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Optimization in progressively Type-II censoring with random sample size based on cost constraint

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Abstract. In this paper, we consider the progressively Type-II censoring and the sample size is assumed as a random variable from a Poisson distribution. The optimal sample size is determined by considering a cost constraint. Towards this end, we first introduce a cost function and then the optimal parameter of Poisson distribution is obtained so that the cost function does not exceed a pre-fixed value. In the following, through a simulation study, the results are evaluated. Finally, the conclusion of the article is presented.

Keywords: Random Sample Size, Cost Function, Optimization.

2020 Mathematics subject classification: 62F30, 62N01.

1. Introduction

The scheme of progressively Type-II censoring is an important method of obtaining data in lifetime studies. It allows the experimenter to remove units from a life test at various stages during the experiment. Under the progressively Type-II censoring scheme, n units are placed on a lifetime test. At the first failure time, R_1 surviving items are randomly withdrawn from the test. At the second failure time, R_2 surviving items are selected at random and taken out of the experiment, and so on. Finally, at the time of the m-th failure, the remaining $R_m = n - m - \sum_{i=1}^{m-1} R_i$ objects are removed. If the failure times are based on an absolutely

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continuous cumulative distribution function (cdf) $F_\theta(\cdot)$ and probability density function (pdf) $f_\theta(\cdot)$, and denote the i-th failure time by $X^{\tilde{R}}_{i:m:n}$, then the random variables $X^{\tilde{R}}_{1:m:n}, \ldots, X^{\tilde{R}}_{m:m:n}$ are called *progressively Type-II censored order statistics* (PCOs) based on censoring scheme $\tilde{R} = (R_1, \ldots, R_m)$, where $n = m + \sum_{j=1}^{m} R_j$. For notational simplicity, hereafter we use $X_{i:m:n}$ instead of $X^{\tilde{R}}_{i:m:n}$, for $1 \le i \le m$. The marginal pdf of the i-th failure time, $X_{i:m:n}$, $1 \le i \le m$, is given by (see for example, Balakrishnan and Aggarwala [3])

$$f_{X_{i:m:n};\theta}(x) = c_{i-1} f_\theta(x) \sum_{t=1}^{i} a_{t,i;n} (\bar{F}_\theta(x))^{\gamma_{t;n}-1},\ F^{-1}(0^+) < x < F^{-1}(1^-), \tag{1.1}$$

where $\bar{F}_\theta(x) = 1 - F_\theta(x)$ is the survival function of X-sample and the quantile function $F_\theta^{-1} : [0,1] \to \mathbb{R}$ is defined by

$$F_\theta^{-1}(y) = \inf\{x : F_\theta(x) \ge y\},\ y \in (0,1),$$

and $F_\theta^{-1}(0^+) = \lim_{y\to 0^+} F_\theta^{-1}(y)$, $F_\theta^{-1}(1^-) = \lim_{y\to 1^-} F_\theta^{-1}(y)$. Also, $n = m + \sum_{j=1}^{m} R_j$, $m, n \in \mathbb{N}$, $\gamma_{i;n} = m - i + 1 + \sum_{j=i}^{m} R_j$, $c_{i-1;n} = \prod_{j=1}^{i} \gamma_{j;n}$ and $a_{t,i;n} = \prod_{j=1, j\ne t}^{i} \frac{1}{\gamma_{j;n} - \gamma_{t;n}}$, $1 \le t \le i \le m$. For a detailed discussion of progressive censoring, we refer the reader to Balakrishnan and Aggarwala [3] Balakrishnan [2], Balakrishnan and Cramer [4] and the references contained therein.

Optimization in censoring schemes is one of the issues that so far has been studied by many researchers. For example, Ebrahimi [12] investigated determining the sample size for a Hybrid life test which minimizes the expected cost. Pham [15] taking into account a cost function based on the time of the experiment, determined the optimal size of samples for the exponential distribution. Ng et al. [14] determined the optimal censoring plan in progressively Type II censoring based on some criteria such as the cost of experiment. Doostparast and Balakrishnan [11] discussed the optimal sample size for estimating the mean based on a criterion involving a cost function as well as the Fisher information based on records arising from a random sample. Cordeiro and Pham [10] introduced a new cost function that, in addition to the duration of the experiment, also considers the reliability of the test. Then, they determined the optimal number of samples for Type-II censoring, with r = 2 failures, for Weibull distribution. The optimization problem of sample size allocation when the competing risks data are from a progressive type-II censoring in a constant-stress accelerated life test with multiple levels, is studied by Huang andWu [13]. Basiri [6] Obtained the optimal number of failures in Type-II censoring by considering Bayesian prediction problem and cost function. Basiri and Beigi [8] obtained optimal censoring scheme in progressively Type-II censoring with binomial removals in Bayesian two-sample prediction problem.

In some applications, such as clinical trials and quality control, it is almost impossible to have a fixed sample size all the time because some observations may be missing for various reasons. In other words, the sample size is a random variable. Some examples in this area can be found in Srivastava [17]. Ahmadi et al. [1] investigated the optimal size of samples based on a cost function for the Bayesian prediction when the information sample size is fixed as well as a random variable. They obtained the parameter of distribution of the information sample size, such that the point predictor of a future order statistic has minimum mean squared prediction error when the total cost of experiment is bounded. Motivated by they work, the aim of this paper is to find the optimal sample size in progressively Type-II censoring so that the cost is bounded. We assume that the sample size is a random variable from a Poisson distribution. Then, we introduce a cost function and then the optimal parameter of the distribution of sample size is determined such that the cost function does not exceed a pre-determined value.

The remainder of this paper is organized as follows. In Section 2, first, we introduce a cost function that is a basic tool for finding the optimal sample size. Then, the optimal parameter of the distribution of sample size is determined such that the cost function does not exceed a pre-determined value. We consider some different censoring schemes and obtain the results. Numerical computations are given in Section 3. Finally, a conclusion is presented in Section 4.

2. Main results

Throughout this paper, let $\tilde{X} = (X_{1:m:N}, \cdots, Xm : m : N)$ be the progressively Type-II censored order statistics from a sample of size N of independent and identically distributed (iid) continuous random variables from the one-parameter exponential distribution, denoted by $Exp(\theta)$, with probability density function (pdf) and cumulative distribution function (cdf) given by

$$f_\theta(x) = \theta e^{-\theta x}, \quad \text{and} \quad F_\theta(x) = 1 - e^{-\theta x}, \quad x > 0, \ \theta > 0, \tag{2.1}$$

respectively. Here, N is assumed to be a Poisson random variable double truncated at points m and M, $(m < M)$, with parameter θ, denoted by $Poi(\theta; m; M)$, i.e.,

$$P(N = n) = \frac{\lambda^n}{n! \sum_{j=m}^{M} \frac{\lambda^j}{j!}}, \ m \le n \le M. \tag{2.2}$$

So, $P(N \ge i) = 1$, for $1 \le i \le m$.

In this paper we consider a cost function as

$$\begin{aligned} EC(N) &= c_0 + c_u E_N(N) - p_s\left(E_N(N) - m\right) + c_t E_N\left(E(X_{m:m:N}|N=n)\right) \\ &\quad + c_v E_N\left(V(X_{m:m:N}|N=n)\right) + c_R\left\{F\left(E_N\left(E(X_{m:m:N}|N=n)\right)\right)\right\}, \end{aligned} \tag{2.3}$$

where c_0, c_u, p_s, c_t, c_v and c_R are the sampling set-up cost or any other related cost involved in sampling, cost per unit, the price of second-hand units, cost of total time on test, cost of expected testing time variance and cost of the risk of testing units fail before the expected test time, respectively. It is important to mention that this cost function is an adopted model proposed by Cordeiro [9] and Cordeiro and Pham [10].

From (1.1), (2.1) and (2.2) and following Basiri and Ahmadi [7], the marginal density function of Xm:m:N, when N is a Poisson random variable, can be written as

$$\begin{aligned} f_{X_{m:m:N};\theta}(x) &= \frac{1}{P(N \geq m)} \sum_{n=i}^{\infty} \sum_{t=1}^{m} c_{m-1;n} f_\theta(x) a_{t,m;n} (\bar{F}_\theta(x))^{\gamma_{t;n}-1} P(N=n) \\ &= \frac{\theta}{\sum_{j=m}^{M} \frac{\lambda^j}{j!}} \sum_{n=m}^{M} \sum_{t=1}^{m} c_{m-1;n} a_{t,m;n} e^{-\theta x \gamma_{t;n}} \frac{\lambda^n}{n!}. \end{aligned}$$

Assuming N = n, from Balakrishnan and Aggarwala [3] we have

$$E(X_{m:m:N}|N=n) = \frac{1}{\theta} g(m,n) \text{ and } V(X_{m:m:N}|N=n) = \frac{1}{\theta^2} h(m,n),$$

where

$$g(m,n) = \sum_{t=1}^{m} \frac{1}{n - \left(\sum_{l=0}^{t-1} R_l\right) - t + 1}, \tag{2.4}$$

and

$$h(m,n) = \sum_{t=1}^{m} \frac{1}{\left(n - \left(\sum_{l=0}^{t-1} R_l\right) - t + 1\right)^2}. \tag{2.5}$$

So, we conclude that

$$E_N\left(E(X_{m:m:N}|N=n)\right) = \frac{1}{\theta} G(\lambda; m, M) \text{ and } E_N\left(V(X_{m:m:N}|N=n)\right) = \frac{1}{\theta^2} H(\lambda; m, M), \tag{2.6}$$

where

$$G(\lambda; m, M) = \frac{1}{\sum_{j=m}^{M} \frac{\lambda^j}{j!}} \sum_{n=m}^{M} g(m,n) \frac{\lambda^n}{n!},$$

and

$$H(\lambda; m, M) = \frac{1}{\sum_{j=m}^{M} \frac{\lambda^j}{j!}} \sum_{n=m}^{M} h(m,n) \frac{\lambda^n}{n!}.$$

Also, it is easy to show that

$$E_N(N) = \lambda \frac{\sum_{j=m-1}^{M-1} \frac{\lambda^j}{j!}}{\sum_{j=m}^{M} \frac{\lambda^j}{j!}} = A(\lambda; m, M) \text{ say.} \tag{2.7}$$

By substituting (2.6) and (2.7) into (2.3), the cost function can be rewritten as

$$\begin{aligned} EC(N) \quad = \quad & c_0 + c_u A(\lambda; m, M) - p_s \left(A(\lambda; m, M) - m\right) + \frac{c_t}{\theta} G(\lambda; m, M) \\ & + \frac{c_v}{\theta^2} H(\lambda; m, M) + c_R \left[1 - exp\{-G(\lambda; m, M)\}\right]. \end{aligned} \tag{2.8}$$

From (2.8) we can see that $EC(N)$ depends on the unknown parameter θ, and therefore it can be replaced by its preliminary estimator based on past experiments and pre-information.

In the sequel, we try to find optimal value for the sample size such that and $EC(N) \leq c^*$, where c^* is a pre-fixed value. To do this, we consider the following m different censoring schemes as

$$\tilde{R}_{(k)} = (R_1, \cdots, R_k, \cdots, R_m),$$

where $R_k = N - m$ and $R_i = 0$, for $i \neq k, i, k = 1, \cdots, m$.

We have computed the values of $EC(N)$ for some selected choices of θ, m, $\tilde{R}$ and λ when $M = 10$, $c_0 = 10$, $c_u = 5$, $p_s = 1$, $c_t = 5$, $c_v = 2$ and $c_R = 1$. The results are reported in Table 1. Also, values of $EC(N)$ for different choices of θ, m, $\tilde{R}_{(k)}$, $k = 1, \cdots, m$, and λ are plotted in Figure 1, when $M = 10$, $c_0 = 10$, $c_u = 5$, $p_s = 1$, $c_t = 5$, $c_v = 2$, $c_R = 1$ and $c^* = 50$. From Table 1 and Figure 1, by an empirical evidence, we find the following points:

- The cost function $EC(N)$ is an increasing function of m but a decreasing function of θ, when other components are fixed, as we expected.
- Considering $\tilde{R}_{(m)} = (0, \cdots, 0, N - m)$, or Type-II censoring, leads to better results than other censoring schemes.

In Table 2, we have presented the values of λ which the condition $EC(N) \leq c^*$ is satisfied, say λ_{opt}, when $M = 10$, $c_0 = 10$, $c_u = 5$, $p_s = 1$, $c_t = 5$, $c_v = 2$, $c_R = 1$ and $c^* = 50$ for different values of θ, m and $\tilde{R}_{(k)}$, $k = 1, \cdots, m$. In Table 2, dash (-) means that there is no λ which satisfies the condition $EC(N) \leq c^*$. Moreover, we find that the optimal values are increasing with respect to θ when all other components are held fixed.

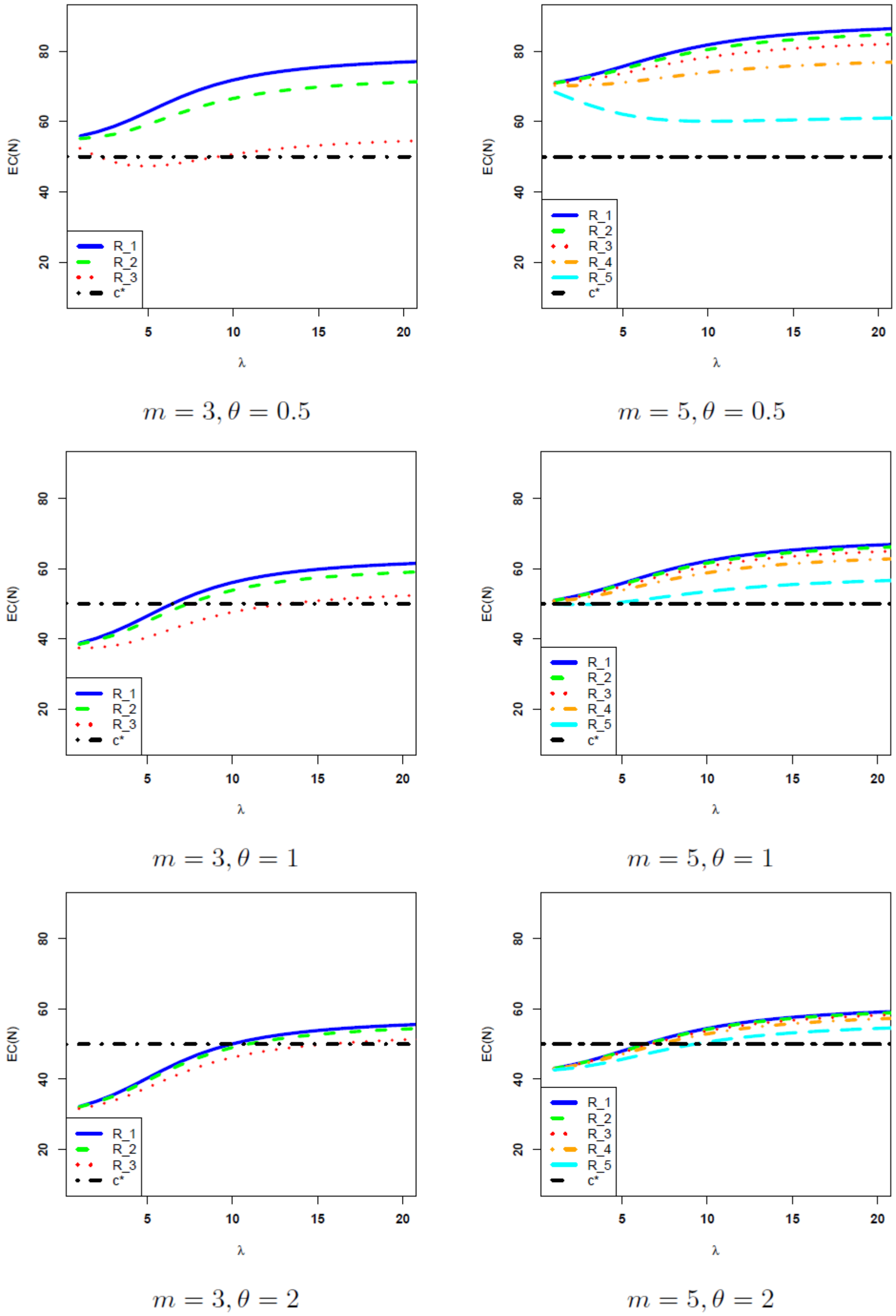

FIGURE 1. Plots of $EC(N)$ for some selected choices of θ, m, $\tilde{R}_{(k)}$ and λ when $M = 10$, $c_0 = 10$, $c_u = 5$, $p_s = 1$, $c_t = 5$, $c_v = 2$, $c_R = 1$ and $c^* = 50$.

TABLE 1. Values of $EC(N)$ for some selected choices of θ, m and $\tilde{R}_{(k)}$ and λ when $M=10$, $c_0=10$, $c_u=5$, $p_s=1$, $c_t=5$, $c_v=2$ and $c_R=1$.

θ	m	$\tilde{R}_{(k)}/\ \lambda$	1	2	3	4	5	6	7	8	9	10
0.5	3	$\tilde{R}_{(1)}$	55.897	57.073	58.646	60.603	62.815	65.073	67.184	69.031	70.581	71.854
		$\tilde{R}_{(2)}$	55.172	55.588	56.400	57.635	59.206	60.936	62.636	64.176	65.502	66.611
		$\tilde{R}_{(3)}$	52.411	50.149	48.469	47.511	47.275	47.610	48.291	49.113	49.942	50.706
	5	$\tilde{R}_{(1)}$	71.115	71.977	73.051	74.320	75.718	77.152	78.531	79.793	80.908	81.871
		$\tilde{R}_{(2)}$	70.993	71.717	72.638	73.745	74.982	76.262	77.503	78.647	79.662	80.544
		$\tilde{R}_{(3)}$	70.776	71.260	71.922	72.760	73.731	74.765	75.788	76.745	77.607	78.363
		$\tilde{R}_{(4)}$	70.292	70.259	70.379	70.672	71.121	71.681	72.293	72.908	73.491	74.023
		$\tilde{R}_{(5)}$	68.430	66.510	64.763	63.270	62.089	61.231	60.664	60.329	60.163	60.113
1	3	$\tilde{R}_{(1)}$	38.751	40.130	41.916	44.083	46.487	48.910	51.153	53.101	54.726	56.055
		$\tilde{R}_{(2)}$	38.455	39.521	40.990	42.852	44.983	47.177	49.240	51.053	52.579	53.835
		$\tilde{R}_{(3)}$	37.431	37.489	38.003	39.007	40.417	42.043	43.682	45.190	46.501	47.605
	5	$\tilde{R}_{(1)}$	50.959	51.871	53.000	54.328	55.787	57.278	58.708	60.016	61.169	62.163
		$\tilde{R}_{(2)}$	50.906	51.756	52.818	54.074	55.460	56.882	58.251	59.505	60.614	61.571
		$\tilde{R}_{(3)}$	50.814	51.561	52.511	53.651	54.922	56.236	57.509	58.681	59.721	60.623
		$\tilde{R}_{(4)}$	50.617	51.154	51.881	52.795	53.847	54.962	56.061	57.087	58.007	58.812
		$\tilde{R}_{(5)}$	49.932	49.767	49.791	50.026	50.451	51.013	51.647	52.293	52.912	53.480
2	3	$\tilde{R}_{(1)}$	32.200	33.662	35.534	37.787	40.271	42.762	45.060	47.050	48.708	50.061
		$\tilde{R}_{(2)}$	32.066	33.385	35.110	37.221	39.576	41.959	44.170	46.095	47.705	49.022
		$\tilde{R}_{(3)}$	31.635	32.520	33.828	35.553	37.576	39.691	41.697	43.471	44.972	46.211
	5	$\tilde{R}_{(1)}$	43.074	44.006	45.159	46.513	47.997	49.513	50.966	52.293	53.463	54.471
		$\tilde{R}_{(2)}$	43.049	43.952	45.073	46.393	47.842	49.325	50.749	52.050	53.198	54.188
		$\tilde{R}_{(3)}$	43.006	43.862	44.931	46.196	47.591	49.023	50.401	51.6633	52.778	53.742
		$\tilde{R}_{(4)}$	42.918	43.679	44.647	45.808	47.103	48.442	49.739	50.933	51.992	52.910
		$\tilde{R}_{(5)}$	42.633	43.099	43.767	44.636	45.656	46.749	47.836	48.855	49.774	50.580

3. Simulation study

In this section, a simulation study is carried out in order to assess the performances of the results in the paper. Based on the algorithm proposed by Balakrishnan and Sandhu [5], we have the following algorithm.

Algorithm 3.1. Suppose θ, m, M, c_0, c_u, p_s, c_t, c_v, c_R, c^* and the censoring scheme $\tilde{R}$ are all given. Then:

(1) Choose λ_{opt} from the condition $EC(N) \le c^*$.
(2) Generate N from the distribution $Poi(\lambda_{opt}; m, M)$.
(3) Generate m iid random variables $W_1, \ldots, W_m$ from the uniform distribution $U(0,1)$.
(4) Take $V_i = W_i^{\frac{1}{i+\sum_{k=m-i+1}^{m} R_k}}$ for $i = 1, \cdots, m$.
(5) Set $U_i = 1 - \prod_{k=m-i+1}^{m} V_k$ for $i = 1, \cdots, m$.

(6) Obtain the progressively Type-II censored order statistics by setting $X_{i:m:n} = F_\theta^{-1}(U_i)$ for $i = 1, \cdots, m$, when $F_\theta^{-1}(\cdot)$ is the quantile function of the exponential distribution with parameter θ.
(7) Repeat the Steps 2-6 for $B = 10000$ times and let $N^{(j)}$ and $X^{(j)}_{i:m:N}$ be the results obtained from Steps 2 and 6 in the jth iteration, $j = 1, \ldots, B$.
(8) Then, calculate the estimated expected cost functions (EEC) as

$$\begin{aligned} EEC(N) &= c_0 + \frac{c_u}{B}\sum_{j=1}^{B} N^{(j)} - p_s\left(\frac{1}{B}\sum_{j=1}^{B} -m\right) + \frac{c_t}{\theta} g'(m,N) \\ &\quad + \frac{c_v}{\theta^2} h'(m,N) + c_R\left[1 - exp\{-g'(m,N)\}\right], \end{aligned}$$

where

$$g'(m,N) = \frac{1}{B}\sum_{j=1}^{B} X^{(j)}_{i:m:N},\ \ k'(m,N) = \frac{1}{B}\sum_{j=1}^{B}\left(X^{(j)}_{i:m:N}\right)^2,$$

and

$$h'(m,N) = k'(m,N) - \left(g'(m,N)\right)^2.$$

Based on Algorithm 3.1 and the results in Table 2, we have computed the values of $EEC(N)$ for different values of θ, m and $\tilde{R}$, when $M = 10$, $c_0 = 10$, $c_u = 5$, $p_s = 1$, $c_t = 5$, $c_v = 2$, $c_R = 1$ and $c^* = 50$. For Step 1 we have considered the largest values of λ_{opt} in Table 2. The results are tabulated in Table 3. From Table 3, it is observed that for most cases $EEC(N) \leq c^*$.

TABLE 2. Values of $EEC(N)$ for different values of θ, m and $\tilde{R}$, when $M = 10$, $c_0 = 10$, $c_u = 5$, $p_s = 1$, $c_t = 5$, $c_v = 2$, $c_R = 1$ and $c^* = 50$.

m	$\tilde{R}_{(k)}$	$\theta = 0.5$	$\theta = 1$	$\theta = 2$
3	$\tilde{R}_{(1)}$	-	48:881	48:648
	$\tilde{R}_{(2)}$	-	50:013	50:202
	$\tilde{R}_{(3)}$	61:250	53:721	53:061
5	$\tilde{R}_{(1)}$	-	-	44:421
	$\tilde{R}_{(2)}$	-	-	44:378
	$\tilde{R}_{(3)}$	-	-	44:298
	$\tilde{R}_{(4)}$	-	-	46:504
	$\tilde{R}_{(5)}$	-	50:426	49:891

TABLE 3. Values of λ_{opt} for some selected choices of $\tilde{R}_{(k)}$, $k = 1, 2, 3$,when $m = 3$, $M = 6$, $c_0 = 10$, $c_u = 5$, $p_s = 1$, $c_t = 5$, $c_v = 2$, $c_R = 1$ and $c^* = 18700$.

$\tilde{R}_{(k)}$	λ_{opt}
$\tilde{R}_{(1)}$	4.293
$\tilde{R}_{(2)}$	1.001
$\tilde{R}_{(3)}$	-

4. Example

In this example, we present the analysis of real data, partially considered in Proschan [16] for illustrative purposes. Records were kept for the time of successive failures of the air conditioning system of each member of a eet of Boeing 720 jet air planes. The intervals between successive failures for a plane are listed in order of occurrence are

$$194,\ 15,\ 41,\ 29,\ 33,\ 181.$$

Ahmadi et al. [1] have used these data set as the exponential random variables with parameter $\theta = 0.0121$. Let us assume in this example, $m = 3$, $M = 6$, $c^* = 18700$, $c_0 = 10$, $c_u = 5$, $p_s = 1$, $c_t = 5$, $c_v = 2$ and $c_R = 1$. Also, we consider the censoring schemes $\tilde{R}_{(k)}$, $k = 1, 2, 3$. Values of λ_{opt} are presented in Table 4. From Table 4 we can observed that for $\tilde{R}_{(3)} = (0, 0, N - m)$ there is no λ which satisfies the condition $EC(N) \leq c^*$.

5. Conclusion

Determining the optimal sample size is one of the issues that has been studied by many researchers so far. In some applications it is almost impossible to have a fixed sample size all the time because some observations may be missing for various reasons. In other words, the sample size is random variable. In this paper, we first introduce a cost function and then assuming the size as a random variable from a Poisson distribution, the optimal sample size is investigated. The results show that considering the Type-II censoring, choosing small values for m and λ but large values for θ leads to better results.

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