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Tauberian theorems for the weighted mean methods of summability in intuitionistic fuzzy normed spaces

Reha Yapali[1] and Harun Polat
Mathematics Department , Mus Alparslan University, Turkey

Abstract. In this paper, weighted mean methods of summability are given in intuitionistic fuzzy normed spaces $IFNS$. Also, some Tauberian conditions are defined for the weighted mean methods of summability in $IFNS$.

Keywords: intuitionistic fuzzy normed space, weighted mean summability, slow oscillation, Tauberian theorem

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1. Introduction

Many mathematicians from various branches have studied fuzzy sets, which were given by Zadeh[1] in 1965 as a generalization of classical sets. In classical sets, elements in the universal set are divided crisply into two groups as members and nonmembers, and partial membership is not allowed. By giving degrees of membership between 1 and 0, fuzzy sets, unlike classical sets, allow partial membership and take into account all items in the universe. Fuzzy sets are used in many real-world situations to deal with problems of ambiguity and indefiniteness because of their ability to handle unclassifiable data. In 1983, Atanassov[2, 3], inspired by fuzzy sets, considered partial non membership and expanded fuzzy sets to intuitionistic fuzzy sets. Following Atanassov's introduction, concepts of intuitionistic fuzzy metric[4] and intuitionistic fuzzy

[1]Corresponding author: reha.yapali@alparslan.edu.tr
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norm (IF-norm)[5, 6] are defined, and topics related to them are researched. Convergence of sequences in $IFNS$ is studied in particular, and various forms of convergence (e.g., statistical and ideal convergence) are applied to sequences in $IFNS$ to grasp the convergence[7, 8, 9, 10].

Talo and Yavuz[11] recently introduced Cesàro summability of sequences in $IFNS$ and provided Tauberian theorems for Cesàro summability process in $IFNS$, by which they initiated summability theory and Tauberian theory in $IFNS$. They also described the definition of slow oscillation in $IFNS$ and gave related theorems in their analysis. After their study, the logarithmic summability of sequences in $IFNS$ is defined and a Tauberian theorem for logarithmic summability method is proved by Yavuz[24]. In this paper, we define the weighted mean summability of sequences in $IFNS$ and prove a Tauberian theorem for the weigted mean summability method. Also, we give slowly oscillating type Tauberian conditions for which weighted mean summability yields convergence in $IFNS$.

Definition 1.1. [6] The triplicate (M, μ, ν) is said to be an $IF-$normed space if M is a real vector space, and μ, ν are fuzzy sets on $M \times \mathbb{R}$ satisfying the following conditions for every $x, y \in M$ and $k, l \in \mathbb{R}$:

(1) $\mu(x, k) = 0$ for $k \leq 0$,
(2) $\mu(x, k) = 1$ for all $k \in \mathbb{R}^+$ if and only if $x = \theta$
(3) $\mu(cx, k) = \mu\left(x, \frac{k}{|c|}\right)$ for all $k \in \mathbb{R}^+$ and $c \neq 0$,
(4) $\mu(x + y, k + l) \geq \min\{\mu(x, k), \mu(y, l)\}$,
(5) $\lim_{k\to\infty} \mu(x, k) = 1$ and $\lim_{k\to 0} \mu(x, k) = 0$,
(6) $\nu(x, k) = 1$ for $k \leq 0$,
(7) $\nu(x, k) = 0$ for all $k \in \mathbb{R}^+$ if and only if $x = \theta$
(8) $\nu(cx, k) = \nu\left(x, \frac{k}{|c|}\right)$ for all $k \in \mathbb{R}^+$ and $c \neq 0$,
(9) $\max\{\nu(x, k), \nu(y, l)\} \geq \nu(x + y, k + l)$,
(10) $\lim_{k\to\infty} \nu(x, k) = 0$ and $\lim_{k\to 0} \nu(x, k) = 1$.

Indicate that we say (μ, ν) an $IF-$norm on M. Additionally, it is obvious that the functions $\mu(x, \cdot)$ and $\nu(x, \cdot)$ are both non-increasing and non-decreasing on $\mathbb{R}$.

Example 1.2. Let $(M, \|\cdot\|)$ be a normed space and μ_0, ν_0 be fuzzy sets on $M \times \mathbb{R}$ defined by

$$\mu_0(x, k) = \begin{cases} 0, & k \leq 0, \\ \frac{k}{k+\|x\|}, & k > 0, \end{cases} \qquad \nu_0(x, k) = \begin{cases} 1, & k \leq 0, \\ \frac{\|x\|}{k+\|x\|}, & k > 0. \end{cases}$$

Then (μ_0, ν_0) is $IF-$norm on M.

Definition 1.3. [6] A sequence (x_n) is said to be convergent to $x \in M$ in the $IF-$normed space (M, μ, ν) and denoted by $x_n \to x$, if for each

$k > 0$ and each $\varepsilon \in (0,1)$ there exists $n_0 \in \mathbb{N}$ such that

$$\mu(x_n - x, k) > 1 - \varepsilon \quad \text{and} \quad \nu(x_n - x, k) < \varepsilon$$

for all $n \geq n_0$.

Definition 1.4. [6] A sequence (x_n) in an $IF-$normed space (M, μ, ν) is said to be Cauchy if for each $k > 0$ and each $\varepsilon \in (0,1)$ there exists $n_0 \in \mathbb{N}$ such that

$$\mu(x_n - x_m, k) > 1 - \varepsilon \quad \text{and} \quad \nu(x_n - x_m, k) < \varepsilon$$

for all $n, m \geq n_0$.

2. Theorems for the weighted mean summability method in intuitionistic fuzzy normed spaces

In $IF-$ normed space, we now give the weighted mean summability and prove the related Tauberian theorems. Other work on weighted mean summability and convergence approaches on fuzzy settings can be found in [21, 22, 16, 17, 18, 15, 14, 12, 19, 13, 20].

Definition 2.1. Let (x_n) be a sequence in $IF-$ normed space (M, μ, ν). The weighted mean t_n of the sequence (x_n) is defined by

$$t_n = \frac{1}{P_n} \sum_{k=0}^{n} p_k x_k \quad \textit{where} \quad P_n = \sum_{k=0}^{n} p_k \ \to \infty \ (n \to \infty).$$

(x_n) is said to be weighted mean summable to $x \in M$ if

$$\lim_{n \to \infty} x_n = x.$$

In order to show that the weighted mean method in $IF-$ normed space is regular, we give the following theorem.

Theorem 2.2. *Let (x_n) be a sequence in (M, μ, ν). (x_n) is weighted mean summable to x, if (x_n) is convergent to $x \in M$.*

Proof. Let the sequence (x_n) converge to $x \in M$. Fix $t > 0$. For $\varepsilon > 0$

- There exists $n_0 \in \mathbb{N}$ such that $\mu\left(x_n - x, \frac{t}{2}\right) > 1-\varepsilon$ and $\nu\left(x_n - x, \frac{t}{2}\right) < \varepsilon$ for $n > n_0$.
- There exists $n_1 \in \mathbb{N}$ such that

$$\mu\left(\sum_{k=1}^{n_0} p_k(x_k - x), \frac{P_n t}{2}\right) > 1 - \varepsilon \quad \text{and} \quad \nu\left(\sum_{k=1}^{n_0} p_k(x_k - x), \frac{P_n t}{2}\right) < \varepsilon$$

for $n > n_1$, since we have

$$\lim_{n \to \infty} \mu\left(\sum_{k=1}^{n_0} p_k(x_k - x), \frac{P_n t}{2}\right) = 1 \quad \text{and} \quad \lim_{n \to \infty} \nu\left(\sum_{k=1}^{n_0} p_k(x_k - x), \frac{P_n t}{2}\right) = 0.$$

Hence we get

$$\begin{aligned}
\mu\left(\frac{1}{P_n}\sum_{k=1}^{n}p_kx_k-x,t\right) &= \mu\left(\frac{1}{P_n}\sum_{k=1}^{n}p_k(x_k-x),t\right)=\mu\left(\sum_{k=1}^{n}p_k(x_k-x),P_nt\right)\\
&\geq \min\left\{\mu\left(\sum_{k=1}^{n_0}p_k(x_k-x),\frac{P_nt}{2}\right),\mu\left(\sum_{k=n_0+1}^{n}p_k(x_k-x),\frac{P_nt}{2}\right)\right\}\\
&\geq \min\left\{\mu\left(\sum_{k=1}^{n_0}p_k(x_k-x),\frac{P_nt}{2}\right),\mu\left(\sum_{k=n_0+1}^{n}p_k(x_k-x),\frac{(P_n-P_{n_0})t}{2}\right)\right\}\\
&\geq \min\left\{\mu\left(\sum_{k=1}^{n_0}p_k(x_k-x),\frac{P_nt}{2}\right),\mu\left(p_{n_0+1}(x_{n_0+1}-x),\frac{P_{n_0+1}t}{2}\right),\cdots,\mu\left(p_n(x_n-x),\frac{P_nt}{2}\right)\right\}\\
&= \min\left\{\mu\left(\sum_{k=1}^{n_0}p_k(x_k-x),\frac{P_nt}{2}\right),\mu\left(x_{n_0+1}-x,\frac{t}{2}\right),\cdots,\mu\left(x_n-x,\frac{t}{2}\right)\right\}\\
&> 1-\varepsilon
\end{aligned}$$

and

$$\begin{aligned}
\nu\left(\frac{1}{P_n}\sum_{k=1}^{n}p_kx_k-x,t\right) &< \max\left\{\nu\left(\sum_{k=1}^{n_0}p_k(x_k-x),\frac{P_nt}{2}\right),\nu\left(x_{n_0+1}-x,\frac{t}{2}\right),\cdots,\nu\left(x_n-x,\frac{t}{2}\right)\right\}\\
&< \varepsilon
\end{aligned}$$

whenever $n>\max\{n_0,n_1\}$, which completes the proof. □

However, as the following example shows, the weighted mean summability of a sequence does not mean convergence in $IF-$ normed space .

Example 2.3. Take $(x_n)=((-1)^{n-1})$ in $IF-$normed space $(\mathbb{R},\mu_0,\nu_0)$ where μ_0 and ν_0 are as in Example 1.2. Sequence (x_n) is weighted mean summable to 0 [11, see Example 3.3], but it is not convergent.

Definition 2.4. [25] A nondecreasing sequence of positive numbers (P_n) is called regularly varying of index $\theta>0$ in the sense of Karamata if,

$$\lim_{n\to\infty}\frac{P_{\lambda_n}}{P_n}=\lambda^{\theta},\lambda>1. \tag{2.1}$$

We now give some Tauberian conditions in which weighted mean summability leads to convergence in $IF-$ normed spaces.

Theorem 2.5. *Let (x_n) be a sequence in (M,μ,ν). If the condition (2.1) is satisfied and (x_n) is weighted mean summable to $x \in M$, then (x_n) converges to x if and only if for each $t>0$*

$$\sup_{\lambda>1} \liminf_{n\to\infty} \mu\left(\frac{1}{P_{\lambda_n}-P_n}\sum_{k=n+1}^{\lambda_n} p_k(x_k-x_n), t\right) = 1 \tag{2.2}$$

and

$$\inf_{\lambda>1} \limsup_{n\to\infty} \nu\left(\frac{1}{P_{\lambda_n}-P_n}\sum_{k=n+1}^{\lambda_n} p_k(x_k-x_n), t\right) = 0. \tag{2.3}$$

Proof. Necessity. Let (x_n) converges to x. For all $\lambda>1$ and large enough n, that is when $P_{\lambda_n}>P_n$, we can write (see [23, Lemma 5.5(i)])

$$x_n - t_n = \frac{P_{\lambda_n}}{P_{\lambda_n}-P_n}(t_{\lambda_n}-t_n) - \frac{1}{P_{\lambda_n}-P_n}\sum_{k=n+1}^{\lambda_n} p_k(x_k-x_n)\cdot \tag{2.4}$$

Since (t_n) is Cauchy, for each $t>0$ we have

$$\lim_{n\to\infty}\mu\left((t_{\lambda_n}-t_n),t\right)=1 \quad \text{and} \quad \lim_{n\to\infty}\nu\left((t_{\lambda_n}-t_n),t\right)=0.$$

Hence, for sufficiently large n such that $\frac{P_{\lambda_n}}{P_{\lambda_n}-P_n} \le \frac{2\lambda^\theta}{\lambda^\theta-1}$ is satisfied, we have

$$\mu\left(\frac{P_{\lambda_n}}{P_{\lambda_n}-P_n}\left(t_{\lambda_n}-t_n\right),t\right) = \mu\left(t_{\lambda_n}-t_n, \frac{t}{\frac{P_{\lambda_n}}{P_{\lambda_n}-P_n}}\right) \ge \mu\left(t_{\lambda_n}-t_n, \frac{t}{\frac{2\lambda^\theta}{\lambda^\theta-1}}\right) \to 1 \quad (n\to\infty)$$

and

$$\nu\left(\frac{P_{\lambda_n}}{P_{\lambda_n}-P_n}\left(t_{\lambda_n}-t_n\right),t\right) = \nu\left(t_{\lambda_n}-t_n, \frac{t}{\frac{P_{\lambda_n}}{P_{\lambda_n}-P_n}}\right) \ge \nu\left(t_{\lambda_n}-t_n, \frac{t}{\frac{2\lambda^\theta}{\lambda^\theta-1}}\right) \to 0 \quad (n\to\infty)$$

revealing that $\frac{P_{\lambda_n}}{P_{\lambda_n}-P_n}\left(t_{\lambda_n}-t_n\right) \to 0$. So, by equation (2.4), we conclude

$$\lim_{n\to\infty}\mu\left(\frac{1}{P_{\lambda_n}-P_n}\sum_{k=n+1}^{\lambda_n}(x_k-x_n)p_k, t\right)=1 \quad \text{and} \quad \lim_{n\to\infty}\nu\left(\frac{1}{P_{\lambda_n}-P_n}\sum_{k=n+1}^{\lambda_n}p_k(x_k-x_n), t\right)=0,$$

which means that (2.2) and (2.3) are satisfied.

Sufficiency. Let conditions (2.2) and (2.3) be satisfied. Let $t>0$ be fixed. For $\varepsilon>0$ we have:

- There exist $\lambda>1$ and $n_0\in\mathbb{N}$ such that

$$\mu\left(\frac{1}{P_{\lambda_n}-P_n}\sum_{k=n+1}^{\lambda_n}p_k(x_k-x_n), \frac{t}{3}\right) > 1-\varepsilon \quad \text{and} \quad \mu\left(\frac{1}{P_{\lambda_n}-P_n}\sum_{k=n+1}^{\lambda_n}p_k(x_k-x_n), \frac{t}{3}\right) < \varepsilon$$

for $n>n_0$.

- There exists $n_1 \in \mathbb{N}$ such that $\mu\left(t_n - x, \frac{t}{3}\right) > 1-\varepsilon$ and $\nu\left(t_n - x, \frac{t}{3}\right) < \varepsilon$ for $n > n_1$.
- There exists $n_2 \in \mathbb{N}$ such that

$$\mu\left(\frac{P_{\lambda_n}}{P_{\lambda_n} - P_n}\left(t_{\lambda_n} - t_n\right), \frac{t}{3}\right) > 1-\varepsilon \quad \text{and} \quad \nu\left(\frac{P_{\lambda_n}}{P_{\lambda_n} - P_n}\left(t_{\lambda_n} - t_n\right), \frac{t}{3}\right) < \varepsilon,$$

for $n > n_2$, since $\frac{P_{\lambda_n}}{P_{\lambda_n} - P_n}\left(t_{\lambda_n} - t_n\right) \to 0$.

Hence, by equation (2.4), we get

$$\begin{aligned}
\mu(x_n - x, t) &= \mu\left(x_n - t_n + t_n - x, t\right)\\
&= \mu\left(\frac{P_{\lambda_n}}{P_{\lambda_n} - P_n}\left(t_{\lambda_n} - t_n\right) - \frac{1}{P_{\lambda_n} - P_n}\sum_{k=n+1}^{\lambda_n}(x_k - x_n)p_k + t_n - x, t\right)\\
&\geq \min\left\{\mu\left(\frac{P_{\lambda_n}}{P_{\lambda_n} - P_n}\left(t_{\lambda_n} - t_n\right), \frac{t}{3}\right), \mu\left(\frac{1}{P_{\lambda_n} - P_n}\sum_{k=n+1}^{\lambda_n} p_k(x_k - x_n), \frac{t}{3}\right), \mu\left(t_n - x, \frac{t}{3}\right)\right\}\\
&> 1 - \varepsilon
\end{aligned}$$

and

$$\begin{aligned}
\nu(x_n - x, t) &< \max\left\{\nu\left(\frac{P_{\lambda_n}}{P_{\lambda_n} - P_n}\left(t_{\lambda_n} - t_n\right), \frac{t}{3}\right), \nu\left(\frac{1}{P_{\lambda_n} - P_n}\sum_{k=n+1}^{\lambda_n} p_k(x_k - x_n), \frac{t}{3}\right), \nu\left(t_n - x, \frac{t}{3}\right)\right\}\\
&< \varepsilon
\end{aligned}$$

for $n > \max\{n_0, n_1, n_2\}$, which completes the proof. □

Theorem 2.6. *Let sequence (x_n) be in (M, μ, ν). If (x_n) is weighted mean summable to $x \in M$, then it converges to x if and only if for each $t > 0$*

$$\sup_{0<\lambda<1} \liminf_{n\to\infty} \mu\left(\frac{1}{P_n - P_{\lambda_n}}\sum_{k=\lambda_n+1}^{n} p_k(x_n - x_k), t\right) = 1$$

and

$$\inf_{0<\lambda<1} \limsup_{n\to\infty} \nu\left(\frac{1}{P_n - P_{\lambda_n}}\sum_{k=\lambda_n+1}^{n} p_k(x_n - x_k), t\right) = 0.$$

Proof. The proof is done similarly to that of Theorem 2.5 by using equation(see [23, Lemma 5.5(ii)])

$$x_n - t_n = \frac{P_{\lambda_n}}{P_n - P_{\lambda_n}}\left(t_n - t_{\lambda_n}\right) + \frac{1}{P_n - P_{\lambda_n}}\sum_{k=\lambda_n+1}^{n} p_k(x_n - x_k) \quad (0 < \lambda < 1)$$

instead of (2.4). □

Now, we introduce the concept of slow oscillation with respect to weighted mean summability in $IFNS$.

Definition 2.7. [11] (x_n) in (M,μ,ν) is said to be slowly oscillating if

$$\sup_{\lambda>1}\liminf_{n\to\infty}\min_{n<k\leq\lambda_n}\mu(x_k-x_n,t)=1 \tag{2.5}$$

and

$$\inf_{\lambda>1}\limsup_{n\to\infty}\max_{n<k\leq\lambda_n}\nu(x_k-x_n,t)=0, \tag{2.6}$$

for each $t>0$. $\sup_{\lambda>1}$ in (2.5) and $\inf_{\lambda>1}$ in (2.6) can be replaced by $\lim_{\lambda\to 1^+}$.

A sequence (x_n) in (M,μ,ν) is slowly oscillating if for each $t>0$ and for all $\varepsilon>0$ there exist $\lambda>1$ and $n_0\in\mathbb{N}$ such that

$$\mu(x_k-x_n,t)>1-\varepsilon \quad \text{and} \quad \nu(x_k-x_n,t)<\varepsilon$$

whenever $n_0\leq n<k\leq\lambda_n$.

Theorem 2.8. *Let sequence (x_n) be in (M,μ,ν). If (x_n) is slowly oscillating then* (2.2) *and* (2.3) *are satisfied.*

Proof. Suppose that (x_n) is slowly oscillating with respect to weighted mean summability. Fix $t>0$. For $\varepsilon>0$ there exist $\lambda>1$ and $n_0\in\mathbb{N}$ such that

$$\mu(x_k-x_n,t)>1-\varepsilon \quad \text{and} \quad \nu(x_k-x_n,t)<\varepsilon$$

whenever $n_0\leq n<k\leq\lambda_n$. Hence, we have

$$\begin{aligned}\mu\left(\frac{1}{P_{\lambda_n}-P_n}\sum_{k=n+1}^{\lambda_n}p_k(x_k-x_n),t\right) &= \mu\left(\sum_{k=n+1}^{\lambda_n}p_k(x_k-x_n),(P_{\lambda_n}-P_n)t\right)\\ &\geq \min\{\mu(p_{n+1}(x_{n+1}-x_n),p_{n+1}t),\dots,\mu(p_{\lambda_n}(x_{\lambda_n}-x_n),P_{\lambda_n}t)\}\\ &> 1-\varepsilon\end{aligned}$$

and

$$\begin{aligned}\nu\left(\frac{1}{P_{\lambda_n}-P_n}\sum_{k=n+1}^{\lambda_n}(x_k-x_n)p_k,t\right) &\leq \max\{\nu(x_{n+1}-x_n,t),\dots,\nu(x_{\lambda_n}-x_n,t)\}\\ &< \varepsilon\end{aligned}$$

for $n\geq n_0$ and this completes the proof. □

In view of Theorem 2.5 and Theorem 2.8 we give the following Tauberian theorem.

Theorem 2.9. *Let sequence (x_n) be in (M,μ,ν) and the condition* (2.1) *be satisfied. If (x_n) is weighted mean summable to $x\in M$ and slowly oscillating, then (x_n) converges to x.*

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