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Perfect 2-colorings of the line graphs of the connected bicubic graphs of order at most 12

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ABSTRACT. Let $G = (V_G, E_G)$ be a graph and let I be a finite set of size $m \geq 1$. A mapping $T : V_G \to I$ is called a perfect m-coloring with a parameter matrix $A = (a_{ij})_{i,j \in I}$ of G if it is surjective and for all i, j, every vertex of color i has a_{ij} neighbors of color j. In this paper, we classify all the realizable parameter matrices of perfect 2-colorings of the line graphs of the connected bicubic graphs of order at most 12.

Keywords: Eigenvalue, Perfect 2-coloring, Line graph, Bicubic graph.

2020 Mathematics subject classification: 05C15; Secondary 05C76.

1. INTRODUCTION

The notion of a perfect m-coloring (also known as an equitable partition into m parts) of a graph arises naturally in graph theory, algebraic combinatorics and coding theory (completely regular codes). Studies of perfect colorings start usually with the case of two colors. This case is the simplest and the most interesting one. It possesses high potential for

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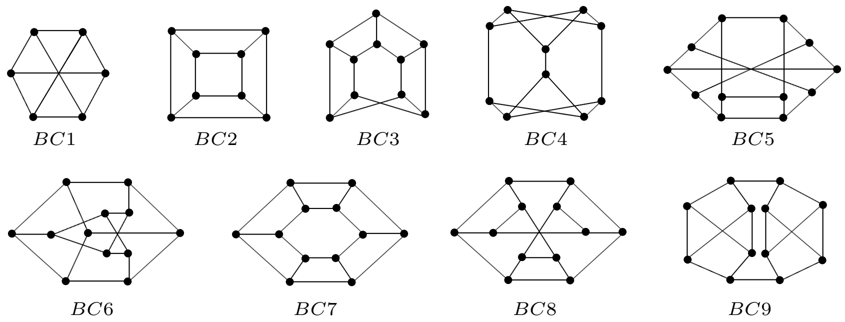
generalization. In recent years, perfect 2-colorings of graphs have been extensively studied. For instance, perfect 2-colorings of the hypercubes, the Johnson graphs including $J(n,2)$, $J(6,3)$, $J(7,3)$, $J(8,3)$, $J(8,4)$ and $J(v,3)$ (v odd), the transitive cubic graphs of order $n \leq 18$, the cubic graphs of order $n \leq 10$, the generalized Petersen graphs, the Platonic graphs, the quartic graphs of order $n \leq 8$ and the bicubic graphs of order $n \leq 12$ have been investigated (see [1, 2, 4, 5, 7, 6, 10, 14, 11, 13, 3]).

For a graph G, a derived graph of G is a graph obtained from G according to some specific rules. One of the most well-known derived graphs is line graph. The line graph of a regular graph obtains many desirable properties from the original graph, such as order, degree, spectrum and so forth.

A bicubic graph is a bipartite cubic graph. The spectrum of a bicubic graphs has a nice properties: 3 is an eigenvalue, the absolute value of every eigenvalue is at most 3 and the eigenvalues are symmetric with respect to the origin.

There are only 9 connected bicubic graphs of order at most 12 [3] (see Figure 1).



$BC1$ $BC2$ $BC3$ $BC4$ $BC5$

$BC6$ $BC7$ $BC8$ $BC9$

Figure 1. The connected bicubic graphs of order at most 12

In this paper, we study perfect 2-colorings of the line graphs of the connected bicubic graphs of order at most 12. In particular, we classify all the realizable parameter matrices of perfect 2-colorings of these graphs.

2. Preliminaries

In the following, we first briefly review some definitions and terminologies related to graphs. For concepts not defined here, we refer to [16]. Throughout this paper, all graphs are finite, simple and undirected. For a graph G, we denote by V_G and E_G the vertex set and the edge set of G, respectively.

The line graph $L(G)$ of a graph G is constructed by taking the edges of G as vertices of $L(G)$ and joining two vertices in $L(G)$ whenever the corresponding edges in G have a common vertex.

For a graph G, we refer to the eigenvalues of its adjacency matrix as the eigenvalues of G. Let G be a connected k-regular graph of order n, size m and with the eigenvalues $\lambda_1 = k^1$, $\lambda_2^{m(\lambda_2)}$, … and $\lambda_s^{m(\lambda_s)}$, where a superscript denotes the multiplicity of the respective eigenvalue. Then, the eigenvalues of $L(G)$ are $(2k-2)^1$, $(k-2+\lambda_2)^{m(\lambda_2)}$, …, $(k-2+\lambda_s)^{m(\lambda_s)}$ and $(-2)^{m-n}$ [8].

Let I be a finite set of size $m \geq 1$. A mapping $T : V_G \to I$ is called a perfect m-coloring with a parameter matrix $A = (a_{ij})_{i,j \in I}$ of a graph G if it is surjective and for all i, j, every vertex of color i has a_{ij} neighbors of color j. If $T : V_G \to \{c_1, \ldots, c_m\}$ is a perfect m-coloring with parameter matrix $A = (a_{ij})_{i,j \in \{1,\ldots,m\}}$ then we assume that the rows and columns of A correspond to the colors in the listed order.

Let T be a perfect m-coloring with the parameter matrix A of a graph G. A number λ is called an eigenvalue of T if it is an eigenvalue of A.

Theorem 2.1 ([12]). *Let T be a perfect m-coloring of a graph G. Then any eigenvalue of T is an eigenvalue of G.*

Let $T : V_G \to \{0, 1\}$ be a perfect 2-coloring of a graph G with the parameter matrix $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$. In this case, the colors 0 and 1 symbolize white and black colors, respectively. Denote by W the set of white vertices of G. Note that [5]

$$|W| = |V_G| \frac{a_{21}}{a_{12} + a_{21}}. \tag{2.1}$$

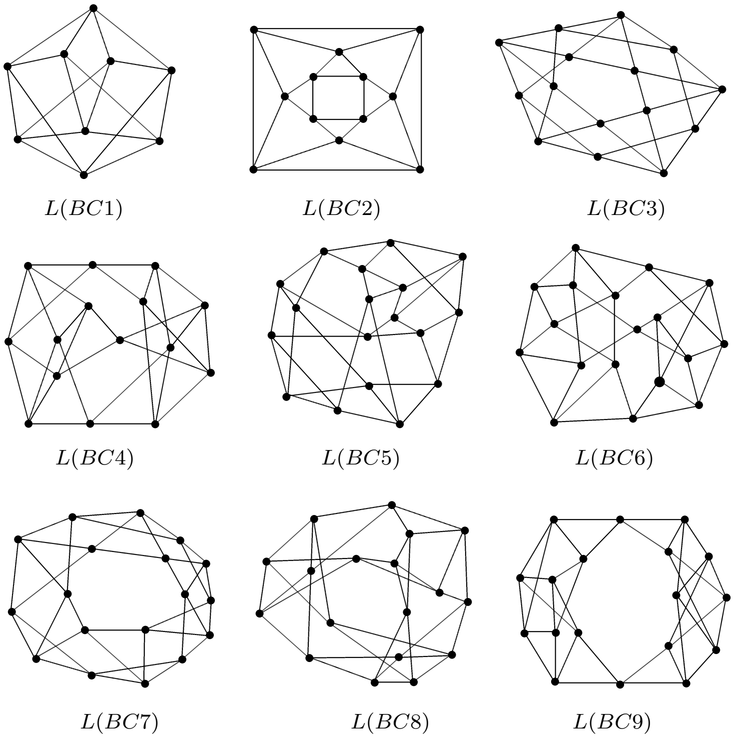
Moreover, if G is a connected k-regular graph then $a_{12} \neq 0$, $a_{21} \neq 0$ and $a_{11} + a_{12} = a_{21} + a_{22} = k$. It is easy to see that k and $a_{11} - a_{21}$ are two eigenvalues of T.

In this paper, we consider all perfect 2-colorings up to renaming the colors; i.e., we identify perfect 2-coloring with the parameter matrix $\begin{pmatrix} a_{22} & a_{21} \\ a_{12} & a_{11} \end{pmatrix}$ obtained by switching the colors with the original coloring. We call the matrix $\begin{pmatrix} a_{22} & a_{21} \\ a_{12} & a_{11} \end{pmatrix}$ is equivalent to the matrix $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$.

Remark 2.2. The eigenvalues of a 2×2 matrix A are $\frac{tr(A) \pm \sqrt{tr(A)^2 - 4\det(A)}}{2}$, where $tr(A)$ is the trace of A.

3. MAIN RESULTS

In this section, we enumerate all parameter matrices of perfect 2-colorings of the line graphs of the connected bicubic graphs of order at most 12. The line graphs of these graphs are shown in Figure 2 [15].



$L(BC1)$ $L(BC2)$ $L(BC3)$

$L(BC4)$ $L(BC5)$ $L(BC6)$

$L(BC7)$ $L(BC8)$ $L(BC9)$

FIGURE 2. The line graphs of the connected bicubic graphs of order at most 12

Let G be a connected 4-regular graph and T be a perfect 2-coloring of G with the parameter matrix $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$. Since G is a connected graph, according to the conditions

$$\begin{aligned} & a_{11} + a_{12} = a_{21} + a_{22} = 4, \\ & a_{ij} \in \{0, 1, 2, 3, 4\}, \qquad \text{for all } i, j = 1, 2, \end{aligned}$$

we obtain the following matrices:

$$\begin{pmatrix} 0 & 4 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 4 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 3 \\ 4 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 4 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 1 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 3 & 1 \\ 4 & 0 \end{pmatrix}.$$

Some of the above matrices are equivalent. So, up to equivalence, a parameter matrix of a perfect 2-coloring of a connected quartic graph may be one of the following matrices:

$$A_1 = \begin{pmatrix} 0 & 4 \\ 1 & 3 \end{pmatrix},\ A_2 = \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix},\ A_3 = \begin{pmatrix} 0 & 4 \\ 3 & 1 \end{pmatrix},\ A_4 = \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix},$$

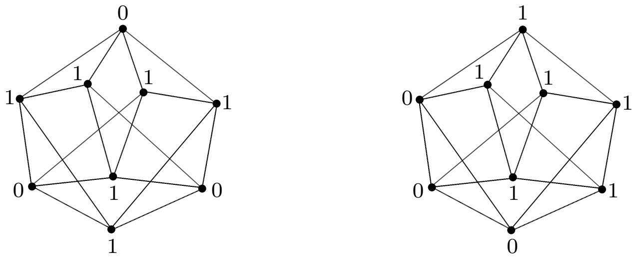
$$A_5 = \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix},\ A_6 = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix},\ A_7 = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix},\ A_8 = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix},$$

$$A_9 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix},\ A_{10} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}.$$

Theorem 3.1. *Let G be a bicubic graph of order 6 or 8. Then, all parameter matrices A for perfect 2-colorings of the line graph of G are only the ones listed in the following table:*

G	A
(i) $BC1$	$A_2,\ A_8$
(ii) $BC2$	$A_2,\ A_9$

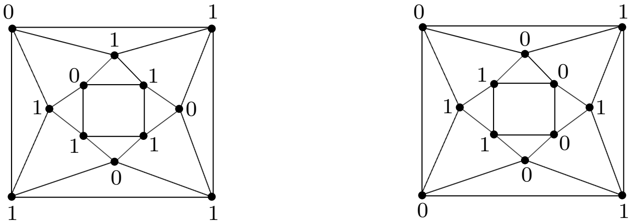
Proof. (i) The eigenvalues of $BC1$ are 3^1, 0^4 and -3^1 [15]. Thus, the eigenvalues of $L(BC1)$ are 4^1, 1^4 and -2^4. By Theorem 2.1 and (2.1), a parameter matrix of a perfect 2-coloring of $L(BC1)$ may be one of the matrices A_2 or A_8. Perfect 2-colorings of $L(BC1)$ with the parameter matrices A_2 and A_8 are shown in Figure 3.



(a) with the matrix $A_2 = \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix}$ (b) with the matrix $A_8 = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$

FIGURE 3. Perfect 2-colorings of $L(BC1)$

(ii) The eigenvalues of $BC2$ are 3^1, 1^3, -1^3 and -3^1 [9]. Thus, the eigenvalues of $L(BC2)$ are 4^1, 2^3, 0^3 and -2^5. By Theorem 2.1 and (2.1), a parameter matrix of a perfect 2-coloring of $L(BC2)$ may be one of the matrices A_2, A_5, A_7, A_9 or A_{10}. Perfect 2-colorings of $L(BC2)$ with the parameter matrices A_2 and A_9 are shown in Figure 4.



(a) with the matrix $A_2 = \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix}$ (b) with the matrix $A_9 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$

FIGURE 4. Perfect 2-colorings of $L(BC2)$

It suffices to show that there is no perfect 2-coloring of $L(BC2)$ with the parameter matrix A_5, A_7 and A_{10}. Let $T : V_{L(BC2)} \to \{0, 1\}$ be a perfect 2-coloring of $L(BC2)$ and H be the subgraph of $L(BC2)$ induced by the white vertices. If A_5 is the parameter matrix of T then by (2.1), $|W| = 12\,\dfrac{1}{3+1} = 3$ and so, H is a 1-regular graph of order 3, a contradiction. If A_7 is the parameter matrix of T then H is a 1-regular graph of order 6, i.e., it is isomorphic to the disjoint union $3K_2$, where K_2 is the complete graph of order 2. Also, if A_{10} is the parameter matrix of T then H is a cubic graph of order 6, i.e., it is isomorphic to the complete bipartite graph $K_{3,3}$ or the triangular prism $C_3\Box K_2$. But $L(BC2)$ has no induced subgraph isomorphic to $3K_2$, $K_{3,3}$ or $C_3\Box K_2$ [15], a contradiction. It completes the proof. $\Box$

Theorem 3.2. *Let G be a bicubic graph of order 10. Then, all parameter matrices A for perfect 2-colorings of the line graph of G are only the ones listed in the following table:*

G	A
(i) $BC3$	A_2
(ii) $BC4$	A_1, A_2

Proof. (i) The eigenvalues of $BC3$ are 3^1, 1.62^2, 0.62^2, -0.62^2, -1.62^2 and -3^1 [15]. Thus, the eigenvalues of $L(BC3)$ are 4^1, 2.62^2, 1.62^2, 0.38^2, -0.62^2 and -2^6. By Theorem 2.1 and (2.1), a parameter matrix of a perfect 2-coloring of $L(BC3)$ may be the matrix A_2. Perfect 2-coloring of $L(BC3)$ with the parameter matrix A_2 is shown in Figure 5.

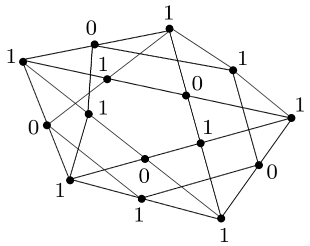
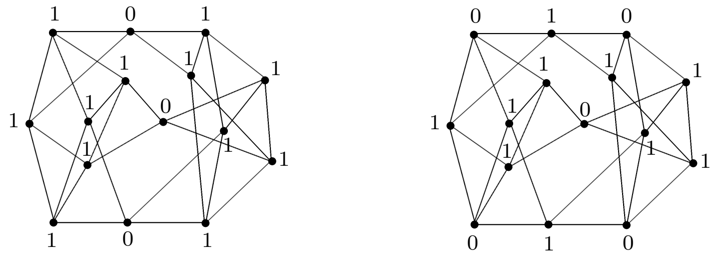


FIGURE 5. Perfect 2-coloring of $L(BC3)$ with the matrix $A_2 = \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix}$

(ii) The eigenvalues of $BC4$ are 3^1, 2^1, 1^2, 0^2, -1^2, -2^1 and -3^1 [15]. Thus, the eigenvalues of $L(BC4)$ are 4^1, 3^1, 2^2, 1^2, 0^2, -1^1 and -2^6. By Theorem 2.1 and (2.1), a parameter matrix of a perfect 2-coloring of $L(BC4)$ may be one of the matrices A_1, A_2, A_6 or A_8. Perfect 2-colorings of $L(BC4)$ with the parameter matrices A_1 and A_2 are shown in Figure 6.



(a) with the matrix $A_1 = \begin{pmatrix} 0 & 4 \\ 1 & 3 \end{pmatrix}$ (b) with the matrix $A_2 = \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix}$

FIGURE 6. Perfect 2-colorings of $L(BC4)$

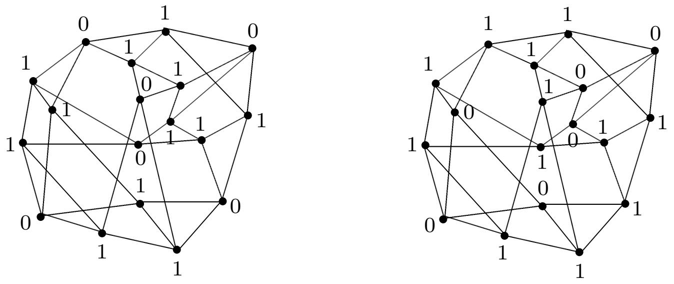
It suffices to show that there is no perfect 2-coloring of $L(BC4)$ with the parameter matrix A_6 and A_8. Let $T : V_{L(BC4)} \to \{0,1\}$ be a perfect 2-coloring of $L(BC4)$. First, suppose that A_6 is the parameter matrix of T. Then, by (2.1), the number of the black vertices is equal to $15 - 15\,\dfrac{2}{3+2} = 15 - 6 = 9$. So, the subgraph of $L(BC4)$ induced by the black vertices is a 2-regular graph of order 9, i.e., it is isomorphic to the cycle C_9 or one of the disjoint unions $3C_3$, $C_4 + C_5$ and $C_3 + C_6$. But $L(BC4)$ has no induced subgraph isomorphic to these graphs [15], a contradiction. Now, suppose that A_8 is the parameter matrix of T. Then, the subgraph of $L(BC4)$ induced by the white vertices is a 2-regular graph of order 5, i.e., it is isomorphic to the cycle C_5. But $L(BC4)$ has no induced subgraph isomorphic to C_5 [15], a contradiction. It completes the proof. □

Theorem 3.3. *Let G be a bicubic graph of order 12. Then, all parameter matrices A for perfect 2-colorings of the line graph of G are only the ones listed in the following table:*

G	A
(i) $BC5$	A_2, A_8
(ii) $BC6$	A_2, A_8
(iii) $BC7$	A_2, A_8
(iv) $BC8$	A_2
(v) $BC9$	A_2, A_8

Proof. (i) The eigenvalues of $BC5$ are 3^1, 2^1, 1.41^2, 1^1, 0^2, -1^1, -1.41^2, -2^1 and -3 [15]. Thus, the eigenvalues of $L(BC5)$ are 4^1, 3^1, 2.41^2, 2^1,

1^2, 0^1, -0.41^2, -1^1 and -2^7. By Theorem 2.1 and (2.1), a parameter matrix of a perfect 2-coloring of $L(BC5)$ may be one of the matrices A_2, A_7, A_8, A_9 or A_{10}. Perfect 2-colorings of $L(BC5)$ with the parameter matrices A_2 and A_8 are shown in Figure 7.



(a) with the matrix $A_2 = \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix}$ (b) with the matrix $A_8 = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$

FIGURE 7. Perfect 2-colorings of $L(BC5)$

Let $T : V_{L(BC5)} \to \{0, 1\}$ be a perfect 2-coloring of $L(BC5)$ and H be the subgraph of $L(BC5)$ induced by the white vertices. If A_7 is the parameter matrix of T then H is a 1-regular graph of order 9, a contradiction. Similarly, if A_{10} is the parameter matrix of T then H is a cubic graph of order 9, a contradiction. Now, suppose that A_9 is the parameter matrix of T. Then H is a 2-regular graph of order 9, i.e., it is isomorphic to the cycle C_9 or one of the disjoint unions $3C_3$, $C_4 + C_5$ and $C_3 + C_6$. But $L(BC5)$ has no induced subgraph isomorphic to C_9, $3C_3$ or $C_4 + C_5$ [15]. Let H be isomorphic to $C_3 + C_6$ (for example, see the subgraph induced by the vertices $v_1, v_2, v_3, v_5, v_6, v_{10}, v_{13}, v_{15}, v_{17}$; Figure 8). Color all the vertices in H with white. Then, there is a black vertex that does not have two white neighbors, a contradiction.

(ii) The eigenvalues of $BC6$ are 3^1, 2.24^1, 1.41^2, 0^4, -1.41^2, -2.24^1 and -3^1 [15]. Thus, the eigenvalues of $L(BC6)$ are 4^1, 3.24^1, 2.41^2, 1^4, -0.41^2, -1.24^1 and -2^7. By Theorem 2.1 and (2.1), a parameter matrix of a perfect 2-coloring of $L(BC6)$ may be one of the matrices A_2, A_7 or A_8. Perfect 2-colorings of $L(BC6)$ with the parameter matrices A_2 and A_8 are shown in Figure 9.

It suffices to show that there is no perfect 2-coloring of $L(BC6)$ with the parameter matrix A_7. Let $T : V_{L(BC6)} \to \{0, 1\}$ be a perfect 2-coloring of $L(BC6)$ with the parameter matrix A_7. Then, by (2.1), the subgraph of $L(BC6)$ induced by the white vertices is a 1-regular graph of order 9, a contradiction.

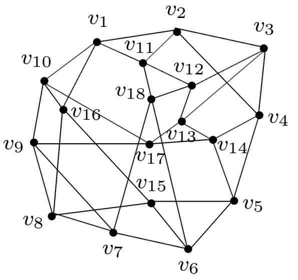
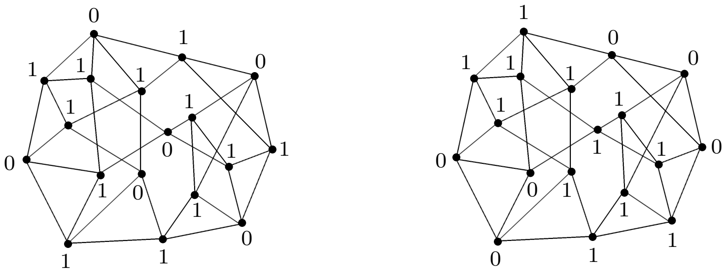


Figure 8.

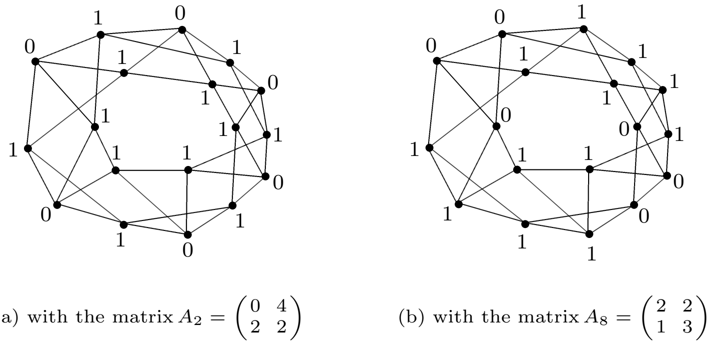


(a) with the matrix $A_2 = \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix}$ (b) with the matrix $A_8 = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$

Figure 9. Perfect 2-colorings of $L(BC6)$

(iii) The eigenvalues of $BC7$ are 3^1, 2^2, 1^1, 0^4, -1^1, -2^2, and -3^1 [15]. Thus, the eigenvalues of $L(BC7)$ are 4^1, 3^2, 2^1, 1^4, 0^1, -1^2 and -2^7. By Theorem 2.1 and (2.1), a parameter matrix of a perfect 2-coloring of $L(BC7)$ may be one of the matrices A_2, A_7, A_8, A_9 or A_{10}. Perfect 2-colorings of $L(BC7)$ with the parameter matrices A_2 and A_8 are shown in Figure 10.

It suffices to show that there is no perfect 2-coloring of $L(BC7)$ with the parameter matrix A_7, A_9 and A_{10}. Let $T : V_{L(BC7)} \to \{0, 1\}$ be a perfect 2-coloring of $L(BC7)$ and H be the subgraph of $L(BC7)$ induced by the white vertices. If A_7 is the parameter matrix of T then by (2.1), H is a 1-regular graph of order 9, a contradiction. Similarly, if A_{10} is the parameter matrix of T then by (2.1), H is a cubic graph of order 9, a contradiction. Now, suppose that A_9 is the parameter matrix of T. Then H is a 2-regular graph of order 9, i.e., it is isomorphic to the cycle C_9 or one of the disjoint unions $3C_3$, $C_4 + C_5$ and $C_3 + C_6$. But $L(BC7)$ has no induced subgraph isomorphic to C_9, $3C_3$ or $C_4 + C_5$ [15]. Let H be isomorphic to $C_3 + C_6$ (for example, see the subgraph induced



(a) with the matrix $A_2 = \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix}$ (b) with the matrix $A_8 = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$

FIGURE 10. Perfect 2-colorings of $L(BC7)$

by the vertices $v_2, v_3, v_6, v_7, v_8, v_{11}, v_{12}, v_{13}, v_{14}$; Figure 11). Color all the vertices in H with white. Then, there is a black vertex that does not have two white neighbors, a contradiction.

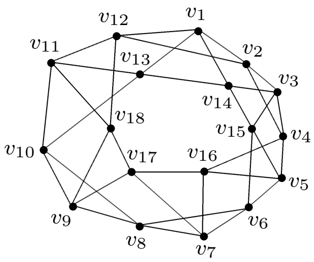


FIGURE 11.

(*iv*) The eigenvalues of $BC8$ are 3^1, 1.73^2, 1^3, -1^3, -1.73^2 and -3 [15]. Thus, the eigenvalues of $L(BC8)$ are 4^1, 2.73^2, 2^3, 0^3, -0.73^2 and -2^7. By Theorem 2.1 and (2.1), a parameter matrix of a perfect 2-coloring of $L(BC8)$ may be one of the matrices A_2, A_7, A_9 or A_{10}. Perfect 2-coloring of $L(BC8)$ with the parameter matrix A_2 is shown in Figure 12.

It suffices to show that there is no perfect 2-coloring of $L(BC8)$ with the parameter matrix A_7, A_9 and A_{10}. Let $T : V_{L(BC8)} \to \{0, 1\}$ be a perfect 2-coloring of $L(BC8)$ and H be the subgraph of $L(BC8)$ induced by the white vertices. If A_7 is the parameter matrix of T then by (2.1), H is a 1-regular graph of order 9, a contradiction. Similarly, if A_{10} is the parameter matrix of T then by (2.1), H is a cubic graph of order 9, a contradiction. Now, suppose that A_9 is the parameter matrix of T.

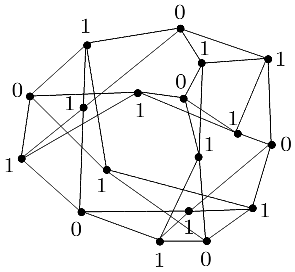


FIGURE 12. Perfect 2-coloring of $L(BC8)$ with the matrix $A_2 = \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix}$

Then H is a 2-regular graph of order 9, i.e., it is isomorphic to the cycle C_9 or one of the disjoint unions $3C_3$, $C_4 + C_5$ and $C_3 + C_6$. But $L(BC8)$ has no induced subgraph isomorphic to C_9, $3C_3$ or $C_4 + C_5$ [15]. Let H be isomorphic to $C_3 + C_6$ (for example, see the subgraph induced by the vertices $v_3, v_6, v_9, v_{10}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}$; Figure 13). Color all the vertices in H with white. Then, there is a black vertex that does not have two white neighbors, a contradiction.

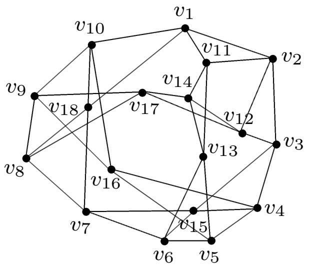
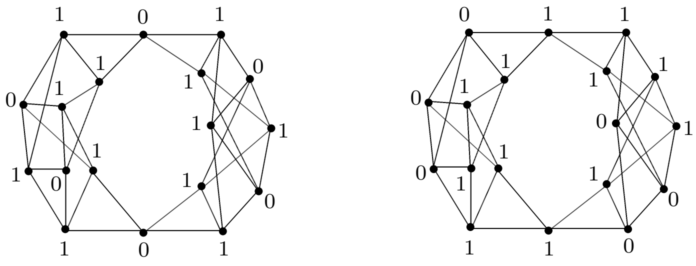


FIGURE 13.

(v) The eigenvalues of $BC9$ are $3^1, 2.56^1, 1.56^1, 0^6, -1.56^1, -2.56^1$ and -3^1 [15]. Thus, the eigenvalues of $L(BC9)$ are $4^1, 3.56^1, 2.56^1, 1^6, -0.56^1$, -1.56^1 and -2^7. By Theorem 2.1 and (2.1), a parameter matrix of a perfect 2-coloring of $L(BC9)$ may be one of the matrices A_2, A_7 or A_8. Perfect 2-colorings of $L(BC9)$ with the parameter matrices A_2 and A_8 are shown in Figure 14.

It suffices to show that there is no perfect 2-coloring of $L(BC9)$ with the parameter matrix A_7. Let $T : V_{L(BC9)} \to \{0, 1\}$ be a perfect 2-coloring of $L(BC9)$ with the parameter matrix A_7. Then, by (2.1), the



(a) with the matrix $A_2 = \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix}$ (b) with the matrix $A_8 = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$

FIGURE 14. Perfect 2-colorings of $L(BC9)$

subgraph of $L(BC9)$ induced by the white vertices is a 1-regular graph of order 9, a contradiction. It completes the proof. □

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