2.07	1.90	842	4	2.00	0.27	8.92
		6	21	2	1.92	2.85	1.69	94	3	1.98	0.35	3.60	69	3	1.95	0.26	8.71
		8	20	2	1.88	1.18	2.05	19	2	1.80	1.12	2.49	14	2	1.98	0.83	6.58
		10	7	1	1.99	6.46	0.87	18	2	1.79	0.55	3.02	13	2	1.92	0.40	7.99
	0.05	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	16	2	4.94	6.87	4.46	69	3	5.00	1.28	8.71	44	3	4.87	0.82	21.09
		6	6	1	4.21	14.22	1.71	13	2	4.88	1.77	7.99	9	2	4.46	1.23	17.39
		8	5	1	4.11	7.09	3.37	13	2	4.38	0.77	7.99	8	2	4.60	0.47	21.12
		10	5	1	3.63	4.66	3.37	5	1	4.01	4.66	3.37	3	1	4.87	2.82	13.08
1.0	0.02	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	12	3	1.85	3.23	1.50	52	4	1.96	0.51	3.41	38	4	1.99	0.37	8.47
		6	5	2	1.48	4.92	0.62	10	3	1.80	0.57	3.02	8	3	1.58	0.46	6.09
		8	4	2	1.72	1.76	1.71	4	2	1.73	1.76	1.71	4	2	1.75	1.76	1.71
		10	4	2	1.55	0.93	1.71	4	2	1.32	0.93	1.71	3	2	1.50	0.70	4.72
	0.05	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	4	2	3.71	11.74	1.71	8	3	4.12	2.16	6.09	5	3	4.56	1.36	17.39
		6	2	1	4.37	17.20	1.16	3	2	3.85	2.98	4.72	2	2	4.21	2.00	13.06
		8	2	1	3.02	10.45	1.16	3	2	3.02	1.32	4.72	2	2	3.32	0.89	13.06
		10	2	1	2.32	6.98	1.16	2	1	4.07	6.98	1.16	1	1	5.00	3.55	10.76

Dashs (-) are presented in the required cells for a large number of groups.

TABLE 4. Optimal $\gamma-$WR plan, $(g^\star, c^\star)$, and the associated risks(%) for selected values of γ, r_0, w_0, f, $r_1 = 1$, $k = 5$ when $\delta = 2$.

			$(w_0,w_1)=(0.2,0.8)$					$(w_0,w_1)=(0.5,0.5)$					$(w_0,w_1)=(0.8,0.2)$				
f	γ	r_0	$g^\star$	$c^\star$	WR	PR	CR	$g^\star$	$c^\star$	WR	PR	CR	$g^\star$	$c^\star$	WR	PR	CR
0.5	0.02	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	27	1	1.90	1.87	1.90	27	1	1.89	1.87	1.90	23	1	1.96	1.59	3.42
		6	26	1	1.84	0.36	2.21	23	1	1.87	0.32	3.42	17	1	1.84	0.24	8.26
		8	7	0	1.94	7.12	0.65	23	1	1.76	0.10	3.42	16	1	1.97	0.07	9.56
		10	6	0	1.85	3.97	1.33	23	1	1.73	0.04	3.42	16	1	1.94	0.03	9.56
	0.05	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	20	1	4.53	1.39	5.32	17	1	4.72	1.18	8.26	11	1	4.60	0.77	19.91
		6	5	0	3.97	8.95	2.73	16	1	4.89	0.22	9.56	10	1	4.72	0.14	23.06
		8	5	0	3.21	5.14	2.73	4	0	4.87	4.13	5.60	3	0	4.79	3.11	11.52
		10	5	0	2.84	3.32	2.73	4	0	4.13	2.66	5.60	3	0	3.91	2.00	11.52
1.0	0.02	2	20	3	1.79	2.86	1.53	105	4	1.95	0.46	3.45	77	4	1.96	0.33	8.47
		4	3	1	1.15	3.15	0.65	3	1	1.90	3.15	0.65	4	2	1.35	0.15	6.16
		6	3	1	0.65	0.65	0.65	2	1	1.95	0.44	3.47	2	1	1.04	0.44	3.47
		8	2	0	1.70	8.11	0.10	2	1	1.81	0.14	3.47	2	1	0.81	0.14	3.47
		10	2	0	1.13	5.26	0.10	2	1	1.76	0.06	3.47	2	1	0.74	0.06	3.47
	0.05	2	5	2	4.34	9.41	3.07	12	3	4.93	1.72	8.13	8	3	4.68	1.15	18.77
		4	2	1	3.20	2.11	3.47	2	1	2.79	2.11	3.47	1	1	4.58	1.06	18.63
		6	1	0	3.93	7.27	3.10	2	1	1.95	0.44	3.47	1	1	3.90	0.22	18.63
		8	1	0	3.31	4.14	3.10	1	0	3.62	4.14	3.10	1	0	3.93	4.14	3.10
		10	1	0	3.01	2.67	3.10	1	0	2.88	2.67	3.10	1	0	2.75	2.67	3.10

Dashs (-) are presented in the required cells for a large number of groups.

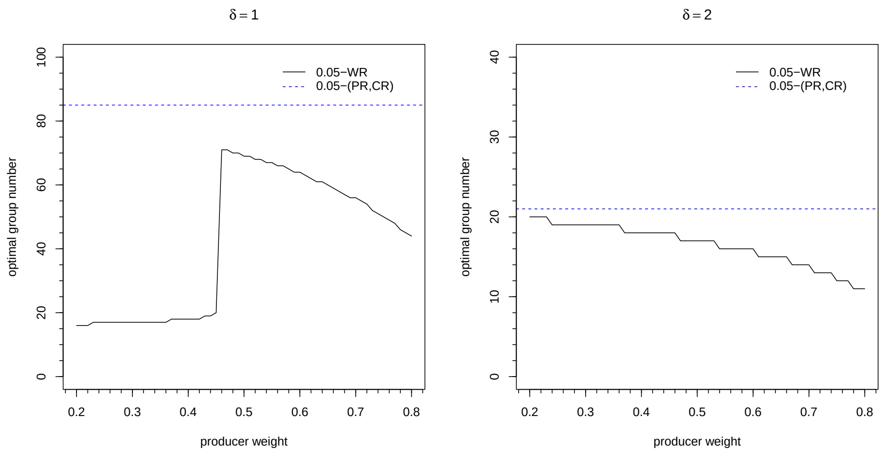

FIGURE 4. Optimal $\gamma-$WR and $\gamma-$(PR,CR) group numbers versus w_0 when $r_0 = 4,\ r_1 = 1,\ k = 5,\ f = 0.5$ when $\delta = 1$ and $\delta = 2$.

TABLE 5. Failure times (in hours) of strands.

1051	1137	1389	1921	1942	2322	3629	4006	4012	4063
4921	5445	5620	5917	5905	5956	6068	6121	6473	7501
7886	8108	8546	8666	8831	9106	9711	9806	10205	10396
10861	11026	11214	11362	11604	11608	11745	11762	11895	12044
13520	13670	14110	14496	15395	16179	17092	17568	17568	

The maximum likelihood estimates of the parameters for this data set are $\hat{\delta} = 1.6407$ and $\hat{\lambda} = 10906.98$. Using (2.3), the mean life can be estimated as $\mu_0 = 8809$. If we consider the termination ratio as $f = 1.0$, then, the termination time is $t_0 = 8809$h. The optimal 0.05-WR plans and the corresponding risks are reported in Table 7 for $k = 5,\ r_0 = 2(2)10$ and $w_0 = 0.2, 0.5, 0.8$. If we consider $r_0 = 4$ and $w_0 = 0.5$, then optimal 0.05-WR plans are $(g^\star, c^\star) = (2, 1)$ with corresponding risks $WR = 4.28\%$, $PR = 5.84\%$ and $CR = 2.72\%$. According to these specifications, a total of $n = 10$ products are needed and five items will be allocated to each of the two groups. We will accept the lot if one failure occurs before $t_0 = 8809$h in each of the two groups.

6. CONCLUDING REMARKS

There are several works in the research literature that focus on *GASP* by determining the optimal group number and acceptance number using the traditional single-point method or two-point method. This study

Table 6. Distribution fit test results of the considered data set.

Model	LL	AIC	BIC	K-S statistic	p-value
GHN	−479.66	963.32	967.11	0.06	0.97
Gamma	−483.14	970.28	974.06	0.11	0.60
Log-normal	−487.87	979.74	983.52	0.14	0.17
Weibull	−480.85	965.70	969.48	0.09	0.87
Birnbaum-Saunders	−488.43	980.86	984.64	0.17	0.07

Table 7. Optimal 0.05−WR plans, $(g^\star, c^\star)$, and the associated risks(%) when $k = 5$ and $\delta = 1.6407$.

		$(w_0, w_1) = (0.2, 0.8)$					$(w_0, w_1) = (0.5, 0.5)$					$(w_0, w_1) = (0.8, 0.2)$				
f	r_0	$g^\star$	$c^\star$	WR	PR	CR	$g^\star$	$c^\star$	WR	PR	CR	$g^\star$	$c^\star$	WR	PR	CR
0.5	2	81	2	4.96	13.32	2.88	614	3	5.00	3.20	6.79	414	3	5.00	2.17	16.31
	4	12	1	4.85	3.90	5.09	12	1	4.49	3.90	5.09	8	1	4.84	2.62	13.73
	6	5	0	4.84	21.29	0.73	10	1	4.62	0.89	8.36	6	1	4.94	0.53	22.56
	8	4	0	3.81	11.24	1.95	10	1	4.35	0.35	8.36	6	1	4.68	0.21	22.56
	10	4	0	3.15	7.93	1.95	4	0	4.94	7.93	1.95	6	1	4.59	0.10	22.56
1.0	2	13	3	4.73	5.54	4.53	14	3	4.76	5.95	3.57	40	4	4.90	0.73	21.61
	4	2	1	3.34	5.84	2.72	2	1	4.28	5.84	2.72	2	2	4.60	0.35	21.61
	6	1	0	4.86	14.00	2.58	2	1	2.19	1.66	2.72	1	1	3.96	0.83	16.49
	8	1	0	3.85	8.93	2.58	2	1	1.69	0.66	2.72	1	1	3.56	0.33	16.49
	10	1	0	3.32	6.26	2.58	1	0	4.42	6.26	2.58	1	1	3.43	0.16	16.49

presents a method for developing the optimal *GASP* for *GHN* distribution using minimization and limiting a weighted-average of risks.

GASPs with minimal WR, (g_c, c) are constructed. Afterwards, the γ-WR plans, $(g^\star, c^\star)$, are determined by solving constrained optimization problems using integer nonlinear programming. It is observed that the optimal γ-WR plans outperform the traditional optimal γ-(PR,CR) plans in terms of sample size. A real data analysis is provided to illustrate the results. The results derived for *GHN* distribution are also valid for any other lifetime distributions.

The methodology outlined in this article may also be utilized in cases where there exists prior knowledge regarding the nonconforming proportion. In this situation, it may be possible to utilize a pre-existing consensus model to integrate expert viewpoints into the development of the most efficient *GASP*, leading to further reductions in group number and acceptance number. The optimal group number and acceptance number can be determined by defining a linear combination of expected produser and consumer risks.

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