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Simplest Equation Method for nonlinear solitary waves in Thomas-Fermi plasmas

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Abstract. The Thomas-Fermi (TF) equation has proved to be useful for the treatment of many physical phenomena. In this paper, the traveling wave solutions of the KdV equation is investigated by the simplest equation method. Also, the effect of different parameters on these solitary waves is considered. The numerical results is conformed the good accuracy of presented method.

Keywords: Simplest equation method; Thomas-Fermi plasmas; KdV equation;Ion acoustic waves.

2014 Mathematics subject classification: 35Q53, 37K40.

1. Introduction

The Thomas-Fermi is a semiclassical statistical method describing electronic potential and densities. It provides a simple way to describe large atoms, solids, and matter at high pressure as in astrophysical objects such as neutron stars. The investigation of the exact solutions of nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. Nonlinear phenomena appear in a

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wide variety of scientific applications such as plasma physics, solid state physics, and fluid dynamics. In order to better understand these nonlinear phenomena, many mathematicians and physical scientists make efforts to seek more exact solutions of them [1, 2, 3, 4, 9, 12].
Indeed, an electron-positron plasma usually behaves as a fully ionized gas consisting of electrons and positrons, as seen in the solar atmosphere as well as in many astrophysical objects (e.g., white dwarfs, neutron stars, near the polar cap of pulsars, in the active galactic nuclei, in the early universe). The success achieved in collecting and keeping positrons and even anti-hydrogen under laboratory conditions has opened up a new field of laboratory anti-matter plasma. The propagation ion-acoustic waves is one the most important subjects in plasma physics and the study of ion acoustic waves in plasmas has received considerable attention, because its key role in understanding the nonlinear-phenomena in laboratory plasmas [13] as well as in space plasmas [14, 19]. There are usually, two methods for investigation ion-acoustic waves in plasmas; one of them is Sagdeev pseudo-potential method [15], where in this method the main properties of arbitrary amplitude ionacoustic waves by obtain an integral energy equation is studied. Another method which is used for studies nonlinear phenomena in plasmas is the reductive perturbation method [11]. In this method nonlinear waves in plasma can be described by different partial differential equations as Korteweg-de-Vries equation [8], modified Kortewegde-Vries (mKdV) equation [18] and the nonlinear Schrodinger equation (NLSE) [6, 17] investigated effects of ion temperature on ion-acoustic waves in a nonthermal plasmas, by using the pseudo-potential approach, which is valid for arbitrary amplitude solitary waves. It has shown that to increase ion temperature the amplitude of the compressive and rarefactive solitary wave's decreases. Linear and nonlinear properties and the modulation of ion-acoustic waves in plasmas with two temperature electrons which having different Boltzmann distributions theoretically, investigated by several authors with different method[5].

2. The simplest equation method

In this section, we present the main idea of the simplest equation method [10], step by step:
Step1. We first consider a general form of nonlinear equation

$$E\left(u, u_t, u_x, u_{tt}, ...\right) = 0. \tag{2.1}$$

Step2. To find the traveling wave solution of Eq. (2.1) we introduce the wave variable $\xi = x - ct$,so that

$$u(x,t) = u(\xi), \tag{2.2}$$

Based on this we use the following changes

$$\begin{array}{l} \frac{\partial}{\partial t}(.) = -c\frac{\partial}{\partial \xi}(.), \\ \frac{\partial}{\partial x}(.) = -c\frac{\partial}{\partial \xi}(.), \\ \frac{\partial^2}{\partial x^2}(.) = -c\frac{\partial^2}{\partial \xi^2}(.), \end{array} \tag{2.3}$$

and so on for other derivatives.
Using (2.3) changes the PDE (2.1) to an ODE

$$\varphi\left(y, \frac{\partial y}{\partial \xi}, \frac{\partial^2 y}{\partial \xi^2}, ...\right) = 0, \tag{2.4}$$

where $y = y(\xi)$ is an unknown function, $\varphi$is a polynomial in the variable $y$and its derivatives.
Step3. The basic idea of the simplest equation method consists in expanding the solutions $y(\xi)$ of Eq. (2.4) in a finite series

$$y(\xi) = \sum_{i=0}^{l} a_i z^i, \qquad a_l \neq 0, \tag{2.5}$$

where the coefficients a_i are independent of $\xi$and $z = z(\xi)$ are the functions that satisfy some ordinary differential equations.
In this paper, we use the Bernoulli equation [6] as simplest equation

$$\frac{dz}{d\xi} = az(\xi) + bz^2(\xi), \tag{2.6}$$

Eq. (2.6) admits the following exact solutions

$$z(\xi) = \frac{a\exp[a(\xi+\xi_0)]}{1 - b\exp[a(\xi+\xi_0)]}, \tag{2.7}$$

for the case $a > 0, b < 0$ and

$$z(\xi) = \frac{a\exp[a(\xi+\xi_0)]}{1 + b\exp[a(\xi+\xi_0)]}, \tag{2.8}$$

for the case $a < 0, b > 0$, where ξ_0 is a constant of integration.

Remark 1. l is a positive integer, in most cases, that will be determined. To determine the parameter l; we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms.

Step4. Substituting (2.5) into (2.4) with (2.6), then the left hand side of Eq. (2.4) is converted into a polynomial in$z\left(\xi\right)$; equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i, a, b, c.
Step5. Solving the algebraic equations obtained in step 4, and substituting the results into (2.5), then we obtain the exact traveling wave solutions for Eq. (2.1).

Remark 2. In Eq.(2.6), when $a = A$ and $b = -1$ we obtain the Bernoulli equation

$$\frac{dz}{d\xi} = Az\left(\xi\right) - z^2\left(\xi\right), \tag{2.9}$$

Eq. (2.9) admits the following exact solutions

$$z\left(\xi\right) = \frac{A}{2}\left[1 + \tanh\left(\frac{A}{2}\left(\xi + \xi_0\right)\right)\right], \tag{2.10}$$

when$A > 0$, and

$$z\left(\xi\right) = \frac{A}{2}\left[1 - \tanh\left(\frac{A}{2}\left(\xi + \xi_0\right)\right)\right], \tag{2.11}$$

when$A < 0$.

Remark3. This method is a simple case of the Ma-Fuchssteiner method [19].

3. Basic equations and derivation KdV equation

We consider two-component plasma consisting of adiabatic ions, described by the fluid-moment equations, and degenerate inertialess electrons, assumed to obey a Thomas-Fermi density distribution [5]. The basic equations for ions in such plasma is given as

$$\frac{\partial n_i}{\partial t} + \frac{\partial\left(n_i u_i\right)}{\partial x} = 0, \tag{3.1}$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial x} + \frac{e}{m_i}\frac{\partial \Phi}{\partial x} + \frac{1}{m_i n_i}\frac{\partial p_i}{\partial x} = 0, \tag{3.2}$$

and the system is closed through Poisson's equation,

$$\frac{\partial^2 \Phi}{\partial x^2} = 4\pi e\left(n_e - n_i\right), \tag{3.3}$$

Where n_i, u_i, Φ and p_i are the ion number density, the ion mean velocity, the electrostatic potential and the ion pressure respectively. For adiabatic ion pressure, we have, $p_i = p_{i0}\left(n_i/n_{i0}\right)^{\gamma}$ where $p_{i0} = n_{i0}k_B T_i$ and

γ is defined as $\gamma = \frac{N+2}{N}$ and N is the number of degrees of freedom (for present work $\gamma = 3$).

The Thomas-Fermi approximation is however valid only when the Fermi wavelength is much less than the wavelength of ion-acoustic waves and this condition enables us to ignore the spatial dispersion of the electron gas, thus we assume that the dependence of the electrons to the electrostatic potential in one- dimensional fully degenerate Fermi gas is given by Thomas-Fermi density distribution as

$$n_e = n_0 \left(1 + \frac{e\phi}{k_B T_{Fe}}\right)^{\frac{1}{2}}, \tag{3.4}$$

where $n_0 = \pi \left(8\pi/3h^3\right) p_F^3$is the unperturbed density in terms of the linear Fermi momentum and h is Plank's constant, k_B is the Boltzmann's constant.Normalizing by appropriate scaling quantities, the electron number density can be written as

$$n_e = (1 + \phi)^{\frac{1}{2}}, \tag{3.5}$$

it is remarked that (3.4), can also be obtained by balancing the electric force and Thomas-Fermi equation of state for non relativistic degenerate inertialess $m_e \to 0$ electrons which in one-dimensional system given [7]

$$p_e = \frac{m_e V_{Fe}^2 n_0}{3} \left(\frac{n_e}{n_0}\right)^3. \tag{3.6}$$

The normalized ion continuity and momentum equation, and Poisson's equation are

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0, \tag{3.7}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial \phi}{\partial x} + \frac{3\sigma}{2} n \frac{\partial n}{\partial x} = 0, \tag{3.8}$$

$$\frac{\partial^2 \phi}{\partial x^2} = 2\left(n_e - n\right), \tag{3.9}$$

Where $\sigma = T_i/T_{Fe}$ is the ratio of ion temperature to electron Fermi temperature. Now, let us introduce the following scaling that are used for the normalized the basic set of equations

$$\begin{aligned} &x = c_s x/\omega_{pi}, t = t/\omega_{pi}, \\ &n_i = n_0 n, u_i = u c_s, \\ &\Phi = \phi\left(k_B T_{Fe}/e\right). \end{aligned} \tag{3.10}$$

Here $\omega_{pi} = \left(e^2 n_0/\varepsilon_0 m_i\right)^{\frac{1}{2}}$ and $c_s = (2K_B T_{Fe}/e)$ are ion plasma frequency and ion sound-speed, respectively.

We are now interested for investigation propagation of ion acoustic waves in an idea plasma with degenerate electrons. So, we shall employ reductive perturbation technique [16]. The independent variables can be stretched as
$\xi=\varepsilon^{\frac{1}{2}}\left(x-\lambda_0 t\right)$ and $\tau=\varepsilon^{\frac{3}{2}}t$. (3.10)
Where ε is small parameter and λ_0 is the unknown linear phase velocity to be determined later. Also, the independent variables $n, u$and ϕ can expanded as follow:

$$\begin{array}{l} n=1+\varepsilon n_1+\varepsilon^2 n_2+\varepsilon^3 n_3+...,\\ u=\varepsilon u_1+\varepsilon^2 u_2+\varepsilon^3 u_3+...,\\ \phi=\varepsilon\phi_1+\varepsilon^2\phi_2+\varepsilon^3\phi_3+...\ . \end{array} \tag{3.11}$$

Substituting (3.10) and (3.11) into (3.7)–(3.9), and isolating distinct orders in ε, for the lowest order in ε, we have

$$\begin{array}{l} -\lambda_0\frac{\partial}{\partial\xi}n_1+\frac{\partial}{\partial\xi}u_1=0,\\ -\lambda_0\frac{\partial}{\partial\xi}u_1+\frac{1}{2}\frac{\partial}{\partial\xi}\phi_1+\frac{3}{2}\sigma\frac{\partial}{\partial\xi}n_1=0,\\ \frac{1}{2}\phi_1-n_1=0, \end{array} \tag{3.12}$$

now, from (3.12), we obtain

$$n_1=\frac{1}{\lambda_0}u_1, \phi_1=2\lambda_0 u_1-3\sigma n_1, n_1=\frac{1}{2}\phi_1, \lambda_0=\sqrt{1+\frac{3\sigma}{2}}, \tag{3.13}$$

for next order of ε, we have

$$\begin{array}{l} \frac{\partial}{\partial\tau}n_1-\lambda_0\frac{\partial}{\partial\xi}n_2+\frac{\partial}{\partial\xi}u_2+\frac{\partial}{\partial\xi}\left(n_1u_1\right)=0,\\ \frac{\partial}{\partial\tau}u_1-\lambda_0\frac{\partial}{\partial\xi}u_2+u_1\frac{\partial}{\partial\xi}u_1+\frac{1}{2}\frac{\partial}{\partial\xi}\phi_2+\frac{3}{2}\sigma\frac{\partial}{\partial\xi}n_2+\frac{3\sigma}{2}n_1\frac{\partial}{\partial\xi}n_1=0,\\ \frac{\partial^2\phi_1}{\partial\xi^2}=\phi_2-\frac{1}{4}\phi_1^2-2n_2, \end{array} \tag{3.14}$$

Finally the KdV equation is derived from (3.13) and (3.14) as

$$\frac{\partial}{\partial\tau}\phi_1+\left(\frac{\left(\lambda_0^2-\frac{3\sigma}{2}\right)}{4\lambda_0}+\frac{\left(\frac{3\lambda_0^2}{2}+\frac{3\sigma}{4}\right)}{2}\right)\phi_1\frac{\partial}{\partial\xi}\varphi_1+\frac{\left(\lambda_0^2-\frac{3\sigma}{2}\right)}{2\lambda_0}\frac{\partial^3\phi_1}{\partial\xi^3}=0. \tag{3.15}$$

4. Solitary wave solutions to the KdV equation

In this section, we will obtain new exact solution for ion acoustic waves in Thomas-Fermi plasmas by simplest equation method. It is seen that the evolution of nonlinear waves described by KdV equation. In this paper we use simplest equation method for obtain analytical solutions of KdV equation.

Now we make a transformation $\eta = \xi - V\tau$ and integrating with respect to η and consider the integral constants to be zero, Eq. (3.15) is transformed as

$$-V\varphi_1 + \frac{1}{2}\left(\frac{\left(\lambda_0^2 - \frac{3\sigma}{2}\right)}{4\lambda_0} + \frac{\left(\frac{3\lambda_0^2}{2} + \frac{3\sigma}{4}\right)}{2}\right)\varphi_1^2 + \frac{\left(\lambda_0^2 - \frac{3\sigma}{2}\right)}{2\lambda_0}\varphi_1^{''} = 0. \quad (4.1)$$

For obtaining the solutions of Eq. (4.1), with the aid of simplest equation method we make the following ansatz

$$\varphi_1(\tau) = \sum_{i=0}^{n} a_i F^i(\tau), \quad (4.2)$$

where a_i are all real constants to be determined, n is a positive integer which can be determined by balancing the highest order derivative term with the highest order nonlinear term. Balancing $\varphi_1^{''}$ with φ_1^2 then gives

$$2n = n + 2 \Rightarrow n = 2.$$

Therefore, we may choose

$$\varphi_1(\eta) = a_2 F^2 + a_1 F + a_0. \quad (4.3)$$

Substituting (4.3) along with (2.7) in Eq. (4.1) and then setting the coefficients of $F^j (j = 0, 1, 2, 3, 4, 5)$ to zero in the resultant expression, we obtain a set of algebraic equations and solving these equations with the aid of Maple we have

$$b = \frac{\lambda_0^2 - \frac{3\sigma}{2} + 2\lambda_0\left(\frac{3\lambda_0^2}{2} + \frac{3\sigma}{4}\right)}{48\lambda_0 V}\sqrt{\frac{4\lambda_0 V}{3\sigma - 2\lambda_0^2}}a_1, \quad a_0 = \frac{2V}{\lambda_0^2 - \frac{3\sigma}{2} + 2\lambda_0\left(\frac{3\lambda_0^2}{2} + \frac{3\sigma}{4}\right)},$$
$$a = \sqrt{\frac{4\lambda_0 V}{3\sigma - 2\lambda_0^2}}, \quad a_2 = \frac{\lambda_0^2 - \frac{3\sigma}{2} + 2\lambda_0\left(\frac{3\lambda_0^2}{2} + \frac{3\sigma}{4}\right)}{48\lambda_0 V}a_1^2. \quad (4.4)$$

Thus, we have the solitary wave solution of the Korteweg-de-Vries equation with taking the solution set (4.4) along with (4.3) and (2.8) we have solutions of (3.15) as follows

$$\varphi_1(\eta) = \left(\frac{\lambda_0^2-\frac{3\sigma}{2}+2\lambda_0\left(\frac{3\lambda_0^2}{2}+\frac{3\sigma}{4}\right)}{48\lambda_0 V}a_1^2\right)\frac{4\lambda_0 V}{3\sigma-2\lambda_0^2}\exp\left[2\sqrt{\frac{4\lambda_0 V}{3\sigma-2\lambda_0^2}}\left(\xi - V\tau+\eta_0\right)\right]\times$$
$$\left[1-\frac{\lambda_0^2-\frac{3\sigma}{2}+2\lambda_0\left(\frac{3\lambda_0^2}{2}+\frac{3\sigma}{4}\right)}{48\lambda_0 V}\sqrt{\frac{4\lambda_0 V}{3\sigma-2\lambda_0^2}}a_1\exp\left[\sqrt{\frac{4\lambda_0 V}{3\sigma-2\lambda_0^2}}\left(\xi - V\tau+\eta_0\right)\right]\right]^{-2}+$$
$$a_1\sqrt{\frac{4\lambda_0 V}{3\sigma-2\lambda_0^2}}\exp\left[\sqrt{\frac{4\lambda_0 V}{3\sigma-2\lambda_0^2}}\left(\xi - V\tau+\eta_0\right)\right]\times$$
$$\left[1-\frac{\lambda_0^2-\frac{3\sigma}{2}+2\lambda_0\left(\frac{3\lambda_0^2}{2}+\frac{3\sigma}{4}\right)}{48\lambda_0 V}\sqrt{\frac{4\lambda_0 V}{3\sigma-2\lambda_0^2}}a_1\exp\left[\sqrt{\frac{4\lambda_0 V}{3\sigma-2\lambda_0^2}}\left(\xi - V\tau+\eta_0\right)\right]\right]^{-1}+$$
$$\frac{2V}{\lambda_0^2-\frac{3\sigma}{2}+2\lambda_0\left(\frac{3\lambda_0^2}{2}+\frac{3\sigma}{4}\right)},$$

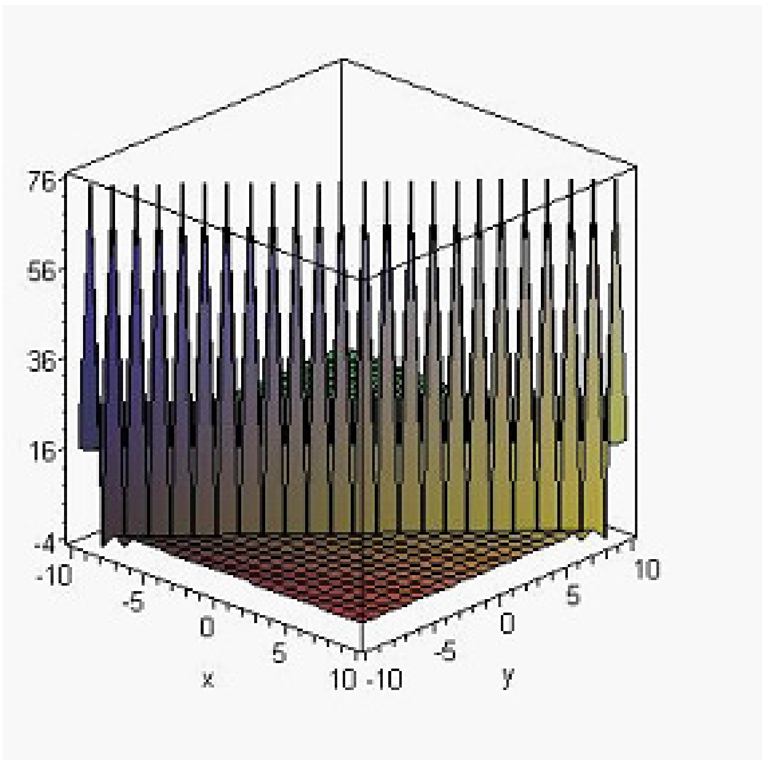

FIGURE 1. (a) Solitary wave solution corresponding to (30) for $a_1 = 1$, $\lambda_0 = 1$, $\sigma = 1$

Substituting (4.3) along with (2.10) in the equation in (4.1) and setting all the coefficients of powers F to be zero, then we obtain a system

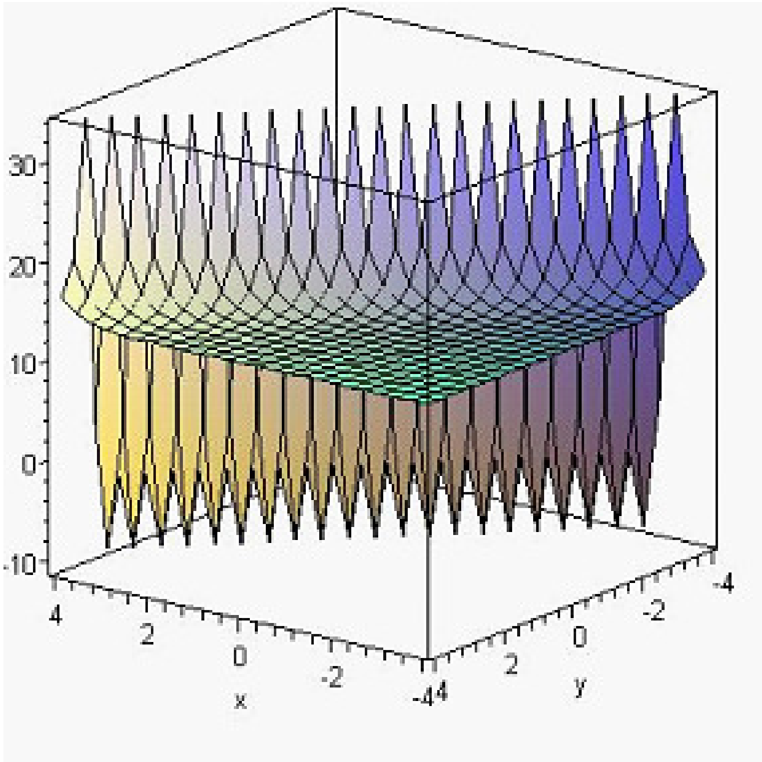

FIGURE 2. (b) Solitary wave solution corresponding to (30) for $a_1 = 1$, $\lambda_0 = 1$, $\sigma = 1$, $\eta_0 = 1$

of nonlinear algebraic equations and by solving it with aid Maple, we obtain

$$
\begin{array}{c}
a_0 = \frac{8\lambda_0 V}{\lambda_0^2 - \frac{3\sigma}{2} + 2\lambda_0\left(\frac{3\lambda_0^2}{2} + \frac{3\sigma}{4}\right)}, a_1 = -\frac{12\left(3\sigma - 2\lambda_0^2\right)\sqrt{\frac{2\lambda_0 V}{\frac{3\sigma}{2} - \lambda_0^2}}}{\lambda_0^2 - \frac{3\sigma}{2} + 2\lambda_0\left(\frac{3\lambda_0^2}{2} + \frac{3\sigma}{4}\right)}, \\
a_2 = -\frac{48\lambda_0 B}{\lambda_0^2 - \frac{3\sigma}{2} + 2\lambda_0\left(\frac{3\lambda_0^2}{2} + \frac{3\sigma}{4}\right)}, \quad A = \frac{4\lambda_0 V}{3\sigma - 2\lambda_0^2}
\end{array}
\tag{4.5}
$$

Now, we have the solitary wave solution of the Korteweg-de-Vries equation with taking the solution set (4.5) along with (4.3) and (2.10)we have solutions of (3.15) as follows:

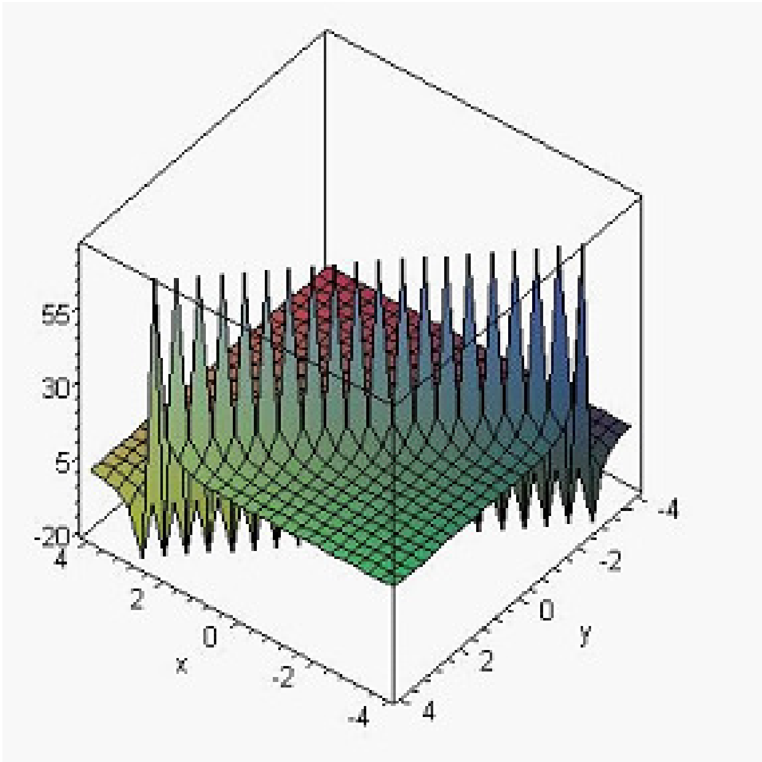

FIGURE 3. (c) Solitary wave solution corresponding to (30) for $a_1 = -1$, $\lambda_0 = 1$, $\sigma = 1$

$$\varphi_1(\eta) = -\frac{48\lambda_0 B}{\lambda_0^2 - \frac{3\sigma}{2} + 2\lambda_0\left(\frac{3\lambda_0^2}{2} + \frac{3\sigma}{4}\right)} \times$$
$$\left[\frac{4\lambda_0^2 V^2}{\left(3\sigma - 2\lambda_0^2\right)^2} + \frac{8\lambda_0^2 V^2}{\left(3\sigma - 2\lambda_0^2\right)^2} \tanh\left(\frac{2\lambda_0 V}{3\sigma - 2\lambda_0^2}\left(\xi + \xi_0\right)\right) + \frac{4\lambda_0^2 V^2}{\left(3\sigma - 2\lambda_0^2\right)^2} \tanh^2\left(\frac{2\lambda_0 V}{3\sigma - 2\lambda_0^2}\left(\xi - V\tau + \eta_0\right)\right)\right] -$$
$$\frac{24\lambda_0 V\sqrt{\frac{2\lambda_0 V}{\frac{3\sigma}{2} - \lambda_0^2}}}{\lambda_0^2 - \frac{3\sigma}{2} + 2\lambda_0\left(\frac{3\lambda_0^2}{2} + \frac{3\sigma}{4}\right)}\left[1 + \tanh\left(\frac{2\lambda_0 V}{3\sigma - 2\lambda_0^2}\left(\xi - V\tau + \eta_0\right)\right)\right] + \frac{8\lambda_0 V}{\lambda_0^2 - \frac{3\sigma}{2} + 2\lambda_0\left(\frac{3\lambda_0^2}{2} + \frac{3\sigma}{4}\right)},$$

In all figures we suppose $\xi = x, \tau = y$.

5. Conclusions

In this paper we have derived the KdV equation for investigation small but finite amplitude solitons. The solutions of KdV equation has obtained by the simplest equation method. The effect of different parameters on solitons has presented by some figures (1-3). In figures the

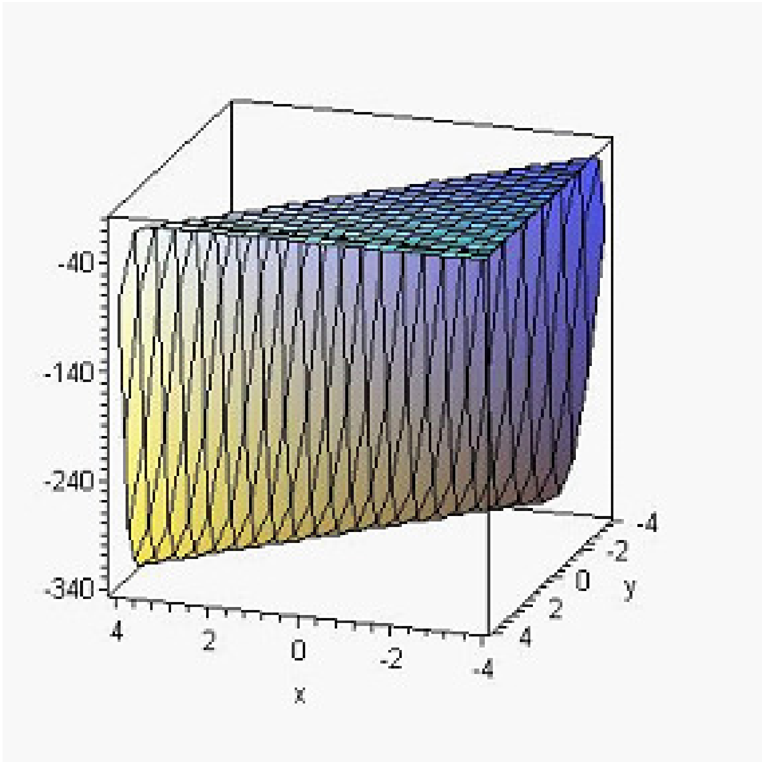

FIGURE 4. (d) Solitary wave solution corresponding to (31) for $\lambda_0 = 1$, $\sigma = 1$

variation of electrostatic potential of ion-acoustic solitary waves with different value of the ratio of ion temperature to electron Fermi temperature σ is drawn. It has seen that the ion temperature have the significant effect on solitary waves, the width, as well as the amplitude of solitary wave decreases with increasing σ. In summary, we have investigated the nonlinear properties of solitary waves in plasma consisting of degenerate electrons and adiabatic ions. Using the reductive perturbation method, the basic set of equations is reduced to KdV equation for lowest order perturbation. Numerical results reveal that compressive solitons has observed. It is essentially important to report in our model of plasma that the amplitude (width) ion-acoustic waves increases as $\lambda\,(\mu)$ increases.

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