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Leader-Following consensus of chaotic fractional-order multi-agent systems using distributed adaptive protocols

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Abstract. In this study, consensus problem of fractional-order multi-agent chaotic systems is considered. By utilizing the fractional-order derivative in the sense of Caputo and classical stability theorem of linear fractional order systems and also algebraic graph theory, sufficient conditions are presented to ensure the consensus for fractional multi-agent systems. Distributed adaptive protocols of each agent using local information is designed and detailed analysis of the leader-following consensus is presented. Some numerical simulation examples are given to demonstrate the effectiveness of the proposed results.

Keywords: Fractional-order, Chaos, Multi-agent systems, Leader-following, Algebraic graph theory.

2020 Mathematics subject classification: 34H10; 34P60; 37N35.

1. Introduction

The last decade, widely and rapidly growing literature concerned with multi-agent systems. As one of the most typical collective behaviors

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of multi-agent systems, consensus problem has attracted numerous researches from various perspectives. Such as formation control [1], flocking [2], sensor network[3], mobile robots[4] and many other fields. Consensus means the agreement of all agents in a multi-agent systems. In a leader-following consensus, the control signals of agents are appropriately selected such that their state trajectories follow the leader state, which can be achieved through local information exchanging among agents. Most of works on consensus of multi-agent systems focus on integer-order dynamics, such as consensus algorithms of first-order dynamic systems [5, 6, 7, 8] as well as second-order dynamics [9] or even high order dynamic systems [10], however many phenomena cannot be accurately described by integer-order dynamics, such as macromolecule fluids and porous media and a typical example of fractional-order model is the relationship between the heat flow and the temperature in the heat diffusion of a semi-infinite solid [11], while each individual agent can be better explained by fractional-order dynamics where the operator order is an arbitary real number as shown in [12]. Also, many systems in the nature can be explained more precisely modeled by a coordinated behavior of agents with fractional-order dynamics. For instance it can be mentioned the diffusion process, such as chemotaxis behavior, food seeking of microbes ,the collective motion of bacteria in lubrications [13, 14] and the effect of the frequency in induction machines [15] amongst others. Moreover, in comparison with integer-order model, fractional-order model provides an excellent method in the description of memory and hereditary properties of various materials and processes. Based on these facts, it is very meaningful to study the consensus problems of fractional-order system. To the best of our knowledge, for this aim was first investigated in [16, 17, 18]. Successively, the convergence speed of consensus for fractional-order multi-agent systems was further researched in [19] and it the following work a varying-order $[0 < \alpha < 1, 1 < \alpha < 2]$ consensus algorithm was presented to expedite the speed of convergence [20]. Considering the consensus problem of fractional-order systems with input delays and heterogeneous multi-agent systems was investigated in [21, 22].

A pariculary interesting topic is the consensus of fractional-order multi-agent systems with static and dynamic leader, where it was reported that the leader-following configuration is an energy saving mechanism, which was found in many biological systems. Through the observer method, consensus of multi-agent systems with leader given by second-order model and the followers depicted by differential order less than two,

was concerned in [23]. In [24], leader-following consensus of fractional-order multi-agent systems under fixed topology has been studied. Moreover, consensus of leader-following multi-agent systems in [25] and [26] which depicted by general linear dynamics without and with input time delay by virtue of the even-triggered control method were studied, respectively.
As we all know, the behavior of some real fractional dynamical systems can be chaotic. Chaotic systems, as nonlinear deterministic systems, describe numerous complex and unusual behaviors. Therefore, it is meaningful to research the cooperative behaviors of fractional-order multi-agent chaotic systems such as fractional-order Lorenz systems [27], fractional-order Rössler systems [28], fractional-order chemical reactor model [29] and another same systems. The main contribution of this paper is an effective way to discuss the leader-following consensus problem of chaotic fractional-order multi-agent systems via adaptive control.
Adaptive control is a technique of applying some systems identification methods to obtain a model and using this model to design a controller [30]. The parameters of the controller are adjusted automatically during the operation.
The rest of this paper is organized as follows. In section 2, the graph theory notations, Caputo fractional operator, some necessary lemmas and theorems and problem statement is introduced. In 3, which includes system model and the main results for leader-following adaptive consensus of chaotic fractional-order multi-agent systems with $0 < \alpha < 1$. In 4, some simulations examples are appeared to as a proof of concepts. Finally, a brief conclusion is given in 5.

2. PRELIMINARIES

In this section, first of all, some basic definitions related to the algebraic graph theory and caputo fractional operator introduced. Then, some necessary lemmas and system model are presented for the use of following several sections.

2.1. **Graph theory.** The graph theorem is utilized to describe the topology of multi-agent systems with a leader and N agents. Let $G = (V, \varepsilon)$ be an undirected graph with the set of nodes $V = (v_0, v_1, v_2, \dots, v_N)$ and $\varepsilon = \{(v_i, v_j), v_i \neq v_j\} \subseteq V \times V$ is a set of edges. An edge denoted by(v_i, v_j) represents that the agent j can transfer its information to the agent i. An undirected graph has the property that $(v_i, v_j) \in \varepsilon$ implies $(v_j, v_i) \in \varepsilon$. The set of neighbors of node v_i is denoted by $N_i = \{v_j \in V : (v_i, v_j) \in \varepsilon\}$. The weighted adjacency matrix $W = [w_{ij}] \in R^{N\times N}$ of G with nonnegative entries is defined as $w_{ii} = 0$, $w_{ij} > 0$ if $(v_i, v_j) \in \varepsilon$ and $w_{ij} = 0$, otherwise. The degree matrix of G is $D = diag(d_1, \dots, d_N) \in R^{N\times N}$ where diagonal elements $d_i > 0$ if the i agent is a neighbor of the leader and $d_i = 0$, otherwise. The Laplacian matrix $L = [l_{ij}] \in R^{N\times N}$ of the weighted graph G is defined as $l_{ii} = \sum_{j\neq i} w_{ij}$ and $l_{ij} = -w_{ij}$ for $i \neq j$.

Lemma 2.1. [24] *For any undirected graph G, matrix $H = L + D$ is positive definite, if there is at least one directed path from v_0 (the leader) to all the other nodes.*

2.2. **Caputo fractional derivative.** In fractional calculus, the traditional definitions of the integral and derivative of a function are generalized from integer orders to real orders. Several definitions exist regarding the fractional operators of order $\alpha \geq 0$, Caputo and Riemann–Liouville (R–L) fractional operators are the two broadly used ones in various fields. But the definition of Caputo has been used in most engineering applications, because this definition of the initial conditions of fractional differential equations with Caputo derivatives take on the same form as for the integer- order differential equations. Therefore, we will apply Caputo fractional derivative in this paper to model the multi-agent systems dynamics and analyze the asymptotic consensus properties under the given consensus controller. The Caputo fractional derivative of $f(t)$ with order α is defined as follows [31]

$$ {}_{at}^{c\,\alpha}D = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}}\,\mathrm{d}\tau \tag{2.1} $$

where $n - 1 < \alpha < n$, $n \in \mathbf{Z}^+$, and $\Gamma(.)$ is the Gamma function

$$ \Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1}\,\mathrm{d}t $$

Because only the Caputo fractional operator is used in this study, a simple notation D^α is chosen as the Caputo fractional derivative operator

Theorem 2.2. [32] *A linear fractional order system $D^\alpha x = Ax$ is asymptotically stable if and only if $|arg(\lambda_i)| > \frac{\alpha\pi}{2}$ is satisfied for all eigenvalues λ_i matrix A. Furthermore, this system is stable if and only if $|arg(\lambda_i)| \geq \frac{\alpha\pi}{2}$ is satisfied for all eigenvalues λ_i matrix A and those critical eigenvalues that satisfy the condition $|arg(\lambda_i)| = \frac{\alpha\pi}{2}$ have geometric multiplicity one.*

3. Main results

In this section, the leader-following consensus problem of chaotic fractional multi-agent system is discussed, and a distributed adaptive protocol is designed to achieve consensus under an undirected interaction fixed graph. We first consider the fractional-order multi-agent system consisting of N agents and a leader. The dynamics of each agent is given by

$$D^{\alpha}x_i = Bx_i + G(x_i) + U \tag{3.1}$$

and the dynamics of the leader(labeled as $i = 0$) is depicted by

$$D^{\alpha}x_0 = Ax_0 + F(x_0) \tag{3.2}$$

where D^{α} means the Caputo fractional derivative of order α, $0 < \alpha < 1$ and $x_i = (x_{i1}, x_{i2}, \dots x_{iN}) \in R^N$, $U = (u_1, u_2, \dots u_N) \in R^N$ and $x_0 = (x_{01}, x_{02}, \dots x_{0N}) \in R^N$ represent the state of ith agent, the control input and the state of the leader, respectively. $A = (a_{ij})_{N\times N}$ is the system matrix and $F(x_0) = (f_1(x_0), f_2(x_0), \dots, f_N(x_0))^T$, $G(x_i) = (g_1(x_i), g_2(x_i), \dots, g_N(x_i))^T$ denote the nonlinear part of leader-following system.
The distributed adaptive control law for agent i is proposed as

$$\begin{aligned} u_i = {} & (a_{ij} - b_{ij})x_0 + f_i(x_0) - g_i(x_0) - \frac{\partial g_i}{\partial x_i}(x_0)(x_i - x_0) \\ & - (b_{ij} + I_{ij})(x_i - x_0) + \gamma \sum_{j\in N_i} w_{ij}(x_j - x_i) + \gamma d_i(x_0 - x_i) \end{aligned} \tag{3.3}$$

where γ is feedback control gain. The term $\sum_{j\in N_i} w_{ij}(x_j - x_i)$ denotes the information exchanges between i agent and those of its neighbors. The term $d_i(x_0 - x_i)$ denotes the information exchanges between i agent and the leader.

Remark 3.1. The leader's dynamic is independent of others. We take the different nonlinear dynamical functions $F(x)$ and $G(x)$ for leader and all the agents, respectively.

Definition 3.2. The leader-following consensus of systems 3.1 and 3.2 will be achieved if for each agent $i \in \{1, 2, \dots, N\}$ there is the appropriate control u_i of $\{x_j : j \in N_i\}$ such that the closed-loop system satisfies

$$\lim_{x\to\infty} \|x_i(t) - x_0(t)\| = 0, \qquad i = 1, 2, \dots, N$$

Now using the control law 3.3, we can mention the following theorem.

Theorem 3.3. *Consider the leader-following multi-agent systems 3.1 and 3.2 under the control law 3.3. And assuming that there is at least one path from the leader to all the other nodes then if*

$$\gamma > \frac{-1}{\lambda_{Max}}$$

where λ_{Max} is the largest eigenvalue of H, γ is feedback control gain, then all the agents follow the leader from any initial conditions.

Proof. Let us define the state error between the agent i and leader as $e_i = x_i - x_0$. Then, the dynamics of e_i is

$$\begin{aligned} D^{\alpha}e_i &= b_{ij}x_i + g_i(x_i) - a_{ij}x_0 - f_i(x_0) + u_i \\ &= b_{ij}x_i - a_{ij}x_0 + g_i(x_i) - f_i(x_0) - b_{ij}x_0 + b_{ij}x_0 + u_i \\ &= b_{ij}(x_i - x_0) + (b_{ij} - a_{ij})x_0 + g_i(x_i) - f_i(x_0) + u_i \end{aligned} \tag{3.4}$$

Now by substituting u_i from 3.3 in 3.4, we have

$$\begin{aligned} D^{\alpha}e_i = {} & b_{ij}(x_i - x_0) + (b_{ij} - a_{ij})x_0 + g_i(x_i) - f_i(x_0) \\ & + (a_{ij} - b_{ij})x_0 + f_i(x_0) - g_i(x_0) - \frac{\partial g_i}{\partial x_i}(x_0)(x_i - x_0) \\ & - (b_{ij} + I_{ij})(x_i - x_0) + \gamma \sum_{j \in N_i} w_{ij}(x_j - x_i) + \gamma d_i(x_0 - x_i) \\ & = b_{ij}(x_i - x_0) + g_i(x_i) - g_i(x_0) - \frac{\partial g_i}{\partial x_i}(x_0)(x_i - x_0) \\ & - (b_{ij} + I_{ij})(x_i - x_0) + \gamma \sum_{j \in N_i} w_{ij}(x_j - x_i) + \gamma d_i(x_0 - x_i) \\ & = b_{ij}e_i + g_i(x_i) - g_i(x_0) - \frac{\partial g_i}{\partial x_i}(x_0)e_i - (b_{ij} + I_{ij})e_i \\ & + \gamma \sum_{j \in N_i} w_{ij}(e_j - e_i) - \gamma d_i e_i \\ & i = 1, 2, \ldots, N \end{aligned} \tag{3.5}$$

By introducing $D = diag(d_1, d_2, \ldots, d_N)^T$, $e = (e_1, e_2, \ldots, e_N)^T$, JG denotes Jacobian matrix G and using the Laplacian L of graph G, we have

$$D^{\alpha}e = Be + G(x_i) - G(x_0) - JG(x_0)e + Ce - \gamma He \tag{3.6}$$

where $C = -(B + I)$ and $H = L + D$.

By using Taylor expansion around $x_i = x_0 + e$ the following statement is obtained

$$G(x_i) = G(x_0) + JG(x_0)e + o(\|e\|^2) \tag{3.7}$$

from 3.6 and 3.7, it can be obtained that

$$D^{\alpha}e = Be + G(x_0) + JG(x_0)e + o(\|e\|^2) \tag{3.8}$$

$$\begin{aligned} & - G(x_0) - JG(x_0)e + Ce - \gamma He + o(\|e\|^2) \\ & = Be + Ce - \gamma He + o(\|e\|^2) \\ & = -(I + \gamma H)e + o(\|e\|^2) \end{aligned} \tag{3.9}$$

Thus, according to theorem 2.2 system 3.9 is asymptotically stable if

$$|arg(\lambda_{i(-I-\gamma H)})| > \frac{\alpha\pi}{2}, \quad \forall i \in \{1, 2, \dots, N\}$$

If we select γ so that the following relation is established

$$-1 - \gamma\lambda_{i(H)} < 0, \quad \forall i \in \{1, 2, \dots, N\}$$

On the other hand, from Lemma 2.1, all eigenvalues of matrix H are positive which implies that

$$-\gamma < \frac{1}{\lambda_{i(H)}}$$

By definition, $\lambda_{Max} = Max(\lambda_i)$, all the agents follow the leader from any initial conditions, it's enough $\gamma > \frac{-1}{\lambda_{Max}}$ □

Corollary 3.4. *Consider the leader-following multi-agent systems 3.1 and 3.2 under the control law 3.3. If*

$$\gamma < -\frac{1}{\lambda_{min}}$$

where λ_{min} is the smallest eigenvalue of H, the agents never follow the leader.

4. Numerical example

In order to demonstrate the effectiveness of the theories, we will give two numerical examples in this section. Let us state the following example.

Example 4.1. Consider a multi-agent of chaotic fractional-order consisting of a leader and three agents and in order to facilitate the solution of our examples, without loss of generality, we assume that $v_0 = (x_0, y_0, z_0)$ and $v_i = (x_i, y_i, z_i)$, $i = 1, 2, \dots, N$ for all examples in this section. They are the state variables of the leader and state variables of the agents satisfying

$$\begin{cases} D^\alpha x_0 = a(y_0 - x_0) \\ D^\alpha y_0 = -x_0 z_0 + c y_0 \\ D^\alpha z_0 = x_0 y_0 - b z_0, \end{cases} \tag{4.1}$$

$$\begin{cases} D^\alpha x_i = a(y_i - x_i) \\ D^\alpha y_i = (c - a)x_i - x_i z_i + c y_i \\ D^\alpha z_i = x_i y_i - b z_i, \end{cases} \tag{4.2}$$

where $a = 35$, $b = 3$, $c = 28$.

Assume the Lü system 4.1 is the leader system [33] and the Chen system 4.2 are the agent systems [34]. Also, suppose the topology is described as in figure 1. For convenience, let $w_{ij} = 1(d_i = 1)$ if $w_{ij} > 0(d_i > 0)$

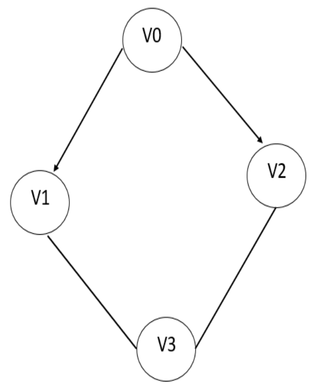


FIGURE 1. The topology of leader-following multi-agent system under the undirected graph.

and $w_{ij} = 0 (d_i = 0)$, otherwise .Thus, the Laplacian L and matrix D are as follows:

$$L = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{pmatrix}, \qquad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4.3}$$

A straight forward calculation shows the largest eigenvalue of $H = L + D$ is $\lambda_{Max} = 3.4142$ and let $\gamma = 3$. In simulation, we choose $\alpha = 0.8$ and the initial conditions of leader system as $(0.5, 2, 1)$ and the initial conditions of agents sytstem as $(-4, 3, 3)$, $(-2, 1.5, 3)$, $(4, 0, 1.5)$. Under the control law 3.3, the state trajectories of x_i, y_i, z_i, $i = 0, 1, 2, 3$ are shown in figure 2. Then, the consensus errors are provided in figure 3. We can see that three agents follow the leader.

Example 4.2. Consider a multi-agent of chaotic fractional-order consisting of a leader and three agents. They are the state variables of the leader and state variables of the agents satisfying

$$\begin{cases} D^{\alpha} x_0 = a x_0 + e x_0^2 - x_0 y_0 - s z_0 x_0^2 \\ D^{\alpha} y_0 = -c y_0 + d x_0 y_0 \\ D^{\alpha} z_0 = -p z_0 + s z_0 x_0^{,} \end{cases} \tag{4.4}$$

$$\begin{cases} D^{\alpha} x_i = -a_1 x_i + y_i + 10 y_i z_i \\ D^{\alpha} y_i = -x_i - 0.4 y_i + 5 x_i z_i \\ D^{\alpha} z_i = -5 x_i y_i + b_1 z_i, \end{cases} \tag{4.5}$$

where $a = 1$, $e = 2$, $b = 1$, $s = 2.7$, $c = 1$, $d = 1$, $p = 3$ and $a_1 = 0.4$, $b_1 = 0.175$.

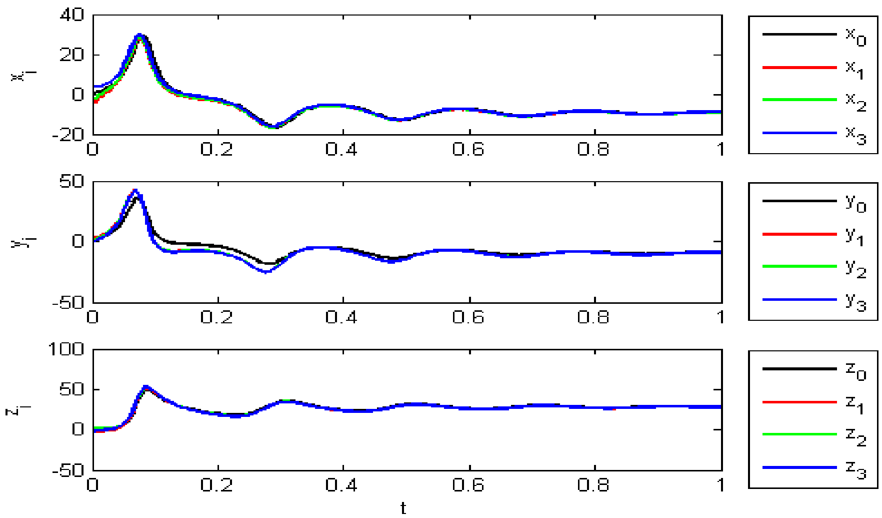
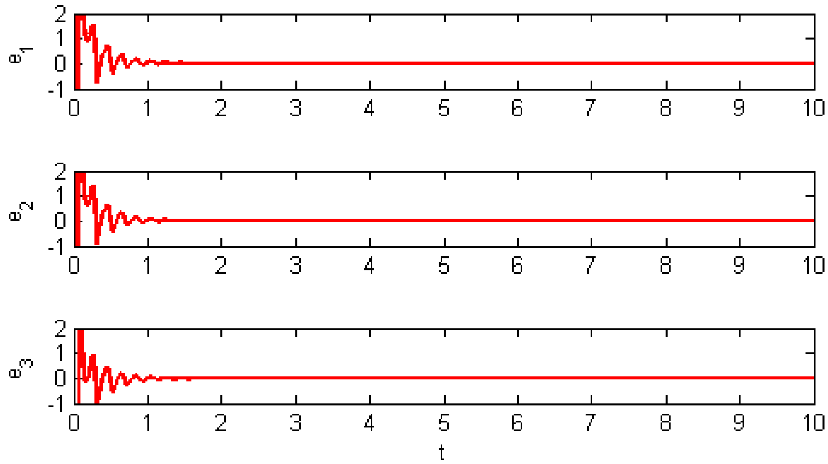
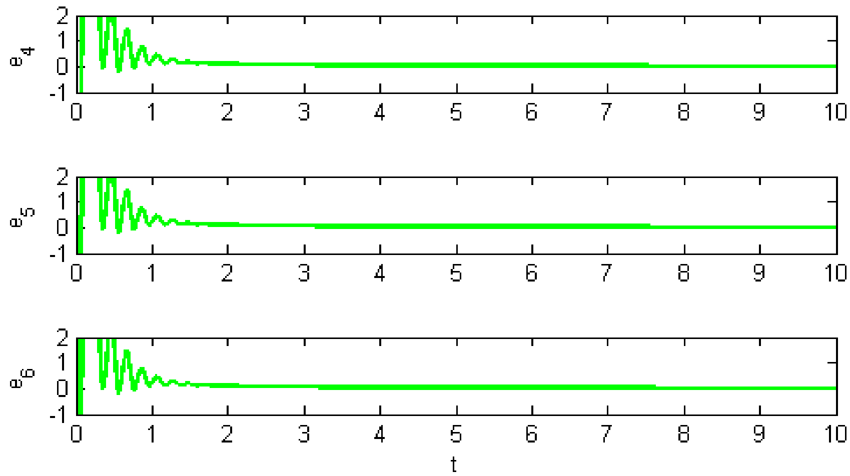


FIGURE 2. The state trajectories of x_i, y_i, z_i, $i = 0, 1, 2, 3$ with $\gamma = 3$ in example 4.1.

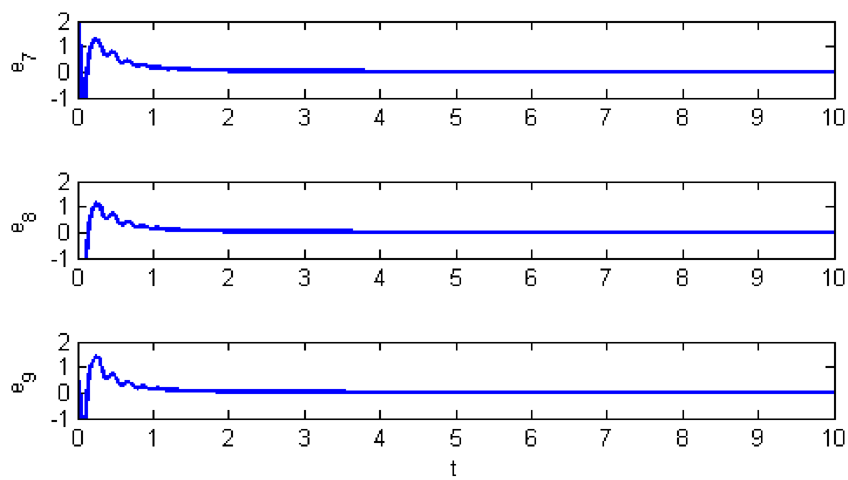
Assume the Lotka-Voltra system 4.4 is the leader system [35] and the Newton-Leipnik system 4.5 are the agent systems [36]. Also, suppose the topology is described as in figure 1, then, the Laplacian L and the matrix D are the same as 4.3. In simulation, we choose $\alpha = 0.99, \gamma = 2$ and the initial conditions of leader sytstem as $(-1, 2.1, 0)$ and the initial conditions of agents sytstem as $(2.19, -0.5, 0.18)$, $(-4, -1, 0.4)$, $(1, 4, -3)$. Under the control law3.3, the state trajectories of x_i, y_i, z_i, $i = 0, 1, 2, 3$ are shown in figure 4. Then,the consensus errors are provided in figure 5.



(a) The error trajectories of v0 and v1



(b) The error trajectories of v0 and v2.



(c) The error trajectories of v0 and v3.

FIGURE 3. The error trajectories of leader and agents in example 4.1.

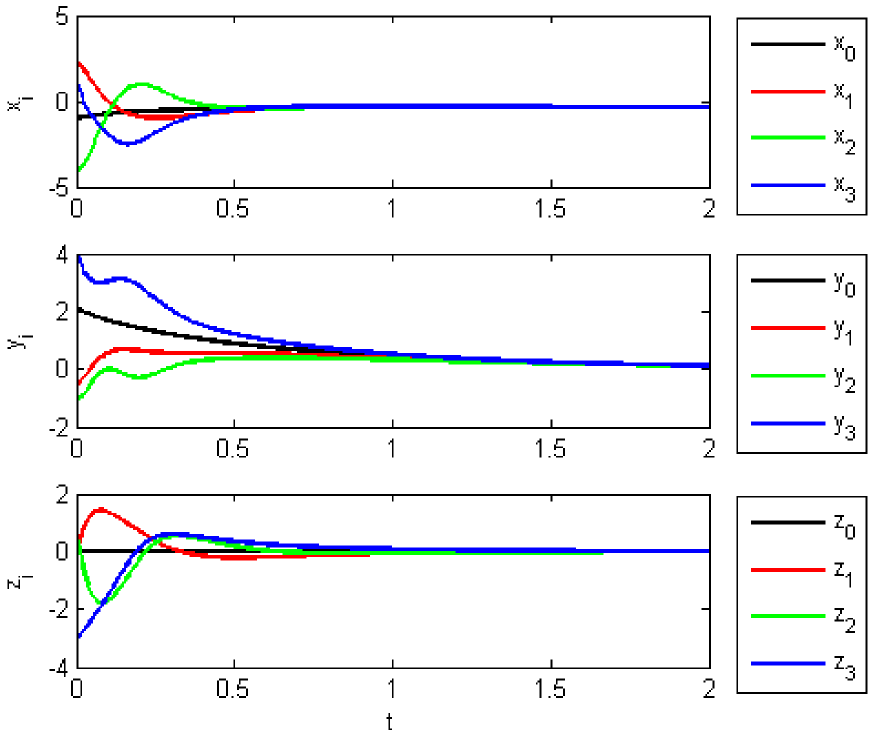
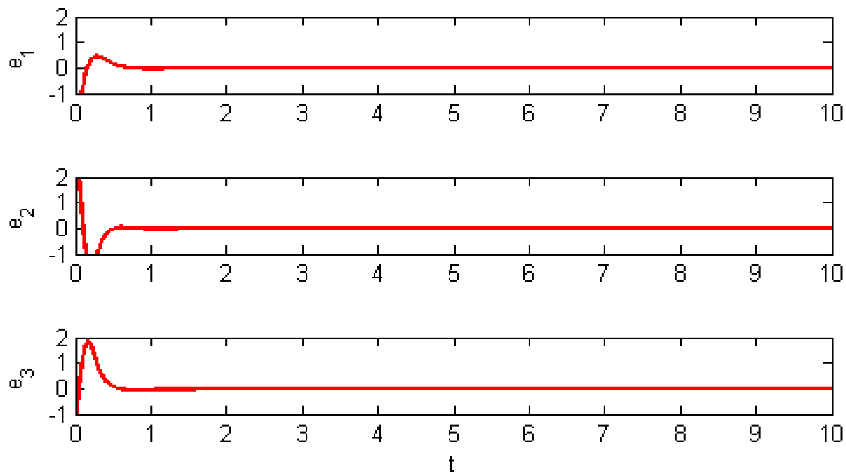
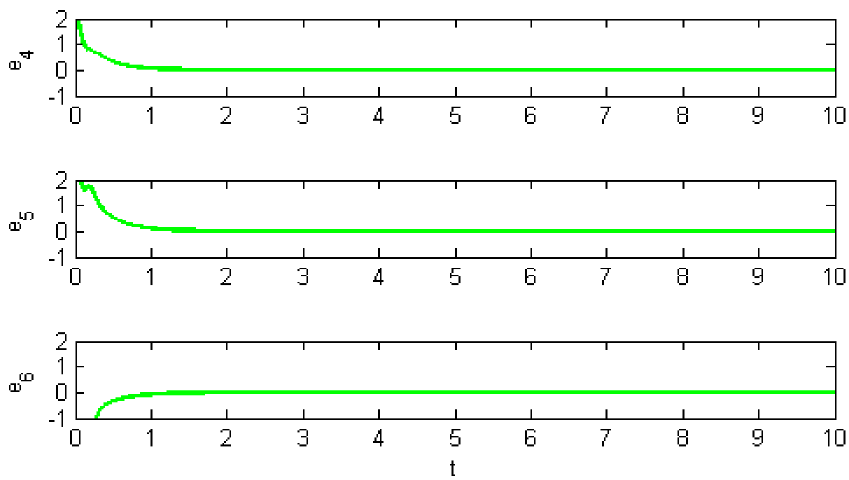


FIGURE 4. The state trajectories of x_i, y_i, z_i, $i = 0, 1, 2, 3$ with $\gamma = 2$ in example 4.2..

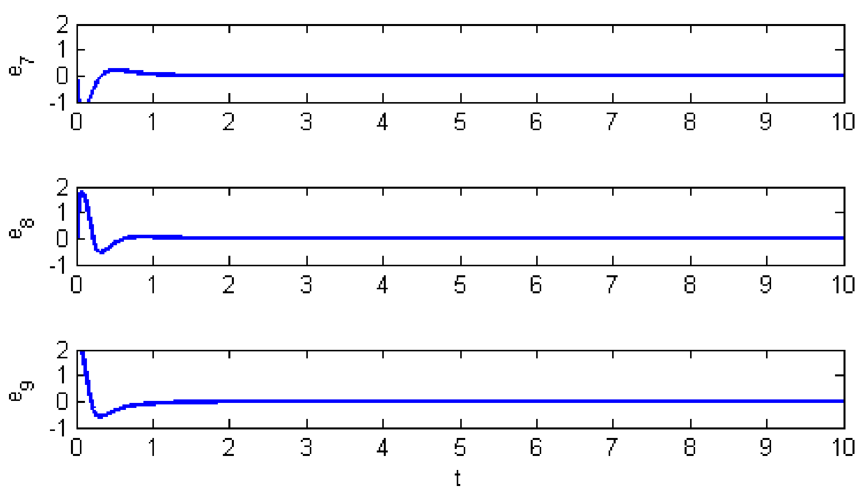
Now, according eigenvalues of H are $\{0.5858, 2, 3.4142\}$ in the above example, if we suppose $\gamma < -\frac{1}{\lambda_{min}}$ for example $\gamma = -2$ we will see that the followers do not follow the leader, which is shown in the figure 6. And this example is presented as a proof of corollary 3.4



(a) The error trajectories of v0 and v1



(b) The error trajectories of v0 and v2.



(c) The error trajectories of v0 and v3.

FIGURE 5. The error trajectories of leader and agents in example 4.2.

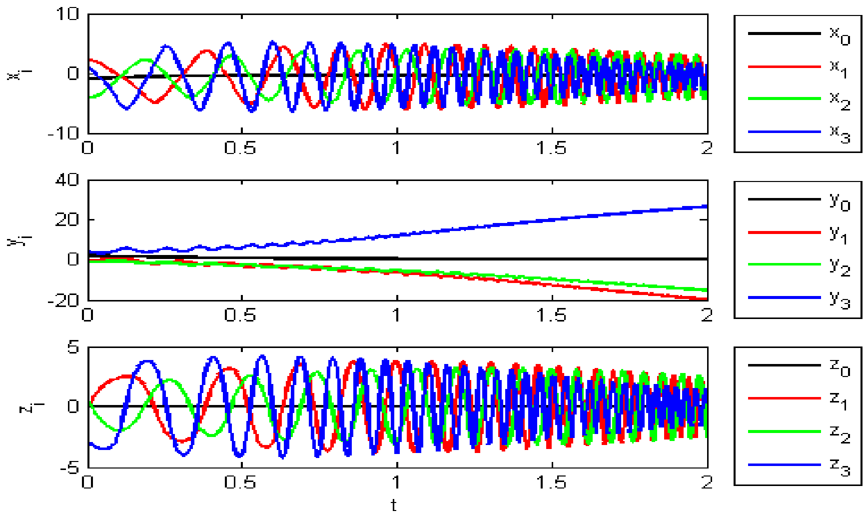


FIGURE 6. The state trajectories of $x_i, y_i, z_i, i = 0, 1, 2, 3$ with $\gamma = -2$ in example 4.2.

5. Conclusion

In this paper, we investigated the leader-following consensus for chaotic fractional-order multi- agent systems with adaptive protocols via an undirected fixed interaction graph. It is interesting to find that the consensus tracking can be attained via only using the position information among the agent and its neighbors for $\alpha \in (0, 1)$.A distributed adaptive protocol is used nonlinear cases, under which consensus is reached for fractional multi-agent systems. Based on the stability theory of fractional-order differential system a sufficient condition ensuring the leader-following consensus of the model is given. Finally, two examples are given to demonstrate the effectiveness of the proposed method.

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