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Complexiton solutions of some nonlinear partial differential equations via modified double sub-equation method

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Abstract. In this study, we focus on extended (3+1)-dimensional Jimbo-Miwa equations in constructing complexiton solutions. On this way, we use modified double sub-equation method which presents different solutions from ones obtained through double sub-equation method. Modified double sub-equation method employs two wave transformations to reach expected solutions. Referred method also enables scientists to reach exponential and trigonometric solutions individually based on parameter choices. In literature, this method is given in a different way and considered as generalization of double sub-equation method.

Keywords: Complexiton solution, Modified double sub-equation method, Extended (3+1)-dimensional Jimbo-Miwa equations.

2020 Mathematics subject classification: 35A25; Secondary 35C07.

1. Introduction

It is important to find complexiton solutions of nonlinear partial differential equations since complexitons have both trigonometric and hyperbolic type functions. On one hand this property provides a novel shape and view to corresponding wave, on the other hand the difficulty of finding complexiton solutions to nonlinear partial differential

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equations makes the process more valuable. So far, some methods have been presented to find analytical solutions of nonlinear partial differential equations, such as, the homogeneous balance method [41], the F-expansion method [56], the tanh function method [36], the sech-function method [30], the extended tanh function method [16, 31, 15, 43], tanh-coth method [44], the first integral method [7, 8], the (G'/G,1/G) expansion method [13] and some other methods [9, 10, 18, 29, 52]. In literature, there are also some other papers on application of newly developed techniques [2, 3, 4, 20, 5, 1, 50, 51].

In former studies, the phrase "complexiton solutions" was used and named in [32] for the first time. Complexiton solutions to the KdV equation was given via its bilinear form [32, 33]. After that, some new techniques in order to obtain complexiton solutions have been developed for nonlinear partial differential equations such as extended transformed rational function method [55], multiple Riccati equations rational expansion method [11], generalized compound Riccati equations rational expansion method [23], generalized sub-equations rational expansion method [24], double sub-equation method [12], technique introduced by Wazwaz and Zhaqilao [45, 46, 39, 40]. Then, Hossen et al. have presented modified double sub-equation method by applying to some partial differential equations [19].

This paper aims to show applicability of modified double sub-equation method to extended (3+1)-dimensional Jimbo-Miwa equations and we present new complexiton solutions different than solutions obtained by previous papers. Categorization of exact solutions by means of spectral parameters, lets us classify them into three different classes. The sign of spectral parameter determines what the solution is among two concepts, positon or negaton. By the way, complex spectral parameter corresponds to complexiton. The terms positon, negaton and complexiton correspond to trigonometric solutions, hyperbolic solutions and both, respectively.

The continuation of this paper has been scheduled as follows: In section 2, we give modified double sub-equation method as it is given in previous papers. Section 3, complexiton solutions to first type extended (3+1)-dimensional Jimbo-Miwa equation are derived. Section 4, second type extended (3+1)-dimensional Jimbo-Miwa equation is considered to derive complexiton waves. Finally, conclusions that have been deduced from our results obtained in this paper are located.

2. Modified Double Sub-equation Method

In [19], the method utilized in this work is given as follows:

Step 1: We start with a nonlinear partial differential equation intended to be solved

$$S(u, u_t, u_x, u_{tt}, u_{tx}, u_{xx}, ...) = 0 \tag{2.1}$$

which is intended to be solved and equipped with exact solutions. u is dependent variable which depends on independent variables x and t. Independent variables in partial differentia equations are also known space and time variables. Not surprisingly, like the processes used in some well-known exact solution procedures, modified double sub-equation method uses homogeneous balance principle to be able to determine form of the solution.

Step 2: The form of solution of Eq.(2.1) is constructed as

$$u(x,t) = a_0 + \frac{a_1\varphi(\xi) + a_2\psi(\eta)}{\lambda_0 + \lambda_1\varphi(\xi)\psi(\eta)} \tag{2.2}$$

where λ_0 and λ_1 are arbitrary constants and $a_0 = a_0(x,t)$, $a_1 = a_1(x,t)$, $a_2 = a_2(x,t)$, $\xi = \xi(x,t)$, $\eta = \eta(x,t)$ are functions of space and time variables. $\varphi(\xi)$ and $\psi(\eta)$ are the functions which satisfy following equations

$$\varphi'(\xi) = q_1 + p_1\varphi^2(\xi) \tag{2.3}$$

and

$$\psi'(\eta) = q_2 + p_2\psi^2(\eta) \tag{2.4}$$

where $\xi = k_1x + w_1t$ and $\eta = k_2x + w_2t$.

Step 3: Riccati Eqs. (2.3) and (2.4) have solutions depending on p_1, q_1, p_2 and q_2. Some of these for Eq. (2.3) can be classified as follows:

$$\varphi'(\xi) = q_1 + p_1\varphi^2(\xi)$$

(i) When $q_1 = 1$, $p_1 = -1$

$$\varphi(\xi) = \tanh(\xi),\ \varphi(\xi) = \coth(\xi), \tag{2.5}$$

(ii) When $q_1 = p_1 = \pm\frac{1}{2}$

$$\varphi(\xi) = \sec(\xi) \pm \tan(\xi), \tag{2.6}$$

(iii) When $q_1 = p_1 = 1$

$$\varphi(\xi) = \tan(\xi), \tag{2.7}$$

(iv) When $q_1 = p_1 = -1$

$$\varphi(\xi) = \cot(\xi), \tag{2.8}$$

(v) When $q_1 = \frac{1}{2}$, $p_1 = -\frac{1}{2}$

$$\varphi(\xi) = \tanh(\xi) \pm isech(\xi),\ \varphi(\xi) = \coth(\xi) \pm csch(\xi), \tag{2.9}$$

(vi) When $q_1 = 0$, $p_1 = 1$

$$\varphi(\xi) = -\frac{1}{\xi + w}, \tag{2.10}$$

Step 4: By the use of Eq.(2.2) into Eq.(2.1) and taking Eqs.(2.3) and (2.4) into account presents us a new expression which consists of φ, ψ. Then, in resulting expression, taking coefficients of the terms $\varphi^m \psi^n$ ($m = 0, 1, 2, ..., \ n = 0, 1, 2, ...$) to zero provides us a system of equations which consists of $a_0, a_1, a_2, k_1, w_1, k_2, w_2, \lambda_0$ and λ_1. Solution and analysis of this system takes us to the solutions of Eq.(2.1).

3. Implementation of Modified Double Sub-equation Method to first type extended (3+1)-dimensional Jimbo-Miwa equation

In this part, we implement the referred method to first type extended (3+1)-dimensional Jimbo-Miwa equation which is very popular in nonlinear sciences and applied sciences and employed in modelling shallow water waves, is expressed in the way that

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3(u_{xz} + u_{yz} + u_{zz}) = 0 \qquad (3.1)$$

in the literature [47]. Then, Manafian presented some solutions for Eq.(3.1) in [34] and Wazwaz also obtained some exact solutions of Eq.(3.1) in paper [21]. In [48], Xu et al. obtained multi-exponential wave solutions to Eq.(3.1). Khalique and Moleleki [22] presented conservation laws and some solutions to (3.1). For other studies made on Eq. (3.1), please see [25, 38, 54, 6, 42, 53, 37, 14, 49].

As we pointed out in section 2, we take the solution in the way that

$$u(x, y, t) = b_0 + \frac{b_1\varphi(\xi) + b_2\psi(\eta)}{b_3 + b_4\varphi(\xi)\psi(\eta)} \qquad (3.2)$$

where b_0, b_1, b_2, b_3, b_4 are arbitrary constants and $\xi = k_1 x + l_1 y + m_1 z + w_1 t$ and $\eta = k_2 x + l_2 y + m_2 z + w_2 t$. Taking Eqs. (2.3)-(2.4) into consideration and using Eq. (3.2) into Eq. (3.1) presents us a new expression which consists of φ, ψ. Then, in resulting expression, taking coefficients of the terms $\varphi^m \psi^n$ ($m = 0, 1, 2, ..., \ n = 0, 1, 2, ...$) to zero provides us a system of equations. Solution and analysis of this system enable us to obtain following two classes of solutions:

Case I:

$$\begin{aligned} &b_1 = 2b_4k_2q_2, \quad b_2 = 2b_4k_1q_1, \quad b_3 = 0, \quad l_2 = -\tfrac{k_2l_1}{k_1} \\ &w_1 = \tfrac{4l_1k_1^3p_1q_1 + 3m_1^2 + 3m_1k_1 + 3m_1l_1}{2l_1}, \\ &w_2 = -\tfrac{-4k_2^4l_1p_2q_2 + 3m_2^2k_1 + 3k_2m_2k_1 - 3k_2l_1m_2}{2k_2l_1} \end{aligned} \qquad (3.3)$$

Case II:

$$b_1 = -2b_3k_1p_1 + 2b_4k_2q_2, \quad b_2 = 2b_4k_1q_1 - 2b_3k_2p_2,$$

$$
\begin{aligned}
l_1 &= -\big[b_3(p_2^2 m_2 k_2 b_3^2 - k_2 p_2 m_1 q_1 b_3 b_4 - 2 m_2 p_2 m_1 q_1 b_3 b_4 \\
&\quad - b_4 m_2 p_2 k_1 q_1 b_3 + b_4^2 q_1^2 m_1 k_1 + p_2^2 m_2^2 b_3^2 + b_4^2 m_1^2 q_1^2)\big]/ \\
&\quad [2 k_1 q_1 k_2 (p_1 p_2 b_3^2 - b_4^2 q_1 q_2)(-b_4 k_1 q_1 + b_3 k_2 p_2)],
\end{aligned}
$$

$$
\begin{aligned}
l_2 &= -\big[b_4(p_2^2 m_2 k_2 b_3^2 - k_2 p_2 m_1 q_1 b_3 b_4 - 2 m_2 p_2 m_1 q_1 b_3 b_4 \\
&\quad - b_4 m_2 p_2 k_1 q_1 b_3 + b_4^2 q_1^2 m_1 k_1 + p_2^2 m_2^2 b_3^2 + b_4^2 m_1^2 q_1^2)\big]/ \\
&\quad [2 k_1 p_2 k_2 (p_1 p_2 b_3^2 - b_4^2 q_1 q_2)(-b_4 k_1 q_1 + b_3 k_2 p_2)],
\end{aligned}
$$

$$
\begin{aligned}
w_1 &= (4 b_3^3 p_1 p_2^2 k_1^3 q_1 m_2 k_2 - 6 b_3^3 p_1 p_2^2 k_1^2 q_1 m_1 k_2^2 + 4 b_3^3 p_1 p_2^2 k_1^3 q_1 m_2^2 \\
&\quad - 6 b_3^3 p_1 p_2^2 m_1^2 q_1 k_1 k_2^2 + 3 b_3^3 p_2^2 m_1 m_2 k_2 + 3 b_3^3 m_2^2 m_1 p_2^2 \\
&\quad - 8 b_3^2 p_1 p_2 b_4 k_1^3 q_1^2 m_2 m_1 - 4 b_3^2 p_1 p_2 b_4 k_1^4 q_1^2 m_2 \\
&\quad + 6 b_3^2 p_1 p_2 b_4 m_1^2 q_1^2 k_1^2 k_2 + 2 b_3^2 p_1 p_2 b_4 k_1^3 q_1^2 k_2 m_1 \\
&\quad - 3 b_3^2 p_2 q_1 b_4 m_1^2 k_2 - 6 b_3^2 p_2 q_1 b_4 m_1^2 m_2 - 3 b_3^2 p_2 k_1 m_2 m_1 q_1 b_4 \\
&\quad + 4 b_3 p_1 b_4^2 k_1^4 q_1^3 m_1 + 4 b_3 p_1 b_4^2 k_1^3 q_1^3 m_1^2 + 6 b_3 p_2 b_4^2 k_2^2 q_2 m_1 q_1^2 k_1^2 \\
&\quad + 6 b_3 p_2 b_4^2 k_1 q_1^2 k_2^2 q_2 m_1^2 + 3 b_3 q_1^2 b_4^2 m_1^3 + 3 b_3 q_1^2 b_4^2 m_1^2 k_1 \\
&\quad - 6 b_4^3 k_1^2 q_1^3 k_2 q_2 m_1^2 - 6 b_4^3 k_1^3 q_1^3 k_2 q_2 m_1)/(2 b_3 (p_2^2 m_2 k_2 b_3^2 \\
&\quad - k_2 p_2 m_1 q_1 b_3 b_4 - 2 m_2 p_2 m_1 q_1 b_3 b_4 - b_4 m_2 p_2 k_1 q_1 b_3 \\
&\quad + b_4^2 q_1^2 m_1 k_1 + p_2^2 m_2^2 b_3^2 + b_4^2 m_1^2 q_1^2)),
\end{aligned}
$$

$$
\begin{aligned}
w_2 &= -(6 b_3^3 p_1 p_2^3 m_2^2 k_2^2 k_1 + 6 b_3^3 p_1 p_2^3 m_2 k_1 k_2^3 - 6 b_3^2 p_1 p_2^2 m_2^2 b_4 k_1^2 q_1 k_2 \\
&\quad - 6 b_3^2 p_1 p_2^2 m_2 b_4 k_1^2 q_1 k_2^2 - 4 b_3^2 p_2^3 m_2 k_2^4 q_2 b_4 - 4 b_3^2 p_2^3 m_2^2 k_2^3 q_2 b_4 \\
&\quad - 3 b_3^2 p_2^2 b_4 m_2^2 k_2 - 3 b_3^2 p_2^2 m_2^3 b_4 + 8 b_3 p_2^2 b_4^2 m_2 m_1 q_1 k_2^3 q_2 \\
&\quad - 6 b_3 p_2^2 m_2^2 k_2^2 q_2 k_1 q_1 b_4^2 - 2 b_3 p_2^2 m_2 b_4^2 k_1 q_1 k_2^3 q_2 \\
&\quad + 4 b_3 p_2^2 b_4^2 q_1 q_2 k_2^4 m_1 + 3 b_3 p_2 q_1 b_4^2 m_2^2 k_1 + 6 b_3 p_2 m_2^2 b_4^2 m_1 q_1 \\
&\quad + 3 b_3 p_2 q_1 b_4^2 m_2 k_2 m_1 - 4 p_2 b_4^3 q_1^2 q_2 k_2^3 m_1 k_1 - 4 p_2 b_4^3 q_1^2 q_2 k_2^3 m_1^2 \\
&\quad + 6 p_2 m_2 b_4^3 k_1^2 q_1^2 k_2^2 q_2 + 6 p_2 m_2^2 b_4^3 k_1^2 q_1^2 k_2 q_2 - 3 q_1^2 b_4^3 m_2 m_1 k_1 \\
&\quad - 3 q_1^2 b_4^3 m_2 m_1^2)/(2 b_4 (p_2^2 m_2 k_2 b_3^2 - k_2 p_2 m_1 q_1 b_3 b_4 \\
&\quad - 2 m_2 p_2 m_1 q_1 b_3 b_4 - b_4 m_2 p_2 k_1 q_1 b_3 + b_4^2 q_1^2 m_1 k_1 \\
&\quad + p_2^2 m_2^2 b_3^2 + b_4^2 m_1^2 q_1^2))
\end{aligned} \tag{3.4}
$$

Using the relations (2.5)-(2.10) with solutions (3.3)-(3.4), many solutions can be given to equation (3.1).
With Fig. 1-Fig. 3, in order to help readers understand better about our solutions, we generate some illustrations of achieved solutions as follows:

Fig. 1: when $q_1 = \frac{1}{2}$, $p_1 = -\frac{1}{2}, q_2 = \frac{1}{2}, p_2 = \frac{1}{2}$ with (3.3),

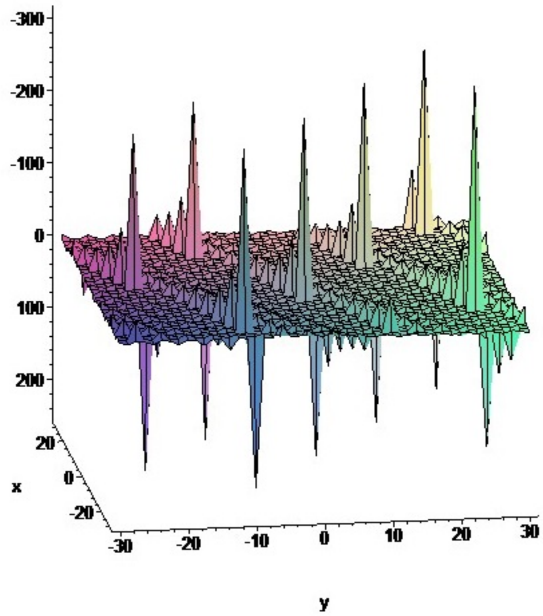

FIGURE 1. $l_1 = 2.8, k_2 = 1.1, k_1 = 1.8, m_2 = 3.6,$
$m_1 = 3.8, z = 2.6, b_0 = 1.8, b_4 = 1.8, t = 2.3$

Fig. 2: when $p_1 = -1$, $q_1 = -1, p_2 = -1, q_2 = 1$ with (3.3),

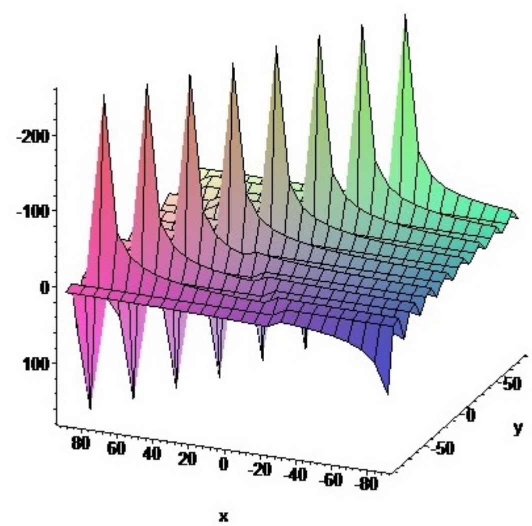

FIGURE 2. $l_1 = 1.4, k_1 = 2.1, m_1 = 1.6, k_2 = 2.4,$
$m_2 = -3, z = 2.8, b_0 = 1.4, b_4 = -3.2, t = 2$

Fig. 3: when $p_1 = -1$, $q_1 = -1, p_2 = -1, q_2 = 1$ with (3.4),

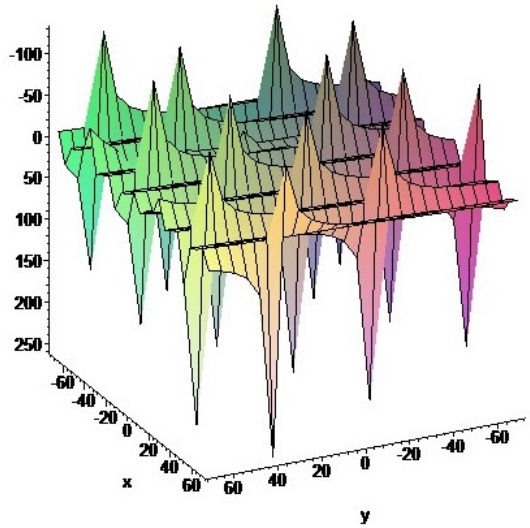

FIGURE 3. $k_2 = 1.2, k_1 = 2.3, m_1 = 1.1, m_2 = 2.3,$
$b_3 = 0.8, z = -4, b_0 = 0.7, b_4 = 3, t = 1$

4. Implementation of Modified Double Sub-equation Method to second type extended (3+1)-dimensional Jimbo-Miwa equation

In this part, we implement the referred method to second type extended (3+1)-dimensional Jimbo-Miwa equation which is very popular in nonlinear sciences and applied sciences and employed in modelling shallow water waves, is expressed in the way that

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2(u_{yt} + u_{xt} + u_{zt}) - 3u_{xz} = 0 \tag{4.1}$$

in the literature [47]. Then, Manafian presented some solutions for Eq. (4.1) in [34] and Wazwaz also obtained some exact solutions of Eq. (4.1) in paper [21]. In [48], Xu et al. obtained multi-exponential wave solutions to Eq. (4.1). For other studies made on Eq. (4.1), please see [25, 38, 6, 53, 26, 27, 28, 17, 35]. In [35], Moleleki et al. presented conservation laws and some solutions to (4.1).

As we pointed out in section 2, we take the solution in the way that

$$u(x, y, t) = b_0 + \frac{b_1\varphi(\xi) + b_2\psi(\eta)}{b_3 + b_4\varphi(\xi)\psi(\eta)} \tag{4.2}$$

where b_0, b_1, b_2, b_3, b_4 are arbitrary constants and $\xi = k_1x + l_1y + m_1z + w_1t$ and $\eta = k_2x + l_2y + m_2z + w_2t$. Taking Eqs. (2.3)-(2.4) into consideration and using Eq. (4.2) into Eq. (4.1) presents us a new expression which consists of φ, ψ. Then, in resulting expression, taking coefficients of the terms $\varphi^m\psi^n$ $(m = 0, 1, 2, ..., n = 0, 1, 2, ...)$ to zero provides us a system of equations. Solution and analysis of this system enable us to obtain following two classes of solutions:

Case I:

$$b_1 = -2b_3k_1p_1, \quad b_2 = -2p_2k_2b_3, \quad b_4 = 0, \quad l_2 = -\frac{k_2l_1}{k_1} \quad (4.3)$$
$$m_1 = -\frac{2\left(2l_1k_1^3p_1q_1 - w_1k_1 - w_1l_1\right)}{3k_1 - 2w_1}, \quad m_2 = \frac{2k_2\left(2k_2^3l_1p_2q_2 + w_2k_1 - l_1w_2\right)}{k_1(3k_2 - 2w_2)}$$

Case II:

$$b_1 = -2b_3k_1p_1 + 2b_4k_2q_2, \quad b_2 = 2b_4k_1q_1 - 2b_3k_2p_2,$$

$$\begin{aligned} l_1 = & -(-4k_1^5p_1^3q_1^3b_3b_4^3m_1 + 6k_2^3p_2^3k_1^2p_1^3b_3^4m_1 + 3p_1^2m_1k_1^3b_3b_4^3q_1^2 \\ & +18k_2p_2k_1^5p_1^3b_3^2b_4^2q_1^2 + 6p_1^2m_1k_1b_3^3b_4p_2^2k_2^2 - 26k_1^4q_1p_1^3k_2^2p_2^2b_3^3b_4 \\ & -38k_1^2p_1^2p_2^3k_2^4b_3^3b_4q_2 + 24k_1p_1p_2^3k_2^5b_3^2b_4^2q_2^2 \\ & -18k_1^2q_1p_1k_2^3p_2^2b_3b_4^3m_2q_2^2 - 22k_1^2q_1p_1k_2^4p_2^2b_3b_4^3q_2^2 \\ & +6m_1p_1^2k_1^4k_2b_4^4q_1^3q_2 + 6m_2k_1^3k_2^2p_2b_4^4p_1q_1^2q_2^2 \\ & +14k_1^4p_1^3q_1^2b_3^2b_4^2m_1p_2k_2 - 16k_2^2p_2^2k_1^3p_1^3b_3^3m_1b_4q_1 \\ & -9p_1^2m_1k_1^2b_3^2b_4^2q_1p_2k_2 - 34k_2^2q_2p_1^2k_1^4p_2b_3b_4^3q_1^2 \\ & -16k_2^4q_2p_1^2k_1p_2^3b_3^3b_4m_1 + 34k_2^3q_2p_1^2k_1^2p_2^2b_3^2b_4^2m_1q_1 \\ & -24k_2^2q_2p_1^2k_1^3p_2b_3b_4^3m_1q_1^2 + 4k_1^5p_1^3q_1^2b_3^2b_4^2p_2m_2 \\ & +4k_2^5q_2^2p_2^3b_3^2b_4^2m_1p_1 - 3k_2^3p_2^2m_1p_1b_3^2b_4^2q_2 + 6m_2p_2^2k_1^2p_1^2b_3^3b_4k_2 \\ & -3m_2p_2k_1^3p_1^2b_3^2b_4^2q_1 + 6b_4^4k_1^3q_1^2k_2^3p_1q_2^2p_2 - 4k_2^6q_2^3p_2^3b_3b_4^3 \\ & +12k_1^3p_1^3p_2^3k_2^3b_3^4 - 4k_2^5q_2^3p_2^3b_3b_4^3m_2 + 6p_1^2k_1^5k_2b_4^4q_1^3q_2 \\ & +6k_1^3p_1^3p_2^3k_2^2b_3^4m_2 + 62k_1^3q_1p_1^2k_2^3p_2^2b_3^2b_4^2q_2 - 22k_1^2p_1^2p_2^3k_2^3b_3^3m_2b_4q_2 \\ & +20k_2^4q_2^2p_2^3b_3^2b_4^2m_2k_1p_1 - 10k_1^4q_1p_1^3k_2p_2^2b_3^3b_4m_2 \\ & +28k_1^3q_1p_1^2k_2^2p_2^2b_3^2b_4^2m_2q_2 - 4k_1^6p_1^3q_1^3b_3b_4^3 + 3k_2^3q_2^2b_4^3p_2^2m_2b_3 \\ & -4k_2^4q_2^2p_2^2b_3b_4^3m_1p_1k_1q_1 + 3k_2^2p_2m_1p_1b_3b_4^3q_2k_1q_1 \\ & -10k_1^4p_1^2q_1^2b_3b_4^3p_2m_2k_2q_2 - 9p_2^2m_2k_2^2q_2b_3^2b_4^2p_1k_1 \\ & +3p_2m_2k_2q_2b_3b_4^3p_1k_1^2q_1)/(p_1b_4k_1k_2(p_1p_2b_3^2 - b_4^2q_1q_2) \\ & \times(-b_4k_1q_1 + b_3k_2p_2)), \end{aligned}$$

$$
\begin{aligned}
w_1 \quad = \quad & p_1k_1^2(-8b_3p_1^3k_1^6q_1^4b_4^3m_1 + 9k_1q_1^2q_2^2b_4^4k_2^3m_1p_2 + 12p_1^2k_1^5q_1^4q_2b_4^4m_1k_2 \\
& +12p_1p_2k_1^4q_1^3q_2^2b_4^4k_2^2m_2 - 20b_3^3p_1^3p_2^2k_1^5q_1^2k_2b_4m_2 - 8b_3p_1^3k_1^7q_1^4b_4^3 \\
& -32b_3^3p_1^3p_2^2k_1^4q_1^2k_2^2m_1b_4 - 32b_3^3p_1^2p_2^3k_1^2q_1k_2^4q_2b_4m_1 \\
& -44b_3^3p_1^2p_2^3k_1^3q_1k_2^3m_2b_4q_2 - 76b_3^3p_1^2p_2^3k_1^3q_1k_2^4b_4q_2 \\
& +3b_3^3p_1^2p_2^2k_1^2q_1m_1b_4k_2^2 + 12b_3^3p_1^2p_2^2k_1^3q_1m_2b_4k_2 + 12p_1^2k_1^6q_1^4q_2b_4^4k_2 \\
& +12p_1p_2k_1^4q_1^3q_2^2b_4^4k_2^3 - 9p_1q_2q_1^3b_4^4k_1^3m_1k_2 + 6b_3b_4^3k_1^4q_1^3p_1^2m_1 \\
& +12b_3^4p_1^3p_2^3k_1^4q_1k_2^2m_2 + 12b_3^4p_1^3p_2^3k_1^3q_1k_2^3m_1 + 24b_3^4p_1^3p_2^3k_1^4q_1k_2^3 \\
& -52b_3^3p_1^3p_2^2k_1^5q_1^2k_2^2b_4 + 9b_3^3k_2^4q_2m_1b_4p_2^3p_1 + 36b_3^2p_1^3p_2k_1^6q_1^3k_2b_4^2 \\
& +8b_3^2p_1^3p_2k_1^6q_1^3b_4^2m_2 - 6b_3^2p_1^2p_2b_4^2k_1^4q_1^2m_2 - 8b_3p_2^3k_1q_1k_2^6q_2^3b_4^3 \\
& -9b_3p_2^2k_2^4q_2^2m_1b_4^3q_1 + 28b_3^2p_1^3p_2k_1^5q_1^3b_4^2m_1k_2 \\
& +68b_3^2p_1^2p_2^2k_1^3q_1^2q_2b_4^2k_2^3m_1 + 56b_3^2p_1^2p_2^2k_1^4q_1^2q_2b_4^2k_2^2m_2 \\
& +124b_3^2p_1^2p_2^2k_1^4q_1^2q_2b_4^2k_2^3 - 9b_3^2p_1^2p_2k_1^3q_1^2m_1b_4^2k_2 \\
& +40b_3^2p_1p_2^3k_1^2q_1k_2^4q_2^2b_4^2m_2 + 8b_3^2p_1p_2^3k_1q_1k_2^5q_2^2b_4^2m_1 \\
& +48b_3^2p_1p_2^3k_1^2q_1k_2^5b_4^2q_2^2 - 18b_3^2p_1p_2^2k_1^2q_1m_2k_2^2q_2b_4^2 \\
& -15b_3^2p_1p_2^2k_1q_1k_2^3m_1b_4^2q_2 - 20b_3p_1^2p_2q_2q_1^3b_4^3k_1^5k_2m_2 \\
& -48b_3p_1^2p_2k_1^4q_1^3q_2b_4^3k_2^2m_1 - 68b_3p_1^2p_2k_1^5q_1^3q_2b_4^3k_2^2 \\
& -36b_3p_1p_2^2k_1^3q_1^2q_2^2b_4^3k_2^3m_2 - 44b_3p_1p_2^2k_1^3q_1^2q_2^2b_4^3k_2^4 \\
& -8b_3p_1p_2^2k_1^2q_1^2q_2^2b_4^3k_2^4m_1 + 6b_3b_4^3k_1^3q_1^2p_1p_2m_2k_2q_2 \\
& +15b_3b_4^3p_2k_2^2q_2k_1^2q_1^2m_1p_1 - 8b_3p_2^3k_1q_1k_2^5q_2^3b_4^3m_2 \\
& +6b_3b_4^3k_1q_1k_2^3p_2^2m_2q_2^2)/(-4k_1^5p_1^3q_1^3b_3b_4^3m_1 + 6k_2^3p_2^3k_1^2p_1^3b_3^4m_1 \\
& +3p_1^2m_1k_1^3b_3b_4^3q_1^2 + 24k_2p_2k_1^5p_1^3b_3^2b_4^2q_1^2 + 6p_1^2m_1k_1b_3^3b_4p_2^2k_2^2 \\
& -32k_1^4q_1p_1^3k_2^2p_2^2b_3^3b_4 - 32k_1^2p_1^2p_2^3k_2^4b_3^3b_4q_2 + 24k_1p_1p_2^3k_2^5b_3^2b_4^2q_2^2 \\
& -18k_1^2q_1p_1k_2^3p_2^2b_3b_4^3m_2q_2^2 - 28k_1^2q_1p_1k_2^4p_2^2b_3b_4^3q_2^2 \\
& +6m_2k_1^3k_2^2p_2b_4^4p_1q_1^2q_2^2 + 20k_1^4p_1^3q_1^2b_3^2b_4^2m_1p_2k_2 \\
& -22k_2^2p_2^2k_1^3p_1^3b_3^3m_1b_4q_1 - 9p_1^2m_1k_1^2b_3^2b_4^2q_1p_2k_2 \\
& -28k_2^2q_2p_1^2k_1^4p_2b_3b_4^3q_1^2 - 10k_2^4q_2p_1^2k_1p_2^3b_3^3b_4m_1 \\
& +28k_2^3q_2p_1^2k_1^2p_2^2b_3^2b_4^2m_1q_1 - 18k_2^2q_2p_1^2k_1^3p_2b_3b_4^3m_1q_1^2 \\
& +4k_1^5p_1^3q_1^2b_3^2b_4^2p_2m_2 + 4k_2^5q_2^2p_2^3b_3^2b_4^2m_1p_1 - 3k_2^3p_2^2m_1p_1b_3^2b_4^2q_2 \\
& +6m_2p_2^2k_1^2p_1^2b_3^3b_4k_2 - 3m_2p_2k_1^3p_1^2b_3^2b_4^2q_1 + 12b_4^4k_1^3q_1^2k_2^3p_1q_2^2p_2 \\
& +56k_1^3q_1p_1^2k_2^3p_2^2b_3^2b_4^2q_2 + 12k_1^3p_1^3p_2^3k_2^3b_3^4 + 6k_1^3p_1^3p_2^3k_2^2b_3^4m_2 \\
& -22k_1^2p_1^2p_2^3k_2^3b_3^3m_2b_4q_2 + 3k_2^2p_2m_1p_1b_3b_4^3q_2k_1q_1 \\
& -4k_2^6q_2^3p_2^3b_3b_4^3 + 3k_2^3q_2^2b_4^3p_2^2m_2b_3 + 6k_2^3p_2q_2^2b_4^4m_1p_1k_1^2q_1^2 \\
& +20k_2^4q_2^2p_2^3b_3^2b_4^2m_2k_1p_1 - 10k_1^4q_1p_1^3k_2p_2^2b_3^3b_4m_2 \\
& +28k_1^3q_1p_1^2k_2^2p_2^2b_3^2b_4^2m_2q_2 - 4k_2^5q_2^3p_2^3b_3b_4^3m_2 \\
& -10k_2^4q_2^2p_2^2b_3b_4^3m_1p_1k_1q_1 - 10k_1^4p_1^2q_1^2b_3b_4^3p_2m_2k_2q_2 \\
& -9p_2^2m_2k_2^2q_2b_3^2b_4^2p_1k_1 + 3p_2m_2k_2q_2b_3b_4^3p_1k_1^2q_1 \\
& -4k_1^6p_1^3q_1^3b_3b_4^3),
\end{aligned}
$$

$$
\begin{aligned}
w_2 \;=\;& p_2k_2^2(-8b_3p_2^3k_2^7q_2^4b_4^3+48b_3^2p_1^3p_2k_2^2k_1^5b_4^2q_1^2q_2-8b_3p_2^3k_2^6q_2^4b_4^3m_2\\
&+68b_3^2p_1^2p_2^2k_1^3q_1k_2^3b_4^2m_2q_2^2+124b_3^2p_1^2p_2^2k_2^4q_2^2b_4^2k_1^3q_1\\
&+56b_3^2p_1^2p_2^2k_2^4k_1^2m_1q_2^2q_1b_4^2+28b_3^2p_1p_2^3k_2^5q_2^3b_4^2m_2k_1\\
&-8b_3p_1^3k_1^5q_1^3b_4^3m_1k_2q_2-8b_3p_1^2p_2k_1^4q_1^2k_2^2b_4^3m_2q_2^2\\
&-36b_3p_1^2p_2k_2^3k_1^3m_1b_4^3q_1^2q_2^2-44b_3p_1^2p_2k_1^4q_1^2k_2^3b_4^3q_2^2\\
&-20b_3p_1p_2^2q_2^3q_1b_4^3k_2^5k_1m_1-68b_3p_1p_2^2k_2^5q_2^3b_4^3k_1^2q_1\\
&-48b_3p_1p_2^2k_1^2q_1k_2^4b_4^3m_2q_2^3-18b_4^2k_1^2q_1k_2^2p_1^2q_2b_3^2m_1p_2\\
&+12p_2^2b_4^4k_2^5q_2^4m_2k_1q_1+12p_1p_2k_2^4q_2^3b_4^4k_1^3q_1^2\\
&+12b_3^4p_1^3p_2^3k_2^4k_1^2m_1q_2+12b_3^4p_1^3p_2^3k_1^3k_2^3m_2q_2\\
&-52b_3^3p_1^2p_2^3k_1^2k_2^5b_4q_2^2+8b_3^2p_1p_2^3k_2^6q_2^3b_4^2m_1+6b_4^3k_2^4p_2^2q_2^3b_3m_2\\
&+36b_3^2p_1p_2^3k_2^6q_2^3b_4^2k_1-8b_3p_1^3k_2k_1^6b_4^3q_1^3q_2+9b_3^3b_4k_1^4q_1p_1^3p_2m_2\\
&-15b_3^3b_4^2k_1^3q_1p_1^2p_2m_2k_2q_2+15b_4^3k_1^2q_1k_2^2p_1q_2^2b_3p_2m_2\\
&+3b_3^3q_2b_4k_1^2p_1^2k_2^2p_2^2m_2-9b_3^2q_2^2b_4^2k_1p_1k_2^3p_2^2m_2\\
&-6b_4^2k_2^4p_2^2q_2^2b_3^2m_1p_1+12p_2^2k_2^6q_2^4b_4^4q_1k_1+24b_3^4p_1^3p_2^3k_1^3k_2^4q_2\\
&+9b_4^4k_1^3q_1^2p_1q_2^2m_2k_2-9b_3k_1^4p_1^2q_1^2b_4^3m_2q_2-9q_2^3q_1b_4^4k_2^3m_2k_1p_2\\
&+12p_1p_2k_2^4q_2^3k_1^2b_4^4m_1q_1^2-44b_3^2p_1^3p_2^2k_2^3k_1^3m_1q_2q_1b_4\\
&-76b_3^3p_1^3p_2^2k_1^4q_1k_2^3b_4q_2-32b_3^3p_1^3p_2^2k_1^4k_2^2m_2q_2q_1b_4\\
&-32b_3^3p_1^2p_2^3k_1^2k_2^4m_2b_4q_2^2-20b_3^3p_1^2p_2^3k_2^5q_2^2k_1b_4m_1\\
&+40b_3^2p_1^3p_2k_2^2k_1^4m_1q_1^2q_2b_4^2+8b_3^2p_1^3p_2k_1^5q_1^2k_2b_4^2m_2q_2\\
&+6b_4^3k_1^3q_1^2k_2p_1^2q_2b_3m_1+12b_3^3q_2b_4k_1p_1^2k_2^3p_2^2m_1\\
&+6b_4^3k_2^3q_2^2m_1p_1k_1q_1p_2b_3)/(-4k_1^5p_1^3q_1^3b_3b_4^3m_1+6k_2^3p_2^3k_1^2p_1^3b_3^4m_1\\
&+3p_1^2m_1k_1^3b_3b_4^3q_1^2+24k_2p_2k_1^5p_1^3b_3^2b_4^2q_1^2+6p_1^2m_1k_1b_3^3b_4p_2^2k_2^2\\
&-32k_1^4q_1p_1^3k_2^2p_2^2b_3^3b_4-32k_1^2p_1^2p_2^3k_2^4b_3^3b_4q_2+24k_1p_1p_2^3k_2^5b_3^2b_4^2q_2^2\\
&-18k_1^2q_1p_1k_2^3p_2^2b_3b_4^3m_2q_2^2-28k_1^2q_1p_1k_2^4p_2^2b_3b_4^3q_2^2\\
&+6m_2k_1^3k_2^2p_2b_4^4p_1q_1^2q_2^2+20k_1^4p_1^3q_1^2b_3^2b_4^2m_1p_2k_2\\
&-22k_2^2p_2^2k_1^3p_1^3b_3^3m_1b_4q_1-9p_1^2m_1k_1^2b_3^2b_4^2q_1p_2k_2\\
&-28k_2^2q_2p_1^2k_1^4p_2b_3b_4^3q_1^2-10k_2^4q_2p_1^2k_1p_2^3b_3^3b_4m_1\\
&+28k_2^3q_2p_1^2k_1^2p_2^2b_3^2b_4^2m_1q_1-18k_2^2q_2p_1^2k_1^3p_2b_3b_4^3m_1q_1^2\\
&+4k_1^5p_1^3q_1^2b_3^2b_4^2p_2m_2+4k_2^5q_2^2p_2^3b_3^2b_4^2m_1p_1-4k_2^6q_2^3p_2^3b_3b_4^3\\
&-3k_2^3p_2^2m_1p_1b_3^2b_4^2q_2+6m_2p_2^2k_1^2p_1^2b_3^3b_4k_2-3m_2p_2k_1^3p_1^2b_3^2b_4^2q_1\\
&+12b_4^4k_1^3q_1^2k_2^3p_1q_2^2p_2-4k_2^5q_2^3p_2^3b_3b_4^3m_2+6k_1^3p_1^3p_2^3k_2^2b_3^4m_2\\
&-10k_2^4q_2^2p_2^2b_3b_4^3m_1p_1k_1q_1+3k_2^2p_2m_1p_1b_3b_4^3q_2k_1q_1\\
&+56k_1^3q_1p_1^2k_2^3p_2^2b_3^2b_4^2q_2-22k_1^2p_1^2p_2^3k_2^3b_3^3m_2b_4q_2\\
&+20k_2^4q_2^2p_2^3b_3^2b_4^2m_2k_1p_1-10k_1^4q_1p_1^3k_2p_2^2b_3^3b_4m_2\\
&+28k_1^3q_1p_1^2k_2^2p_2^2b_3^2b_4^2m_2q_2+12k_1^3p_1^3p_2^3k_2^3b_3^4-4k_1^6p_1^3q_1^3b_3b_4^3\\
&-10k_1^4p_1^2q_1^2b_3b_4^3p_2m_2k_2q_2-9p_2^2m_2k_2^2q_2b_3^2b_4^2p_1k_1\\
&+3k_2^3q_2^2b_4^3p_2^2m_2b_3+6k_2^3p_2q_2^2b_4^4m_1p_1k_1^2q_1^2\\
&+3p_2m_2k_2q_2b_3b_4^3p_1k_1^2q_1),
\end{aligned}
$$

$$
\begin{aligned}
l_2 \;=\; & (-4k_1^5p_1^3q_1^3b_3b_4^3m_1 + 6k_2^3p_2^3k_1^2p_1^3b_3^4m_1 + 3p_1^2m_1k_1^3b_3b_4^3q_1^2 \\
& +24k_2p_2k_1^5p_1^3b_3^2b_4^2q_1^2 + 6p_1^2m_1k_1b_3^3b_4p_2^2k_2^2 \\
& -38k_1^4q_1p_1^3k_2^2p_2^2b_3^3b_4 - 26k_1^2p_1^2p_2^3k_2^4b_3^3b_4q_2 + 18k_1p_1p_2^3k_2^5b_3^2b_4^2q_2^2 \\
& -24k_1^2q_1p_1k_2^3p_2^2b_3b_4^3m_2q_2^2 - 34k_1^2q_1p_1k_2^4p_2^2b_3b_4^3q_2^2 \\
& +20k_1^4p_1^3q_1^2b_3^2b_4^2m_1p_2k_2 - 22k_2^2p_2^2k_1^3p_1^3b_3^3m_1b_4q_1 \\
& -9p_1^2m_1k_1^2b_3^2b_4^2q_1p_2k_2 - 22k_2^2q_2p_1^2k_1^4p_2b_3b_4^3q_1^2 \\
& -10k_2^4q_2p_1^2k_1p_2^3b_3^3b_4m_1 + 28k_2^3q_2p_1^2k_1^2p_2^2b_3^2b_4^2m_1q_1 \\
& -18k_2^2q_2p_1^2k_1^3p_2b_3b_4^3m_1q_1^2 + 4k_1^5p_1^3q_1^2b_3^2b_4^2p_2m_2 \\
& +4k_2^5q_2^2p_2^3b_3^2b_4^2m_1p_1 - 3k_2^3p_2^2m_1p_1b_3^2b_4^2q_2 + 6m_2p_2^2k_1^2p_1^2b_3^3b_4k_2 \\
& -3m_2p_2k_1^3p_1^2b_3^2b_4^2q_1 + 6b_4^4k_1^3q_1^2k_2^3p_1q_2^2p_2 + 6b_4^4k_2^4p_2^2q_2^3m_2k_1q_1 \\
& -4k_2^6q_2^3p_2^3b_3b_4^3 + 12k_1^3p_1^3p_2^3k_2^3b_3^4 - 4k_2^5q_2^3p_2^3b_3b_4^3m_2 \\
& +6k_1^3p_1^3p_2^3k_2^2b_3^4m_2 + 62k_1^3q_1p_1^2k_2^3p_2^2b_3^2b_4^2q_2 - 16k_1^2p_1^2p_2^3k_2^3b_3^3m_2b_4q_2 \\
& +14k_2^4q_2^2p_2^3b_3^2b_4^2m_2k_1p_1 - 16k_1^4q_1p_1^3k_2p_2^2b_3^3b_4m_2 \\
& +34k_1^3q_1p_1^2k_2^2p_2^2b_3^2b_4^2m_2q_2 - 4k_1^6p_1^3q_1^3b_3b_4^3 + 6b_4^4k_2^5p_2^2q_2^3k_1q_1 \\
& +3k_2^3q_2^2b_4^3p_2^2m_2b_3 - 10k_2^4q_2^2p_2^2b_3b_4^3m_1p_1k_1q_1 \\
& +3k_2^2p_2m_1p_1b_3b_4^3q_2k_1q_1 + 6k_2^3p_2q_2^2b_4^4m_1p_1k_1^2q_1^2 \\
& -4k_1^4p_1^2q_1^2b_3b_4^3p_2m_2k_2q_2 - 9p_2^2m_2k_2^2q_2b_3^2b_4^2p_1k_1 \\
& +3p_2m_2k_2q_2b_3b_4^3p_1k_1^2q_1)/(6k_1p_2k_2b_4(p_1p_2b_3^2 - b_4^2q_1q_2) \\
& \times(-b_4k_2q_2 + b_3k_1p_1)(-k_2^2q_2p_2 + p_1k_1^2q_1))
\end{aligned}
$$

With same process used in previous section, using the relations (2.5)-(2.10) with solutions (4.3) and (4.4), many complexiton solutions can be given to equation (4.1).

With Fig. 4 and -Fig. 5, in order to help readers understand better about our solutions, we generate some illustrations of achieved solutions as follows:

Fig. 4: when $q_1 = \frac{1}{2}$, $p_1 = -\frac{1}{2}, q_2 = \frac{1}{2}, p_2 = \frac{1}{2}$ with (4.3),

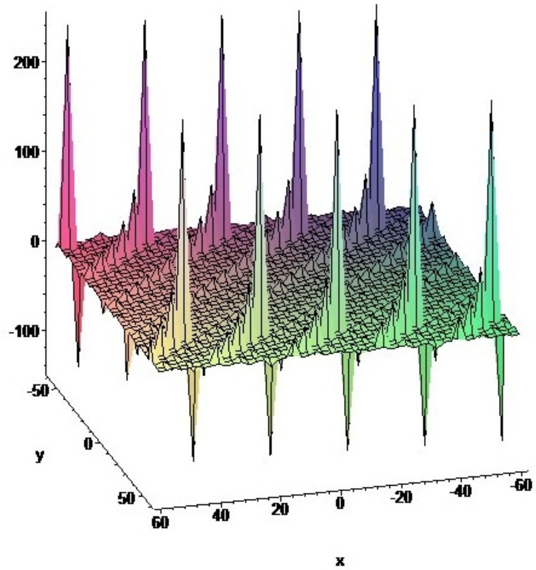

FIGURE 4. $l_1 = 1, k_2 = 0.8, k_1 = -0.1, w_1 = -1.7,$
$w_2 = -1.4, b_3 = 1.4, z = 0.3, b_0 = -0.2, t = 0.2$

Fig. 5: when $p_1 = -1$, $q_1 = -1, p_2 = -1, q_2 = 1$ with (4.4),

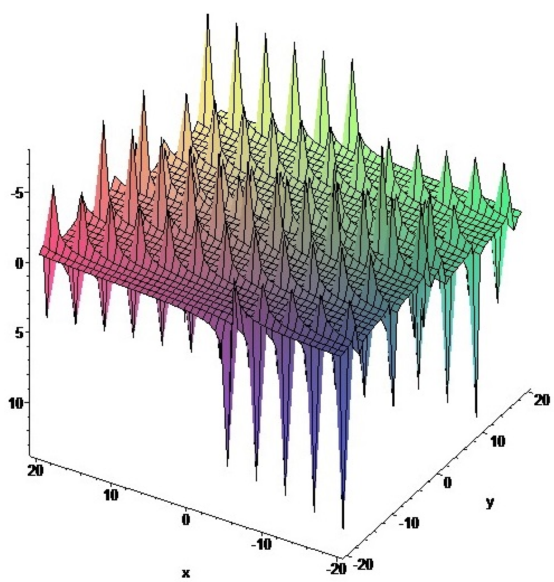

FIGURE 5. $k_2 = 0.3, k_1 = 0.1, m_2 = 0.2, m_1 = 0.4,$
$b_3 = 0.4, z = 0.3, b_0 = 0.4, b_4 = 0.7, t = 0.1$

5. Conclusion

Referred extended equations are pretty new and need to be investigated and solved more than other commonly known nonlinear partial differential equations. Throughout the paper, we presented complexiton waves to extended (3+1)-dimensional Jimbo-Miwa equations through modified double sub-equation method which presents a way for obtaining two different types of function solutions together in solution. Obtained set of solutions are really wide and with some specific choices of given parameters, more concise ones can easily be obtained. Form of solution which is rational and abundance of parameters in solution makes the method more comprehensive than many other techniques used in literature. With proper choices, method also enables us to obtain exponential and trigonometric wave solutions individually. We hope our results, graphs and deductions to be useful to scientists in their future studies will be made in applied sciences.

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