Caspian Journal of Mathematical Sciences (CJMS)
University of Mazandaran, Iran
http://cjms.journals.umz.ac.ir
ISSN: 2676-7260
CJMS. **11**(2)(2022), 495-507 (Research paper)

Hypergroups associated to dominating sets

Tahere Nozari [1] and Fateme Movahedi
Department of Mathematics,
Faculty of Sciences, Golestan University, Gorgan, Iran

Abstract. The study of hyperstructures derived from particular mathematical objects is very important and interesting. Graph theory has been established as a fundamental and important tool for solving practical problems in other branches of mathematics. This paper can be considered as one of the connections between hyperstructures and graph theory. In this way, using the dominating set notion of a graph, we define a hyperoperation on vertices of graph and study its properties and then we construct a hypergroup based on this hyperoperation. This hypergroup is presented for some classes of graphs.

Keywords: Semihypergroup, hypergroup, graph, domonating set.

2020 Mathematics subject classification: 43A62; 05C69

1. Introduction and Preliminaries

The theory of hyperstructures is introduced in 1934 by Marty [8]. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Basic definitions and propositions about the hyperstructures theory are found in [2]. Nowadays, hyperstructures are studied from the theoretical point of view because they are helpful in many subjects of pure and applied mathematics such as geometry, topology, graph and hypergraph, the theory of fuzzy sets, etc. A recent book on

[1]Corresponding author: t.nozari@gu.ac.ir
Received: 19 September 2020
Revised: 21 January 2022
Accepted: 01 February 2022

these topics is " Applications of Hyperstructure Theory" by P. Corsini and V. Leoreanu [3].

Since the last century, Graph Theory has been an important mathematical tool in different fields like Algebra, Geometry and so on. An important concept in graph theory is the study of the dominating sets in a graph. A dominating set is a subset D of the vertices set of a graph such that every vertex is either in D or adjacent to a vertex in D. Domination in graphs has been extensively researched as one of the branches of graph theory. In 1958, for the first time, the concept of the domination number of a graph was defined [1]. The studies of dominating sets in graph theory began around 1960. Domination theory has many applications in sciences and technology [4]. An excellent treatment of the fundamentals of domination can be found in the book by Haynes et. al [4]. A survey of several advanced topics in domination is given in the book by Haynes et al. [5]. The aim of this paper is to provide examples of hypergroups in graph theory by dominating sets.

2. Preliminaries

Let H be a set and $P^*(H)$ be the family of all nonempty subsets of H and $\circ$ a *binary hyperoperation* or *join operation*, that is $\circ$, is a map from $H \times H$ to $P^*(H)$. If $(a, b) \in H \times H$, its image under $\circ$ is denoted by $a \circ b$ or ab. A join operation can be extended to subsets of H in a natural way so that $A \circ B$ or AB is given by $AB = \cup\{ab \mid a \in A, b \in B\}$. The notions aA and Aa are used for $\{a\}A$ and $A\{a\}$ respectively. Generally, the singleton $\{a\}$ is identified by its element a.
A hypergroupoid is a hyperstructure $(H, \circ)$. A hypergroupoid is called a quasihypergroup if for all $a \in H$ we have $a \circ H = H \circ a = H$ and it is called a semihypergroup if $x \circ (y \circ z) = (x \circ y) \circ z$, $\forall x, y, z \in H$.

Definition 2.1. A hypergroupoid $(H, \circ)$ which is both a semihypergroup and a quasihypergroup is called a hypergroup.

H_v-structures are a generalization of algebraic hyperstructures such that some axioms concerning the hyperstructures like associative law, the distributive law and so on are replaced by their corresponding weak axioms [9].

Definition 2.2. A hypergroupoid $(H, \circ)$ is called a H_v- semigroup and also a quasihypergroup $(H, \circ)$ is called a H_v- group if for all $x, y, z \in H$, $x \circ (y \circ z) \cap (x \circ y) \circ z \neq \varnothing$.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. Let, for any vertex $v \in V(G)$, $N_G(v) = \{u \in V | uv \in E(G)\}$ denote the neighbours of the vertex v. A leaf of G is a vertex of degree 1, while a support vertex of G is a vertex adjacent to a leaf. A dominating set of

a graph G is a set D of vertices of G such if every vertex not in D is adjacent to a vertex in D. The domination number of G, denoted by $\gamma(G)$, is the minimum size of dominating set of G.
For example, given the graph G shown in Figure 1. The vertex set is labeled as $\{1, 2, 3, 4, 5, 6\}$. A dominating set of graph G is $D = \{3, 5\}$.

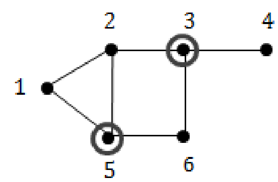


FIGURE 1. A dominating set of graph G.

There are several different ways for computing the dominating set with cardinality $\gamma(G)$. Therefore, the dominating set with cardinality $\gamma(G)$ is not unique in graph G. For example, in Figure 1, another dominating set of graph G can select $\{1, 3\}$.

We recall the domination number for certain graphs in the following Lemma.

Lemma 2.3. [1] *Let P_n and C_n be the path and cycle of order n, respectively. Then,*

$$\gamma(P_n) = \gamma(C_n) = \lceil \frac{n}{3} \rceil.$$

3. EXAMPLES ON SOME CERTAIN GRAPHS

In this section, we propose examples of hypergroups in dominating set theory by investigating the dominating set of two certain graphs as paths and cycles and some families of graphs, namely, Corona graph and Helm graph.

Suppose that G is a graph, V is a non-empty vertex set of $V(G)$ and $\mathcal{P}^*(V)$ is the set of all non-empty subsets of $V(G)$ and D be a fixed dominating set of graph G with $|D| = \gamma(G)$.

We consider a map $f : V \times V \to \mathcal{P}^*(V)$ and obtain examples of hypergroups in some certain graphs by this map.
Let graph P_n be a path of order $n \geq 2$. The following result is clearly for $n = 2, 3$.

Example 3.1. Let us consider P_2 and P_3. We set $D_1 = \{1\}$ and $D_2 = \{2\}$ as dominating sets, respectively. For any couple (x, y) of vertices of $P_n (n = 2, 3)$, define $f(x, y) = D_n$. Then (P_2, f) and (P_3, f) are semihypergroups.

□

FIGURE 2. The labeled graph path P_n.

According to Figure 2, the set $\{v_1, \ldots, v_n\}$ is the vertices set of path P_n of order $n \geq 4$. Using Lemma 2.3, since $\gamma(P_n) = \lceil \frac{n}{3} \rceil$, one can consider the dominating set of graph P_n for different cases of n in the following:

(i) If $n \equiv 0 \,(mod\, 3)$, then $D = \bigcup_{i=0}^{\lceil \frac{n}{3} \rceil - 1} \{v_{3i+2}\}$.

(ii) If $n \equiv 1 \,or\, 2 \,(mod\, 3)$, then $D = \bigcup_{i=0}^{\lceil \frac{n}{3} \rceil - 1} \{v_{3i+1}\}$.

Now, we have the following result.

Existence of a hyperoperation via a specific dominating set in graph P_n:
Let P_n be the path of order $n \geq 4$. Then, there is a well-defined binary hyperoperation f on the vertices set $V(P_n)$, such that for any $x, y \in V(P_n)$, $0 < |f(x, y)| \leq 2$.

(i) Assume $n \equiv 0 \,(mod\, 3)$. So $D = \bigcup_{i=0}^{\lceil \frac{n}{3} \rceil - 1} \{v_{3i+2}\}$ is a dominating set and $|D| = \lceil \frac{n}{3} \rceil$. According to the structure of D, since the distance of any two vertices in D is 3, there is not any vertex in graph P_n that is adjacent to two vertices in D. Thus for any $x, y \in V(P_n)$, we have the following cases.
Case 1: If $x, y \in D$, then set $f(x, y) = \{x, y\}$.
Case 2: If $x \in D$ and $y \notin D$, then according to the definition of D, the vertex y is dominated by just one of the vertices in D. Thus, there is the vertex $a \in D$ in a way that a is adjacent to y. Then set, $f(x, y) = \{x, a\}$. If $a = x$, then $|f(x, y)| = 1$, otherwise $|f(x, y)| = 2$.
Case 3: If $x \notin D$ and $y \in D$, then similar to the last case , for

$x \notin D$ there is the vertex $b \in D$ so that b is adjacent to x and set $f(x, y) = \{y, b\}$, $0 < |f(x, y)| \leq 2$.

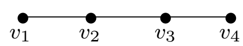
Case 4: If $x, y \notin D$, then according to the definition of D, the vertex y is dominated by just one of the vertices in D like b and the vertex x is dominated by just one of the vertices in D like a, therefore set $f(x, y) = \{a, b\}$. If $a = b$, $|f(x, y)| = 1$, otherwise $|f(x, y)| = 2$.

(ii) Assume $n \equiv 1\, or\, 2\, (mod\, 3)$. So, consider $D = \bigcup_{i=0}^{\lceil \frac{n}{3} \rceil - 1} \{v_{3i+1}\}$ as dominating set of P_n. We consider the above cases for this n and it is easy to show that $0 < |f(x, y)| \leq 2$.

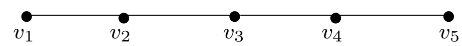
Therefore, the result completes.

□

Example 3.2. Let us consider the following graphs:



P_4



P_5

For both of these graphs, we consider $D = \{v_1, v_4\}$, therefore we obtain these tables:

TABLE 1. (P_4, f)

f	v_1	v_2	v_3	v_4
v_1	v_1	v_1	v_1, v_4	v_1, v_4
v_2	v_1	v_1	v_1, v_4	v_1, v_4
v_3	v_1, v_4	v_1, v_4	v_4	v_4
v_4	v_1, v_4	v_1, v_4	v_4	v_4

TABLE 2. (P_5, g)

g	v_1	v_2	v_3	v_4	v_5
v_1	v_1	v_1	v_1, v_4	v_1, v_4	v_1, v_4
v_2	v_1	v_1	v_1, v_4	v_1, v_4	v_1, v_4
v_3	v_1, v_4	v_1, v_4	v_4	v_4	v_4
v_4	v_1, v_4	v_1, v_4	v_4	v_4	v_4
v_5	v_1, v_4	v_1, v_4	v_4	v_4	v_4

It is not difficult to check that (P_4, f) and (P_5, g) are semihypergroups and $(P_4, f|_D)$ and $(P_5, g|_D)$ are hypergroups.

□

We propose another certain graph. Let graph C_n be a cycle of order $n \geq 3$ with the vertices v_i for $i = 1, \dots, n$ (see Figure 3). One can consider the dominating set of graph C_n as the set $D = \bigcup_{i=0}^{\lceil \frac{n}{3} \rceil -1} \{v_{3i+1}\}$. The following result is clearly in the cycle C_n for $n = 3, 4, 5$.

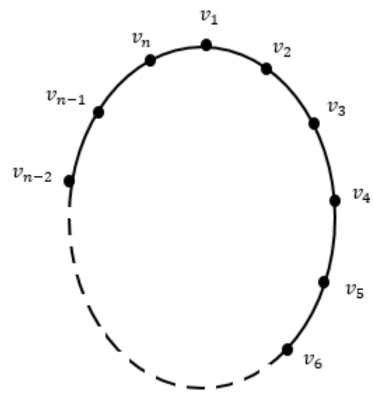


FIGURE 3. The labeled graph path C_n.

Existence of a hyperoperation via a specific dominating set in graph C_n:

Let C_n be the cycle of order $n \geq 3$. Then, there is a well-defined binary hyperoperation f on the vertices set $V(C_n)$, such that for any $x, y \in V(C_n)$,

(i) if $n = 3, 4, 5$, then $0 < |f(x, y)| \leq 2$,

(ii) if $n \geq 6$ and $n \equiv 0 \, (mod \, 3)$, then $0 < |f(x, y)| \leq 2$,

(iii) if $n \geq 6$ and $n \equiv 1 \, or \, 2 \, (mod \, 3)$, then $0 < |f(x, y)| \leq 3$.

If $n = 3$, then according to Figure 3, consider $D = \{v_1\}$ as dominating set and define $f(x, y) = D$, for any couple (x, y) of vertices of C_3. Assume $n = 4, 5$. In these cases, set $D = \{v_1, v_4\}$ and according to Tables 3 and 4, f is a well defined and $0 < |f(x, y)| \leq 2$.

TABLE 3. (C_4, f).

f	v_1	v_2	v_3	v_4
v_1	v_1	v_1	v_1, v_4	v_1, v_4
v_2	v_1	v_1	v_1, v_4	v_1, v_4
v_3	v_1, v_4	v_1, v_4	v_4	v_4
v_4	v_1, v_4	v_1, v_4	v_4	v_4

TABLE 4. (C_5, f).

f	v_1	v_2	v_3	v_4	v_5
v_1	v_1	v_1	v_1, v_4	v_1, v_4	v_1, v_4
v_2	v_1	v_1	v_1, v_4	v_1, v_4	v_1, v_4
v_3	v_1, v_4	v_1, v_4	v_4	v_4	v_1, v_4
v_4	v_1, v_4	v_1, v_4	v_4	v_4	v_1, v_4
v_5	v_1, v_4	v_1, v_4	v_1, v_4	v_1, v_4	v_1, v_4

For $n \geq 6$, let $D = \bigcup_{i=0}^{\lceil \frac{n}{3} \rceil - 1} \{v_{3i+1}\}$ as dominating set. Using Lemma 2.3, $|D| = \lceil \frac{n}{3} \rceil$.

(ii) If $n \equiv 0\,(mod\,3)$, since the distance of any two vertices in D is 3, there is not any vertex in graph C_n that is adjacent to two vertices in D. Thus, in this case, any vertex of $V(C_n)$ is adjacent to only one vertex of D. Therefore, it is the same as graph P_n, for different cases, $0 < |f(x, y)| \leq 2$.

(iii) Assume $n \equiv 1\,(mod\,3)$. According to the definition of set D, $\{v_1, v_n\} \in D$. For any couple $(v_i, v_j) \in V(C_n)$ we study the following cases.

Case 1: Let $v_1, v_n \in V(C_n)$. Then, set $f(v_1, v_n) = \{v_1, v_n\}$.

Case 2: We consider the couple $(v_1, v_j) \in V(C_n)$ such that $v_j \neq v_n$ or $(v_i, v_n) \in V(C_n)$ where $v_i \neq v_1$. Without loss of generality, we suppose $(v_1, v_j) \in V(C_n)$ such that $v_j \neq v_n$. In such a case, if $v_j \in D$ then set, $f(v_1, v_j) = \{v_1, v_j, v_n\}$. Let $v_j \notin D$ and $v_j = v_2$, then $f(v_1, v_j) = \{v_1, v_n\}$. Otherwise, there is the vertex $a \in D$ that a is adjacent to v_j. So, we can consider the set $f(v_1, v_j) = \{v_1, a, v_n\}$.

Case 3: If $v_i, v_j \notin D$. Then, there are $a, b \in D$ such that the vertices a and b dominate v_i and v_j, respectively. Then set $f(v_i, v_j) = \{a, b\}$. If $a = b$, then $|f(v_i, v_j)| = 1$. Otherwise, $|f(v_i, v_j)| = 2$.

Case 4: If at least one of v_i and v_j are in D, then with a similar discussion $|f(v_i, v_j)| \leq 2$.

(iii) Assume $n \equiv 2\,(mod\,3)$. According to the definition of set D, $\{v_1, v_{n-1}\} \in D$. Since, for any vertex $v_i \in V(C_n)$, there is a vertex $v_k \in D$ in a way that v_i is adjacent to v_k. First consider couple (v_i, v_n) for $v_i \in V(C_n)$. There are the following cases.
Case 1: If $v_i \neq v_1, v_{n-1}$, then there is the vertex $a \in D$ such that a is adjacent to v_i. Set, $f(v_i, v_n) = \{v_1, v_{n-1}, a\}$.
Case 2: Suppose that $v_i = v_1$ or $v_i = v_{n-1}$. Without loss of generality, we consider the couple (v_1, v_n). In this case, set $f(v_1, v_n) = \{v_1, v_{n-1}\}$.
Other cases similar to (ii) are proved.

So, the result is completed. □

Example 3.3. Let consider C_4, C_5 and C_6. For these cycles, $D = \{v_1, v_4\}$ is dominating set. Clearly $(C_4, f), (C_5, f)$ and (C_6, f) are H_v-groups and $(D, f|_D)$ in each case is a hypergroup. Also about C_7, C_8 and C_9, the set $D = \{v_1, v_4, v_7\}$ is the dominating set. It is not difficult to check that $(C_7, f), (C_8, f)$ and (C_9, f) are semihypergroups and $(D, f|_D)$ in each case is a hypergroup.

□

Corollary 3.4. *For every $C_n (n \geq 3)$, a dominating set is given in table 5. It is obviously for every C_n, (C_n, f) is a H_v- semigroup and $(D, f|_D)$ is a hypergroup. As a sample for C_{16}, C_{17}, C_{18}, the hypergroup $(D = \{v_1, v_4, v_7, v_{10}, v_{13}, v_{16}\}, f|_D)$ is shown in table 6. By this method for every $n \geq 3$, we obtain a commutative hypergroup or a commutative H_v- group with n elements.*

Table 5. The dominating sets of cycles.

Cycles	*D= Dominatig set*
C_3	v_1
C_4, C_5, C_6	v_1, v_4
C_7, C_8, C_9	v_1, v_4, v_7
C_{10}, C_{11}, C_{12}	v_1, v_4, v_7, v_{10}
C_{13}, C_{14}, C_{15}	$v_1, v_4, v_7, v_{10}, v_{13}$
C_{16}, C_{17}, C_{18}	$v_1, v_4, v_7, v_{10}, v_{13}, v_{16}$
$\vdots$	$\vdots$

TABLE 6. The hypergroup derived of C_{16}.

f	v_1	v_4	v_7	v_{10}	v_{13}	v_{16}
v_1	v_1, v_{16}	v_1, v_4, v_{16}	v_1, v_7, v_{16}	v_1, v_{10}, v_{16}	v_1, v_{13}, v_{16}	v_1, v_{16}
v_4	v_1, v_4, v_{16}	v_4	v_4, v_7	v_4, v_{10}	v_4, v_{13}	v_1, v_4, v_{16}
v_7	v_1, v_7, v_{16}	v_4, v_7	v_7	v_7, v_{10}	v_7, v_{13}	v_1, v_7, v_{16}
v_{10}	v_1, v_{10}, v_{16}	v_4, v_{10}	v_7, v_{10}	v_{10}	v_{10}, v_{13}	v_1, v_{10}, v_{16}
v_{13}	v_1, v_{13}, v_{16}	v_4, v_{13}	v_7, v_{13}	v_{10}, v_{13}	v_{13}	v_1, v_{13}, v_{16}
v_{16}	v_1, v_{16}	v_1, v_4, v_{16}	v_1, v_7, v_{16}	v_1, v_{10}, v_{16}	v_1, v_{13}, v_{16}	v_1, v_{16}

□

The Corona of two graphs G_1 and G_2 is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2, where the ith vertex of G_1 is adjacent to every vertex in the ith copy of G_2. The Corona $G \circ K_1$, in particular, is the graph constructed from a copy of G, where for each vertex $v \in V(G)$, a new vertex v' and a pendant edge vv' are added [6]. We consider the Corona graph $P_n \circ K_1$.

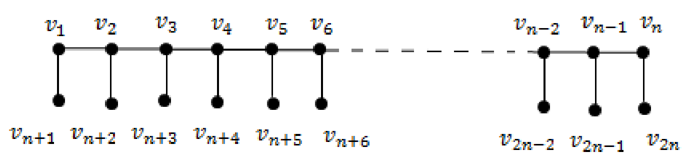


FIGURE 4. The labeled graph $P_n \circ K_1$.

Existence of a hyperoperation via a specific dominating set in graph $P_n \circ K_1$:

Let G_n be the Corona $P_n \circ K_1$ for $n \geq 6$. Then, there is a well-defined binary hyperoperation f on the vertices set $P_n \circ K_1$, such that for any $x, y \in V(P_n \circ K_1)$, $2 \leq |f(x, y)| \leq 6$.

According to Figure 4, the vertices of the graph G_n are labeled with v_i for $i = 1, 2, \ldots, 2n$ so that the graph G_n contains n vertices leaves. Since all of the vertices of degree 1 must be dominated by the dominating set, one can select the set $D = \{v_1, v_2, \ldots, v_n\}$ as the dominating set of the graph G_n. It is clear to see that $|D| = n$. For every couple (x, y) of $V(G_n)$, there are the following cases.

Case 1: If $x, y \notin D$, then the vertices x and y are end-vertices of

two different pendant edges. So, there are the vertices $a, b \in D$ that dominate these two vertices. Then set, $f(x, y) = \{a, b\}$.
Case 2: If $x \notin D\,(or\, y \notin D)$ and $y \in D\,(or\, x \in D)$. Then $f(x, y)$ contains y (*or* x) and all neighbours of x, y that are in D. It is easy to investigate for different cases of the position of vertex y (*or* x) $\in D$ on the path P_n and the number of common neighbours of x and y in D, $|f(x, y)| = 2, 3,$ *or* 4
Case 3: If $x, y \in D$, $f(x, y)$ contains x, y and all neighbours of x, y that are in D. Then according to neighbours of vertices x and y on the path P_n, there are the following cases.

(i) If $x = v_1$ and $y = v_n$ and x and y don't have any common neighbours, then $|f(x, y)| = 4$.
(ii) If $x \neq v_1$ or $y \neq v_n$ and x and y have common neighbours, then $|f(x, y)| = 5$.
(iii) If $x \neq v_1$ and $y \neq v_n$ and x and y don't have any common neighbours, then $|f(x, y)| = 6$.

For other cases, it is easy to see that $2 \leq |f(x, y)| \leq 6$. This completes the proof. □

If we apply this method for the vertices set $V(P_n \circ K_1)$ where $2 \leq n \leq 5$, then for couple $(x, y) \in V(P_n \circ K_1)$, $2 \leq |f(x, y)| \leq 5$.

Example 3.5. Let G_n be the Corona $P_n \circ K_1$. The dominating set for every $G_n (n \in \mathbb{N})$, is given in table 7. It is easy to check, every (G_n, f) is a quasi-hypergroup and $(D, f|_D)$ is a H_v-hypergroup. As a sample for G_6, the H_v-group $(D = \{v_1, v_2, v_3, v_4, v_5, v_6\}, f|_D)$ is shown in table 8. By this method for every $n \in \mathbb{N}$, we obtain a commutative H_v-group or a hypergroup with n elements.

Table 7. The dominating sets of Corona graphs.

Corona graphs	D= Dominatig set
G_1	v_1
G_2	v_1, v_2
G_3	v_1, v_2, v_3
$\vdots$	$\vdots$
G_n	$v_1, v_2, v_3, \ldots, v_n$

□

Now, we consider another family of graphs as the Helm graph. The helm graph H_n is the graph obtained from a wheel graph with n vertices

TABLE 8. The hypergroup derived of G_6.

f	v_1	v_2	v_3	v_4	v_5	v_6
v_1	v_1, v_2	v_1, v_2, v_3	v_1, v_2, v_3, v_4	v_1, v_2, v_3, v_4, v_5	v_1, v_2, v_4, v_5, v_6	v_1, v_2, v_5, v_6
v_2	v_1, v_2, v_3	v_1, v_2, v_3	v_1, v_2, v_3, v_4	v_1, v_2, v_3, v_4, v_5	$v_1, v_2, v_3, v_4, v_5, v_6$	v_1, v_2, v_3, v_5, v_6
v_3	v_1, v_2, v_3, v_4	v_1, v_2, v_3, v_4	v_2, v_3, v_4	v_2, v_3, v_4, v_5	v_2, v_3, v_4, v_5, v_6	v_2, v_3, v_4, v_5, v_6
v_4	v_1, v_2, v_3, v_4, v_5	v_1, v_2, v_3, v_4, v_5	v_2, v_3, v_4, v_5	v_3, v_4, v_5	v_3, v_4, v_5, v_6	v_3, v_4, v_5, v_6
v_5	v_1, v_2, v_4, v_5, v_6	$v_1, v_2, v_3, v_4, v_5, v_6$	v_2, v_3, v_4, v_5, v_6	v_3, v_4, v_5, v_6	v_4, v_5, v_6	v_4, v_5, v_6
v_6	v_1, v_2, v_5, v_6	v_1, v_2, v_3, v_5, v_6	v_2, v_3, v_4, v_5, v_6	v_3, v_4, v_5, v_6	v_4, v_5, v_6	v_5, v_6

by adjoining a pendant edge at each vertex of the cycle [7]. According to Figure 5, vertices of H_n are labeled with v_i for $i = 1, 2, \cdots, 2n-1$ in a way that graph H_n contains $n-1$ leaves, cycle C_{n-1} and the central vertex v_{2n-1}.

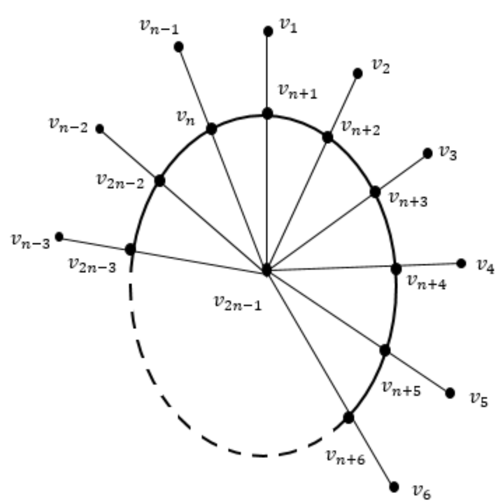


FIGURE 5. The labeled graph H_n.

At first, we obtain the domination number of graph H_n in following Lemma.

Lemma 3.6. *Let H_n be Helm graph. Then, the domination number of H_n equals:*

$$\gamma(H_n) = n - 1.$$

Proof. Let H_n be Helm graph of order $2n-1$ with the set L contains $n-1$ leaves with the vertices $\{v_1, \ldots, v_{n-1}\}$, cycle C_{n-1} with the vertices set $\{v_n, \ldots, v_{2n-2}\}$ and the central vertex v_{2n-1} and also D be the dominating set of graph H_n. So, one can select the set $\{v_n, \ldots, v_{2n-2}\}$ on cycle C_{n-1} of graph H_n as the dominating set. Thus, $|D| \leq n-1$. Assume, $|D| \leq n-2$. Since graph H_n contains $n-1$ leaves, there is at least one leaf in the graph that cannot be dominated by D. Thus, it is a contradiction and we have $|D| = n-1$. □

According to the definition of map f, we have the following result.

Existence of a hyperoperation via a specific dominating set in graph H_n:

Let H_n be the helm graph of order $2n-1$ for $n \geq 4$. Then, there is a well-defined map f on the vertices set of H_n such that for $x, y \in H_n$, $0 < |f(x, y)| \leq n-1$.

For every couple (x, y) of $V(H_n)$, there are the following cases.

Case 1: If $x, y \in D$, then $f(x, y)$ contains x, y and all of neighbours of x and y that are in D.

Case 2: If $x \in D\,(or\, y \in D)$ and $y \notin D\,(or\, x \notin D)$, then $f(x, y)$ contains $x(\,or\, y)$ and all of neighbours of x and y that are in D.

Case 3: If $x, y \notin D$, then $f(x, y)$ contains all of neighbours of x and y that are in D.

On the other hand, for couple (v_{2n-1}, v_i) that $v_i \in V(H_n)$, we consider $f(v_{2n-1}, v_i) = \{v_n, \ldots, v_{2n-2}\} = D$. Because the central vertex v_{2n-1} is adjacent to all of the vertices on C_{n-1} in graph H_n. Thus $|f(v_{2n-1}, v_i)| = n-1$.

□

Using this procedure, we present some examples of hyperstructures by a map f on the vertices set of graph H_n.

Example 3.7. The dominating set for $H_n (n \geq 4)$ is given in table 9. It is easy to check, every (H_n, f) is a H_v- semigroup, and $(D, f|_D)$ is a hypergroup.

TABLE 9. The dominating sets of H_n.

Helm graphs	D= Dominatig set
H_4	v_4, v_5, v_6
H_5	v_5, v_6, v_7, v_8
$\vdots$	$\vdots$
H_n	$v_n, v_{n+1}, \ldots, v_{2n-2}$

As a sample we consider the Helm graph H_5. There is 9 vertices, $V = \{v_i : 1 \leq i \leq 9\}$, and in this case $D = \{v_5, v_6, v_7, v_8\}$ and $|D| = 4$. The hypergroup $(D = \{v_5, v_6, v_7, v_8\}, f|_D)$ is shown in table 10. By this method for every $n \in \mathbb{N}$, we obtain a commutative hypergroup or a commutative H_v- group with n elements.

TABLE 10. The hypergroup derived of H_5.

f	v_5	v_6	v_7	v_8
v_5	v_5, v_6, v_8	v_5, v_6, v_7, v_8	v_5, v_6, v_7, v_8	v_5, v_6, v_7, v_8
v_6	v_5, v_6, v_7, v_8	v_5, v_6, v_7	v_5, v_6, v_7, v_8	v_5, v_6, v_7, v_8
v_7	v_5, v_6, v_7, v_8	v_5, v_6, v_7, v_8	v_6, v_7, v_8	v_5, v_6, v_7, v_8
v_8	v_5, v_6, v_7, v_8	v_5, v_6, v_7, v_8	v_5, v_6, v_7, v_8	v_5, v_7, v_8

□

REFERENCES

[1] C. Berge, Theory of Graphs and its applications, *Methuen, London, 1962.*

[2] P. Corsini, Prolegomena of hypergroup Theory, *second eddition Aviani, editor, 1993.*

[3] P. Corsini, V. Leoreanu, Applications of hyperstructure theory, *Kluwer Academic Publications, 2003.*

[4] T. W. Haynes, S. T. Hedetniemi, P. J. Slater, Fundamentals of Domination in Graphs, *Marcel Dekker Inc., New York, 1998.*

[5] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs - Advanced Topics, *Marcel Dekker Inc., New York, 1998.*

[6] R. Frucht and F. Harary, On the corona of two graphs, *Aequationes Math. 4 (1970) 322-324.*

[7] J. A. Gallian, Dynamic Survey DS6: Graph Labeling, *Electronic J. Combinatorics, 6 (2007) 1-58.*

[8] F. Marty, Surnue generalization de la notion de group, 8^i*em course Math. Scandinaves Stockholm (1934) 45-49.*

[9] T. Vougiuklis, Hyperstructures and their representations, *Hardonic, Press, Inc. (1994).*